Comparing Hg_{199} and Tl, the magnetic moment of Hg_{199} should be that of a neutron, while that of Tl is due to a proton. If now the neutron is thought of as a sphere of positive electricity imbedded in a sphere of negative electricity which is very much larger, the entire structure rotating with one angular velocity, and the moment of the whole being $\frac{1}{2}\, h/2\pi$, its magnetic moment will be of the same order as that of the proton, and the approximate equality of the $g(I)$ factors of Hg_{199} and Tl become intelligible.

Next considering Tl and Pb, the term corresponding to the spin $\frac{1}{2}$ in the case of Pb may be $^4D_{\frac{1}{2}}$ or $^4P_{\frac{1}{2}}$ or $^2P_{\frac{1}{2}}$, so that the g factor may be 0 or 8/3 or 2/3. If the term is taken to be $^2P_{\frac{1}{2}}$, the ratio of the g factors of Tl and Pb comes out to be (magnetic moment of the proton) / (one-third of magnetic moment of the neutron), that is 4, if the magnetic moment of the neutron is assumed to be 0·75 times that of the proton on the basis of our previous comparison of Hg_{199} and Tl. The value deduced by McLennan is between 3·7 and 5 (*loc. cit.* p. 666), thus agreeing with the theoretical value.

To interpret the ratio between the $g(I)$ factors of Tl and Bi deduced by McLennan (from 3·2 to 4·4; *loc. cit.*, p. 665) we have to consider the spin $4\frac{1}{2}$ of the Bi nucleus as due to a $^3G_5$ term with the spin of the proton oppositely directed. Then the magnetic moment of the Bi nucleus $= \frac{6}{5} \times 5 \times \frac{3}{4} - 1 = \frac{7}{2}$ so that its g factor $\frac{7}{2} \times \frac{2}{9} = \frac{7}{9}$. Hence the ratio of $g(I)_{Tl}$ to $g(I)_{Bi} = 18/7 = 2 \cdot 6$. In this case the numerical agreement is not so good as before, but considering the uncertainties in the value deduced by McLennan, as also in the ratio between the magnetic moments of the proton and the neutron, exact numerical coincidence cannot be expected. Considerations of a similar nature may be expected to lead to an understanding of the extremely small value of the $g(I)$ factor in the case of elements like chlorine.

A Note on the Special Theory of Relativity.

By Prof. A. C. Banerji, M.A., M.Sc., I.E.S., *Allahabad University.*

IT has been pointed out (*Current Science*, **1**, 160, 1932) that if there are two particles A and B of rest masses m_1 and m_2 (with respect to each other) moving with a relative velocity v, the total mass of the system can be calculated in two different ways. If m_1 be assumed to be at rest then the total mass of the system is found to be $m_1 + \frac{m_2}{\sqrt{1-\frac{v^2}{c^2}}}$ where c is the velocity of light. On the other hand, if m_2 is supposed to be at rest the total mass of the system becomes $m_2 + \frac{m_1}{\sqrt{1-\frac{v^2}{c^2}}}$. Clearly, these two expressions for the total mass are different.

In the first case the total energy of the system apart from the interaction energy (which, if any, will be the same in both the cases) is

$$m_1c^2 + \frac{m_2c^2}{\sqrt{1-\frac{v^2}{c^2}}}\,;\ i.e.,\ m_1c^2 + m_2c^2 + \tfrac{1}{2}\, m_2v^2$$

neglecting terms of higher order of small quantities. In the second case the total energy apart from the interaction energy would become $m_2c^2 + \frac{m_1c^2}{\sqrt{1-\frac{v^2}{c^2}}}$;

$i.e.,\ m_2c^2 + m_1c^2 + \frac{1}{2}\, m_1v^2$

neglecting terms of higher order of small quantities. These two expressions for energy are different.

We also see that according to the observer A the total linear momentum of the system is $\frac{m_2v}{\sqrt{1-\frac{v^2}{c^2}}}$ and according to the observer B it is $\frac{-m_1v}{\sqrt{1-\frac{v^2}{c^2}}}$. These two expressions are evidently numerically different.

If there are two or more observers we can show more generally that the total energy of a system of particles becomes different when measured by different observers; and the law of conservation is not true in this sense, and the total energy is not an absolute property of the system. However, it is quite possible that for each particular observer the total energy may remain constant throughout the motion, but it is no new principle. The above remarks apply equally well to the case of total linear momentum.

We shall see presently that the principle of relativity creates another difficulty to which attention has not been drawn before, *viz.*, failure of the concept of the centre of mass as a definite point. It is necessary to call attention to this fact, as when dealing with a number of particles, the concept of the centre of mass has sometimes been used.

Let us take, as before, two particles A and B having the rest masses m_1 and m_2 and let them start moving with respect to each other with the velocity v.

Let B′ be the point which is at rest with respect to A but which momentarily coincides with B at the instant t measured by A. Let AB be equal to r as measured by A. Now, for the observer at A the problem is reduced to a statical case of finding out the centre of mass of two masses m_1 and $\frac{m_2}{\sqrt{1-\frac{v^2}{c^2}}}$ at A and B′ respectively. Let the centre of mass be G_1 as found by A. Then according to the measurement of A

$$AG_1 = \frac{m_2 r}{\sqrt{1-\frac{v^2}{c^2}}} \Bigg/ \left[m_1 + \frac{m_2}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

$$= \frac{m_2 r}{m_1+m_2} + \tfrac{1}{2}\,\frac{m_1 m_2 v^2 r}{(m_1+m_2)c^2} \text{ (neglecting small quantities of higher orders).}$$

$$B\,G_1 = -\left\{\frac{m_1 r}{m_1+m_2} - \frac{\frac{1}{2} m_1 m_2 v^2 r}{(m_1+m_2)c^2}\right\} \text{ (neglecting small quantities of higher orders).}$$

Similarly, take A′ to be the point which is at rest with respect to B but which momentarily coincides with A at the instant t' measured by B. In order that BA′ may be numerically equal to AB′ we have to take t and t' suitably related.

We have $AB' = a + vt$, *i.e.* when $t = 0$ as measured by A the distance between the particles was 'a' according to A. Further, $BA' = -(b + vt')$, *i.e.* when $t' = 0$ as measured by B the distance between the particles was $(-b)$ according to B so that in order that $(AB') = -(BA')$ (as measured by A and B respectively) we get $a + vt = b + vt'$ *i.e.*, $t' - t = \frac{a-b}{v}$. We see that a and b depend upon the initial conditions of the problem.

Now for the observer at B the problem is reduced to a statical case of finding the centre of mass of masses $\frac{m_1}{\sqrt{1-\frac{v^2}{c^2}}}$ and m_2 at A′ and B respectively. Let the centre of mass be G_2 as found by B. Then according to the measurements of B

$$BG_2 = \frac{-m_1 r}{\sqrt{1-\frac{v^2}{c^2}}} \Bigg/ \left[m_2 + \frac{m_1}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

$$= -\left\{\frac{m_1 r}{m_1+m_2} + \tfrac{1}{2}\,\frac{m_1 m_2 v^2 r}{(m_1+m_2)c^2}\right\}$$

(neglecting small quantities of higher order);

also $A'G_2 = \frac{m_2 r}{m_1+m_2} - \frac{1}{2}\,\frac{m_1 m_2 v^2 r}{(m_1+m_2)c^2}$ (neglecting small quantities of higher order).

Expressions for AG_1 and $A'G_2$ are different.

We know that if two systems of reference A and B move with a relative velocity v then to an observer on A the unit of length of A along the line of relative motion appears to be in the ratio $\sqrt{1-\frac{v^2}{c^2}} : 1$ to that of B while to an observer on B the unit of length of B along the line of relative motion appears to be in the ratio $\sqrt{1-\frac{v^2}{c^2}} : 1$ to that of A. To the observer A the distance $A'G_2$ will appear to be

$$\frac{m_2 r}{m_1+m_2} \cdot \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - \tfrac{1}{2}\,\frac{m_1 m_2 v^2}{(m_1+m_2)c^2} \cdot \frac{r^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

i.e., $\frac{m_2 r}{m_1+m_2}$ (neglecting small quantities of higher order).

Even for observer A, G_1 and G_2 are different points. The concept that the centre of mass is a definite point with respect to any configuration of particles fails.

Now m_1 and m_2 are the masses of two particles A and B when they are relatively at rest with respect to each other. Eddington calls them "proper masses" or "invariant masses" and assumes that they have absolute inertial properties and remain unaltered throughout the vicissitudes of their history (Eddington's *Mathematical Theory of Relativity*, p. 30). Let us examine Eddington's assumption a little more carefully. There are two possibilities:—

(*a*) The rest masses m_1 and m_2 of any two particles A and B with respect to each other have the same values in presence of other bodies whatever be their common relative velocity with respect to each of these bodies.

(*b*) The values of the rest masses of A and B with respect to each other may change in the presence of other bodies by amounts which depend on the magnitude of their common relative velocity with respect to each of the other bodies.

Let us now examine the first possibility. Let there be three particles A, B and C. According to the hypothesis the rest masses of A and B between themselves remain the same irrespective of the presence of the third body C. Similarly, the rest masses of the particles A and C between themselves remain the same in spite of the third body B. Now let three bodies A, B and C be relatively at rest with one another and their rest masses with respect to one another be m_1, m_2 and m_3. According to the hypothesis, the rest masses m_1 and m_2 between A and B have not altered due to C and also the rest masses m_1 and m_3 between A and C are not altered due to B. If there is any other particle D we find that the rest masses between A and D are m_1 and m_4 irrespective of the presence of other bodies. So it follows that m_1 is an absolute property of the particle A if the first possibility is true. Let us see if this is borne out by facts.

Let M_1, M_2, M_3, etc. be the masses of the particles A, B, C, etc. and v_1, v_2, v_3, etc. be their relative velocities as measured by an observer S and let M'_1, M'_2, M'_3, etc. be the masses of the same particles and v'_1, v'_2, v'_3, etc. be their velocities as measured by another observer S′. Let u be the velocity of S with respect to S′. Then

$$M_1\sqrt{1-\frac{v_1^2}{c^2}} = M'_1\sqrt{1-\frac{v_1'^2}{c^2}}\,,$$

as each of them is equal to m_1 in virtue of the first possibility. We have similar relations for other particles. Therefore we have

$$\Sigma M_1 v_1 = \Sigma M'_1 v_1 \frac{\sqrt{1-\frac{v_1'^2}{c^2}}}{\sqrt{1-\frac{v_1^2}{c^2}}}$$

Now $$\frac{v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \cdot \frac{(v'_1-u)}{\sqrt{1-\frac{v_1'^2}{c^2}}}$$

(see p. 31, *Mathematical Theory of Relativ ty*, Eddington).

So we get

$$\Sigma M_1 v_1 = \frac{\Sigma M'_1 v'_1}{\sqrt{1-\frac{u^2}{c^2}}} - \frac{u\,\Sigma M'_1}{\sqrt{1-\frac{u^2}{c^2}}} \quad \ldots(A)$$

Similarly we also get

$$\Sigma M'_1 v'_1 = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\,\Sigma M_1 v_1 + \frac{u}{\sqrt{1-\frac{u^2}{c^2}}}\,\Sigma M_1 \quad \ldots(B)$$

Eddington has assumed, it appears rather arbitrarily, that the equation (A) is satisfied and has then come to the conclusion that the rest masses m_1, m_2, etc. are absolute properties of the particles. There does not seem to be any justification for such an assumption.

Clearly $\Sigma M'_1 v'_1$ is not equal to $\Sigma M_1 v_1$. When there are two or more observers total linear momentum of a system of particles becomes different for different observers, and the law of conservation is not true in this sense, and the total linear momentum is not an absolute property of a system of particles or bodies.

From (A) and (B) it is evident that if for each particular observer the total mass is conserved, then for him total linear momentum will also be conserved.

There is one serious difficulty, when we talk of any conservation theorem in connection with a number of particles in the theory of Relativity, as we have to bring in forces existing between them. This involves the idea of the distance, and the quantity giving the total energy or linear momentum becomes ambiguous. Hence it appears that we cannot talk of any conservation theory existing between a number of particles in the theory of Relativity.

Under the second possibility the rest masses are clearly not the absolute properties of the particles. Moreover, if m_1 is the rest mass of A with respect to B, then m_1 would not generally be the rest mass of A with respect to another particle C. It would be some other quantity m'_1. Each observer has his own particular world and measures the masses of the particles, their total energy and linear momentum in his own particular way. Unless some absolute property of each particle independent of the observer is conserved, there cannot be any correlation between the above quantities measured by different observers. Without any such correlation between measurements made by different observers the theory of Relativity cannot make much progress in explaining natural phenomena. So we see that some such postulate as the rest mass of a particle remains invariant throughout the vicissitudes of its history has become necessary.