Logic of evidence-based inference propositions

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Assertions about the content and context of an object of concern guide reasoning about its form, functions and consequences. Equivocation is minimized by the pro-logic state of an assertion affirmed by independent evidence. Four logic states are possible for proposition with two assertions, *it exists* (*A*) and *it is undescribable* (*U*) – is true (*T*) if affirmed *A* is consistent with not affirmed *U*; it is false (*F*) if *U* is affirmed and *A* is not; it is doubtful (*D*) if both *A* and *U* are affirmed, and it is empty or null (*X*) if both are not affirmed. This two-step epistemology (Saptbhangi) adopted in suitable languages provides a common basis for the logics that identify and resolve equivocation in semantic arguments and paradoxes to form a degree of belief constrained by evidence. As first proposed by G. N. Ramachandran, within limits this formalism is reduced to a vector-matrix description of the binary logic.

Keywords: Assertion, evidence-based inference, quantum logic, optical computing, vector-matrix algebra.

ACCORDING to the *Nay* (Prakrit term for tools and rules of reasoning), paradigm shared awareness of the content and context of an object of concern (*pramey*) affirmed by independent evidence (*praman*) is normative of argumentation. Such objects include entities, events, sets, variables, sentences, propositions, hypothesis and their claims. The purpose of reasoned conversation (*vacch nay*) is to identify assertions and claims within the constraints of evidence, and to minimize equivocation by identifying doubt (*syad*) introduced by incomplete information, evidence and logical processes.

As outlined in Figure 1, inference propositions about the form, functions and consequences of an object are derived from descriptions of cognized sense inputs. As a two-step syllogism, the logic state (degree of belief, validity, certainty, probability, truth value) of a proposition is inferred from the pro-logic status of its assertions affirmed, verified and calibrated with independent evidence. The pro-logic status as affirmed (+) or not affirmed (–) assertion does not allow for the binary semantics of true or not true as false. A not-affirmed assertion is not necessarily negated unless the negation is independently affirmed. Also, absence of evidence is not the evidence of absence, and non-existence lacks criteria for evidence.

As elaborated in this article, the binary logic is a limiting case of Saptbhangi Syad syllogism. The Greek logic is based on the primitive language of *all* (1) or *none* (0). In the propositional binary logic (PBL), true ($T = 1$) is complemented by its literal negation not true as false ($F = 0$) (ref. 3). Boolean algebra of $T = 1 - F$ guides deduction and it is implemented with 1 (*on*) or 0 (*off*) state of a signal bit for digital computing. Both strengths and limitations of the binary logic are due to the complementation condition.

Saptbhangi (Sapthabhangi) calibrates validity of an inference and improves granularity of resolved assertions with evidence to reduce equivocation. Valid inference within the bounds of assumptions and knowledge identify, conceptualize, represent, verify and formulate tentative propositions with identified basis for doubt to conserve information that may be deduced as false or discarded as not true. The eight propositions in Table 1 equivocate existence of an object in the first assertion (1) *it exists* (*A*) as an observable and measurable entity (*asti*), with the second (2) *it is undescribable* (*U*) if its content

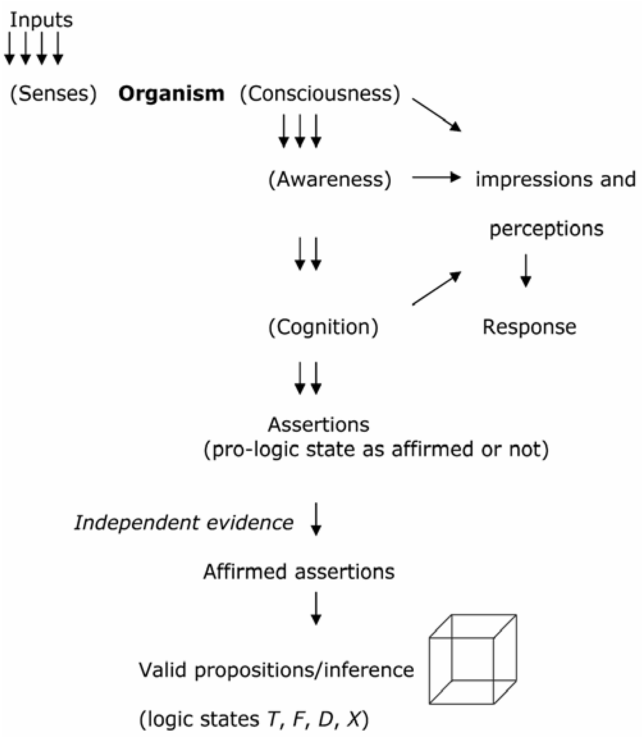

Figure 1. Operational relations of the terms used in the present article.

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Table 1. Propositions with *N*, *A* and *U* assertions affirmed (+) or not affirmed (–) by evidence

	N (does not exist)	*A* (exists)	*U* (undescribable)	Bit map
1	–	–	–	0 0 0
2	–	+	–	0 1 0
3	–	–	+	0 0 1
4	–	+	+	0 1 1
5	+	–	–	1 0 0
6	+	+	–	1 1 0
7	+	–	+	1 0 1
8	+	+	+	1 1 1

does not elicit awareness for a description (*avaktavya*) and the third (3) *it does not exist* (*N*) if it lacks context-dependent action and behaviour consequences (*nasti*). Congruence of the asserted inputs in a proposition may be inferred as consistent (true), inconsistent (false), doubtful or null. *N*–*A*–*U*–(0 0 0) node of the three not-affirmed assertions is maximally noncommittal null. The other seven propositions, called the Saptbhangi, with one or more affirmed assertions have a basis for interpretation. *N*–*A*+*U*–(0 1 0) with congruence of *A*+ with orthogonal and inversely complementary *N*– and *U*– provide a consistent cognitive basis for a description of the observable and measurable form, function and consequences of the object.

The focus of the present article is the two-step syllogism to infer the logic state of *NAU* propositions from the pro-logic status of orthogonal *N*, *A* and *U* assertions. In the first step, the prologic state of an assertion is partitioned to resolve equivocation with independent evidence. In the second step, affirmed assertions form the logical basis for the inference that minimizes liabilities and provide insights into the origins of paradoxes, falsity, undecidability, incompleteness, nothingness, contradiction, as well as existential, emotive and cognitive doubts associated with incomplete information and knowledge. The second part of this article is an outline of the underlying assumptions in the historical context. In the third part, *A*, *U* and *N* assertions as orthonormal basis vectors are formalized with the vector matrix algebra of logic proposed by Ramachandran[1,2]. Overall, two-step syllogism has the kernel of a general theory of evidence-based inference that can be adopted for logics and formalized for filters and quantum gates.

Words anchor awareness

Vocalizations express awareness of sense inputs. Utterances acknowledge fear and joy. Narratives mirror mind's grasp of reality nuanced by perceptions. Words in such speak are labels for tangible parts of experience that may otherwise be too complicated to deal with all at once. Equivocation is inevitable in word communication as meaning is modulated by what one knows, intends and wishes. The English language has well over 200 words to convey equivocation. A grammatical sentence may not elicit meaningful awareness of the underlying reality, and a logical truth may be dubious and inconsequential. Faulty memory and recall, in association with partially cognized sense inputs, mark the fiction of implied and embellished claims of flickers of insight guided by faith and stream of consciousness. One cannot trust whose trust is undiscriminating. Contradictions in narratives remain a source of cognitive dissonance, and the resulting inability to discriminate actions that always result in failure amounts to insanity. Not all words are created equal. The need to scrutinize inputs and outputs is greatest if a crafted narrative fails to make sense, or if the message creates little awareness of the content and function of the object of reasoning.

Conventions of a structured symbolic language facilitate fine-grained resolution of equivocation. Story-tellers weave parables to explicate cognitive awareness of the content by partitioning equivocation to identify, filter and recycle information communicated by the words. Word strings assert awareness of a meaningful part of perceived world, but they remain self-referential fiction unless affirmed by independent evidence. Assertions like *I am*, *I act*, *I feel* and *I think* conceptualize awareness of the internalized and interpreted parts of complex reality. Propositions, descriptions and narratives interweave and integrate assertions to provide awareness of an object in terms of its physical and notional forms, and of the relations of functions and consequences. Such interpretations guided by the awareness of inputs cognized for word expressions and thought map are like crossing a river towards an unknown destination. Heuristics for choices and decisions require mid-course corrections, no matter how clearly the path is charted. Methods, ideas and tools of interpretations along the way remain tentative to be revisited, scrutinized, reinterpreted, reexamined and revised.

Reasoning to reduce doubt

Lack of evidence to affirm an assertion cannot be used to negate it and independent evidence is required to assert true and also to assert false. PBL overlooks unknown, imagined, nonexistent, inconsistent, skeptical, meaningless, mistaken states prefixed with non-, in-, un-, a-, an-, de- or dis-. Also, an axiomatic truth may not be as infallible as claimed by the authority of generalization, tradition, revelation, divination, wisdom, intuition, justified true belief, or common sense. The Roman Church used Biblical truth to judge the solar system of Copernicus as 'false doctrine', and accused Galileo of 'false opinion'. Aristotle suggested that housefly has four legs, possibly motivated by a commonsense belief that *flies are animals* *with wings*, although a fly can be seen to support its weight on six limbs. Like all animal species, horse and donkey are offspring of their own kind. It may suggest that zebra are offspring of neither horse nor donkey; which implies nothing about mule (horse mother) and hinnie (donkey mother) as cross-bred offspring of horse and donkey.

The circularity and self-reference in the binary deduction of not true as false is not unlike the options in the fairy tales where inane perfection of *T* with artifice of *F* can be a source of liabilities, fallacies, contradictions and paradoxes. In the self-referential unary proposition *I am a liar*, the assertion (*a*) contradicts the content (*c*). Such $c = a$ propositions lead to paradoxical inference *x* = *not x* because if *c* is *T* then *a* is not-*T*, and if *c* is *F* then *a* is not-*F*. Impasse of the type *if x is true then x is not true*, *if x is not true then x is true*, *not x implies x*, or *x implies not x* are encountered in paradoxes and proofs of incompleteness theorem for predicate logic[4,5]. Such circularity is avoided if not-*T* unless affirmed as *F* is interpreted as *D* or *X* (Table 2).

Logic state of a proposition follows from the pro-logic status of its assertions. The four logic states in Table 2 result from the pro-logic status of *A* (*it exists*) and *U* (*it is undescribable*) affirmed (+) or not (–) by independent evidence: *it is so* (*T*) for the consistency of *A*+ with *U*-; *it is not so* (*F*) for the inconsistency of *U*+ with *A*–; *it is and it is not* makes *A*+*U*+ doubtful (*D*); *it neither is nor is not* for the null (*X*) of *A*–*U*–. Note that *F* for *U*+*A*– is for the falsehood (*mithya*) of affirmed undescribability (*U*+) of not-affirmed existence (*A*–). Thus not-$T = F + D + X$ precludes binary deduction.

Inference from multiple assertions

No assertion is entire of itself. Descriptions of the cognized awareness of an object continue to evolve with additional attributes and relations (*anekant*). Evolving nature of inference with additional information is aptly illustrated in the parable of encounter of six blind men with an unknown beast. Conundrum breaks out as each interacts with a different part and *sees* (infers) the whole differently. It is not an uncommon experience when faced with unknowns of infinite to infinitesimal worlds around us, whether an elephant facing the blind friends, or a distant object such as the sun, or potential of abstractions such as alphabets, numbers, genetic and cyber codes.

Table 2. Truth table for the logic states of a proposition from the pro-logic status of two assertions *A* and *U*. Compare it with Table 3 for binary AND

AND	*A*+	*A*–
U+	*D*	*F*
U–	*T*	*X*

Consider the evidence required to infer that air exists. Existence of invisible air is inferred from the behaviour consequences of its presence versus absence. It is visualized as bubbles that leave before water enters an apparently empty bottle being submerged in a bucketful of water. Such observables show that air lacks attributes of solids and liquids. Air as a gas has measurable relations of volume, pressure, flow rate, mass and composition that are adequately accounted for by the kinetic theory of gases. Lower pressure and density of air at higher altitudes predicts a finite thickness of the air layer in the earth's atmosphere.

Evidence-based assertions access the underlying realities. Inference (*anuman*) of fire from the sight of smoke is consistent with the generalization about invariance of the smoke–fire events in the past. However, validity of the inference is in the concomitance of smoke and fire with the burn characteristics of the fuel in the real time[6]. Concomitance of evidence to an asserted inference is like a lamp that illuminates itself and others. Such frames of reference balance abstractions with particulars to suggest hypotheses that remain coupled and cohere to all valid inferences. Successful hypotheses that remain falsifiable but are not falsified permit prediction, innovation and evolution of shared knowledge.

Inference-based hypotheses validated with multiple criteria are an antidote against paradoxes and fallacies of circular reasoning with self-referential propositions that invariably lead null of *neither is nor is not* (*X*). Like the emperor's clothes without a cognitive basis in independent reality, there is little to explore in miracles, dreams and hallucinations which may happen, but one cannot build on. Assertions like *if God did not create the world then who did* are self-referential and meaningless, where neither the actor nor the action is independently established. Certitude of ad hoc that contradicts facts of its own reality does not affirm existence, no matter how expedient, believable, useful, purposeful and meaningful they appear. Versions of omniscient, omnipresent or omnipotent are indistinguishable from the 'nothingness' of empty space, not even as a node of not-affirmed assertions. All together one of the goals of Saptbhangi strategy is to identify an entity that exists with demonstrable consequences of its presence versus absence, and to distinguish it from non-existence that is without such consequences. The crux of resulting atheism (*na-astik*) is that even without an observable basis for existence (*A*), an entity could be cognized from meaningful descriptions (*U*) that map consequences of its presence versus absence (*N*).

Structured template of propositions

An object of concern postulated to exist as *let there be x* is elaborated with affirmed assertions about its attributes and relations. The pro-logic status of such assertions determines the logic status of the resulting propositions. As in sculpting a rock, criteria-based identity of *x* as a particular and as a member of a class emerges by carving away extraneous to resolve inconsistencies, eliminate contradictions and minimize equivocation. Each of the eight (2^3) propositions in Table 1 may be interpreted in terms of the logic status of *it does not exist* (*N*), *it exists* (*A*), and *it is undescribable* (*U*):

1. Maybe *it is emptiness of nothing* or null with no affirmed assertion. [*N*–*A*–*U*–] as a node (0 0 0) accommodates affirmed *A*, *N* and *U* in the other proposition.
2. Maybe *it exists* is a true (*T*) proposition of affirmed existence (*A*+) supported by not affirmed non-existence and is not asserted as undescribable: (0 1 0) or [*N*–*A*+*U*–].
3. Maybe *it is undescribable* is a false (*F*) proposition because existence or non-existence is not affirmed: (0 0 1) or [*N*–*A*–*U*+].
4. Maybe *it exists asserted as undescribable* is a doubtful (*D*) proposition for affirmed existence and not affirmed non-existence: (0 1 1) or [*N*–*A*+*U*+].
5. Maybe *it does not exist* because affirmed non-existence is consistent with not affirmed existence and is not undescribable: (1 0 0) or [*N*+*A*–*U*–].
6. Maybe *it is a contradiction* of affirmed existence and affirmed non-existence that is not affirmed as undescribable: (1 1 0) or [*N*+*A*+*U*–].
7. Maybe *it does not exist* with not affirmed existence, but not affirmed non-existence and undescribable: (1 0 1) or [*N*+*A*–*U*+].
8. Maybe *it is a contradiction* of affirmed existence and affirmed non-existence that makes it undescribable: (1 1 1) or [*N*+*A*+*U*+].

Ancient roots

Based on the author's interpretations of the ancient texts[7] s*aptbhangi syad* evolved with the evolution of the Jain thought in India. As its cornerstone, the conservation principle उप्पानेई वा विगमेई वा धुवेई वा (tangible reality is the net of inputs and outputs) is attributed to Rishabhnath (ca. 3000 BC). By the time of Parshvanath (ca. 850 BC), the above conservation principle was invoked to draw inference from real-world analogies. As evolved later, the key assumptions for the relations in Table 1 are: (1) The world in *front of the eyes* (*pratyakch*) is what it is, it does what it does, it is neither created from nothing nor does it disappear into nothing. (2) A conscious (*chetana*) organism extracts information about phenomenal world from sense inputs. Such images are interpreted as perceptions (*itthi*) by the internal world *behind the eyes* (*parokch*, mind). (3) Awareness of such images is cognized in relation to other inputs and beliefs. Criteria-based descriptions (*anugam*) of the cognized parts provide information and evidence to represent, reason, interpret, assert and evaluate consequences. The external world is real and its content is conserved as net balance of inputs and outputs. Its complexity may be daunting and its behaviour unpredictable, but it is never contradictory. (4) As spectator, actor and decision maker, an organism interprets perceived parts of inputs to make choices that may be life-altering and make one happy, anxious or regretful. (5) Organisms bear consequences of individual and collective actions. Such interdependence calls for reasoned conversation to resolve conflict to arrive at a rational basis for coexistence, including a social contract for *live*, *let live*, and *thrive*.

Mahaveer (599–527 BC) revitalized the *Nay* methods with the belief that all organisms interpret their experience to address their concerns. Humans distinguish themselves with their ability to reason and deliberate, and the gulf between belief and words is further minimized by practice. If common sense aligns inputs with perceptions, it takes reasoned uncommon sense to align perceptions with the independent reality of the phenomenal world. Scrutiny of the content and context of propositions with identified assumptions encourages an open-ended search for certainty that proves and improves as *some uncertainty goes away with each day*. In response to a query from his discussion leader Indrabhuti Gautam (607–515 BC), Mahaveer emphasized that a belief is inferred not only from the content and context of what one knows and how it came to be known, but to realize its full potential it is also necessary to know what one does not know, what else is needed, and what may falsify and contradict it.

Saptbhangi Syad Nay is elaborated in several written works that go back 2000 years[7]. It evolved from the core assumption that assertions supported by independent evidence not only affirm but also identify areas of doubt and contradictions. The role of evidence in support of reasoning (*up-nay*) and decision (*nir-nay*) is elaborated in Gautam's *Nyay Sutr* compiled by Akchapad (ca. AD 100). This text does not mention the word *Nyay*. Apparently, it come in the title through the *Nyay Bhasya* commentary by Vatsyayan (ca. AD 400), where the word *Nyay* appears in the text only once in an insignificant context. Apparently by AD 500 evidence-based *Nay* reasoning had morphed under the influence of *Naiyayik* beliefs into *Nyay darshan* based on the evidence from scriptures. Current usage of *Nyay* connotes evidence-based judgement with an authority of rule. Soon the limitations of the scriptural evidence and of the logic of true and false (tark) were widely recognized.

Bhadrabahu I (350 BC) emphasized the four inferred logic states as *it is* (*T*), *it is not* (*F*), *it is both* (*D*), or *it is neither* (*X*). Umaswami (ca. AD 200) noted that *the authority of an affirmed assertion for reasoning is in the evidence* (प्रमाणनयैरधिगमः). Evidence affirms a certain aspect of the object as a particular or as a class, or its functional state or current state, or as addressed in the past. An inference is valid within bounds of all of its assertions affirmed in real time. Samantbhadra (ca. AD 300) emphasized that evidence-based validity is necessarily incomplete unless the remaining doubt, if any, is also resolved. Siddhsen Divakar (ca. AD 500) reiterated that reasoning is not possible unless assertions about content and context relations of the object are affirmed by evidence. Buddhists surmised nothingness (*shoonyata*) as the ultimate reality against which perceptions are transitory constructs of mind. It was rebutted by Akalank (AD 670) in a decisive debate in Kanchi: *shoonyata* as a state *without a basis in the content and context of an object is also without value for reasoning*. Hemchandra (ca. AD 1050) emphasized:

विना प्रमाणं परवत्र शून्यः

Unless supported by evidence an assertion is no different than nothing. Note that shoonyata is a blank platform to represent and interpret sense experience.

Gunratn (ca. AD 1435) reiterated reliance on criteria-based assertions affirmed by independent evidence as antidote against omniscience of ad hoc. More recently, Hiraiynna[8] noted that the four *syad* states, *is* (*asti*) and *is not* (*nasti*) with *both is or is not* and *neither is nor is not*, challenged the dichotomy of true or false in the faith-based Vedic absolutism. They identified contradiction of the undifferentiated Upnishadic reality of *it is so*, and *also it is not so* (*eti eti*, *neti neti*). Such interpretations of explicit assertions about an object of reasoning, inferred as the syad states are not red herrings of relativism, skepticism or deviant logic, nor the metaphysics of *four-cornered truth*[9].

Reasoning with abstractions

Sense organs may not perceive abstract objects, yet awareness of their space–time relations is a necessary part of the fight-or-flight response. Granularity of a natural language also permits equivocation of alternatives. Capacity of the mind to form, project and interact with abstractions allows us to represent objects with rule-bound use of symbols that adhere to and conserve reality. As prisoners of words, we venture out of literals by retelling tales in altered contexts. We have come to rely on alphabets and numbers to concisely and clearly communicate cognized awareness for reasoning to expose and identify deeper structures and relations in the inputs. Such representations are remarkably effective means of communication to liberate awareness and develop a conceptual grasp from cognized abstractions, say, of money with social, cultural, political and personal consequences.

The core of Saptbhangi epistemology is to constrain the degree of belief to arrive at an inference consistent with the sum total of the assertions and claims affirmed by independent evidence. Quantitative interpretation for an inference is possible with language of probability or of algebra, if the assertions are closed under the formation of complements and finite unions. Such rule-bound abstractions with logical and mathematical symbols are not unlike those for word communication. Their purpose is not as much to mimic real-world complexities, but to simulate the context-dependent action and behaviour consequences with meaningful parts and relations of a concern.

Limitations of the linearity of language are overcome with tools such as tables, figures, charts, flow diagrams, models, matrices and equations. Abstract and logical objects and spaces share many of the attributes and relations of physical counterparts, and much more. Individual and class identity of objects is conserved as their content and context adhere to the real-world relations and behaviours. They are not created from nothing, nor do they disappear into nothing. They occupy only one place at a time, and no other object can be in that place at the same time. However, abstract objects and space can have as many dimensions and attributes as minimally required. Not only do they move, rotate and transform in multidimensional spaces, their spaces and dimensions also change while objects remain stationary.

Reasoning is meant to resolve uncertainty. Mathematical tools to identify specific origins of uncertainty are not unlike the sum of the series $1 - 1 + 1 - 1 + 1 - 1\cdots$ as 1 or 0 depending on odd or even number of units. Quadratic[10,11], space–time[12] and other relations[13] also have alternative solutions. Certainty with residual equivocation from unresolved assertions is expressed as statistical probability p in 0 to 1 range[14]. Theories that identify and measure uncertainty as $1 - p$ include predicate, modal, fuzzy and many-valued logics. However, epistemology of Saptbhangi is closer to the objective interpretation of Bayes theorem[15], where the degree of belief that equivocates with undecided outcomes is updated with evolving evidence.

In the game theory[16], uncertainties remain constrained in the Nash equilibrium of the available choices. Such abstractions provide a basis for asking the right questions, to form beliefs without expectations, to develop models for predictions, to identify desirable outcomes and to evaluate their relevance. Mathematical profiles of such formalisms without psychological assumptions mimic, model and extrapolate the essential strategic features of a problem to tentative propositions that conserve information with the goal that sure loss is to be avoided if sure gain is not guaranteed.

Inference space of the Saptbhangi propositions

Figure 2 gives an overview of the relations among the eight *NAU* propositions. Starting from the null *N–A–U–* in row 1, 12 steps track the hierarchy of pathways, forks and dead-ends to the other seven propositions. Each proposition in row 2 has one affirmed assertion, those in row 3 have two, and the only one in row 4 has three affirmed assertions. Each path from *N–A–U–* to *N+A+U+* has three edges for the order in which the affirmed assertions are introduced. These relations show that the Saptbhangi template is a partially ordered set that can be treated as a lattice, electrical circuit or neural net[17]. Its vector matrix (VM) description provides a quantitative basis for logic[1,2].

Inference cube

The cube in Figure 3 represents the relations of the *NAU* propositions with three mutually perpendicular normalized axes that intersect at each of the eight corners (vertices). With the node *X* for *N–A–U–*, the other seven are separated by 1, 2 or 3 edges for the affirmed assertions. The vertices on the front face of the cube for the four *AU*(*N*–) propositions are interpreted in Table 2 to infer the four logic states *X*, *T*, *F* or *D*. The inference space for *n*-orthogonal assertions is a *n*-dimensional hypercube with *n* orthonormal axes and 2^n vertices. *X–T* axis will overlap for each pair of orthogonal assertions. *U*+ for each *F* will not project on *X–T*. As assertions converge to a single valid (*T*) proposition, resolution of the remaining *D* may require a paradigm shift[18]. Note that *T* and *F* are interpreted from independently affirmed orthogonal assertions. A projection from a point on the *FT* diagonal to the *X–T* axis is a measure of the partial truth value or the probability of certainty (p) and of uncertainty ($1 - p$). The space outside the *FT* line may be assigned diffused or

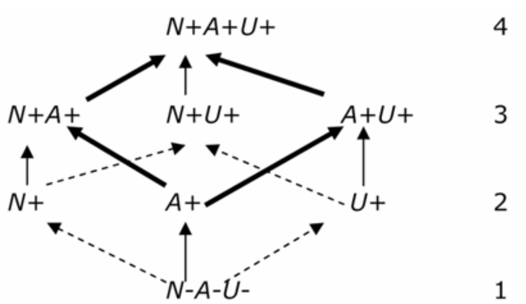

Figure 2. Hasse diagram of the eight *NAU* propositions.

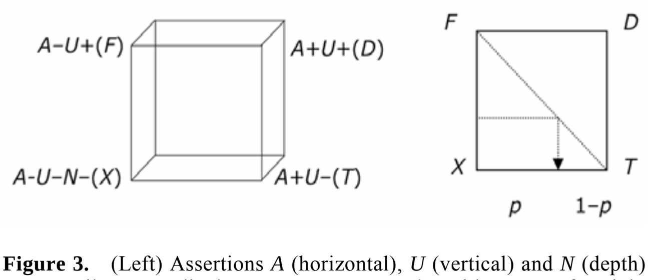

Figure 3. (Left) Assertions *A* (horizontal), *U* (vertical) and *N* (depth) as mutually perpendicular axes generate a cube with corners for eight propositions. (Right) The front face is bounded by *T*, *F*, *X* and *D* logic states.

fuzzy boundaries of the logic states attributed to randomness, imprecision, vagueness or unknownness.

The cubic universe of the set of eight discrete propositions (subsets) from three assertions (cardinal number) is not unlike the three-dimensional Hilbert space bounded by three orthonormal basis vectors[19]. Logical relations of a set of objects in Hilbert space can be modelled with VM algebra. Dirac modelled the quantum behaviour of the subatomic particles in terms of the interactions of the bra and ket forms of the basis vectors with suitable operator matrices[20–22]. The Boolean algebra of *T* and *F* scalars recast as the VM algebra of *T* and *F* basis vectors provides a formalism for two-valued PBL, and extended to two-valued predicate logic[1,2]. This groundbreaking work of Ramachandran[1,2] on Boolean vector matrix formulation (BVMF) is not acknowledged in later publications that provide additional insights into PBL and other binary logics[23–27] for the design of logic gates and filters corresponding to suitable connectives[28–31].

Limits of PBL

Two-valued logic is remarkably powerful for wide-ranging applications where complementation of not-*T* as *F* is the basis of deduction of scalar *T* (1) or *F* (0) output from scalar *T* or *F* inputs. A binary proposition $z = xLy$ is an ordered Boolean algebraic relationship of the logic variables (x, y, z) and connective L (NOT, OR, AND). In the truth table for (L=) AND^{xy}, the output for $z = 1$ with *T* inputs for both x and y, and $z = 0$ for the other three pairs of inputs for x and y is shown in Table 3.

The VM algebra of the relations of the orthonormal *T* (0 1) and *F* (1 0) basis vectors is isomorphous with the Boolean algebra of 1 and 0 scalars. VM formula for a binary $z = \langle x|[\text{AND}^{xy}]|y\rangle$ proposition is the inner and outer product of the variables as basis vectors with an operator matrix. Horizontal bra ($x1$ $x2$) matrix of $\langle x|$ acts from the left, and the vertical ket $\begin{pmatrix} y1 \\ y2 \end{pmatrix}$ matrix of $|y\rangle$ acts from the right of the operator matrix $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ for $[\text{AND}^{xy}]$ from its truth table (Table 3). Sixteen (2^4) 2×2 matrices of 0 and 1 make up the set of 16 binary connectives. The outer products of *T* and *F* vectors give four matrices from which the other 12 are algebraically derived. All other connectives can be expressed with connective NAND (NOT-AND, negation of the conjunction as in not (p and q) or not both) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, or NOR (NOT-OR, neither nor) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ alone.

Table 3. Binary truth table for the connective AND

AND^{xy}	$X = F$	T
$y = F$	0	0
T	0	1

Table 4. Truth tables of the binary connectives are not reversible

x	OR^{xy}	y	=	$\mathbf{z}$	→	$\mathbf{z}$	OR^{zy}	y	=	x	x	OR^{xz}	$\mathbf{z}$	=	y
0		0		0		0		0		0	0		0		0
0		1		1		0		1		*X*	0		1		1
1		0		1		1		0		1	1		0		*X*
1		1		1		1		1		*D*	1		1		*D*
x	AND^{xy}	y	=	$\mathbf{z}$	→	$\mathbf{z}$	AND^{zy}	y	=	x	x	AND^{xz}	$\mathbf{z}$	=	y
0		0		0		0		0		*D*	0		0		*D*
0		1		0		0		1		0	0		1		*X*
1		0		0		1		0		*X*	1		0		0
1		1		1		1		1		1	1		1		1

Table 5. Reversible truth tables *U* and *V* with *T*, *F*, *D* and *X* inputs

xUy	*T*	*F*	*D*	*X*	*xVy*	*T*	*F*	*D*	*X*	*x*NOT*y*	
T	*T*	*D*	*T*	*T*	*T*	*T*	*X*	*T*	*X*	*T*	*F*
F	*D*	*F*	*D*	*F*	*F*	*X*	*F*	*F*	*X*	*F*	*T*
D	*D*	*D*	*D*	*D*	*D*	*T*	*F*	*D*	*X*	*D*	*D*
X	*T*	*F*	*D*	*X*	*X*	*X*	*X*	*X*	*X*	*X*	*X*

Table 6. A 3 × 8 bitmap for Toffoli–Fredkin gate

C	I_1	I_2		*C*	O_1	O_2
0	0	0		0	0	0
0	0	1		0	0	1
0	1	0		0	1	0
0	1	1	→	0	1	1
1	0	0		1	0	0
1	0	1		1	0	1
1	1	0		1	1	1
1	1	1		1	1	0

T and *F* basis vectors intersect at the null *X*, and all other points in the *T*–*F* space are for *D*. Variables in a VM formula are input as normalized *T* (0 1) or *F* (1 0) vectors in bra or ket form. Normalized *X* (0 0) and *D* (1 1) vectors cannot be used as inputs. The unary formula with negation or equivalence connective gives only the *T* and *F* vector output, and the other 14 connectives give *T*, *F*, *D* or *X* vector outputs. Eight *D* and eight *X* outputs are obtained from the 28 possible right unary $|z\rangle = [L]|y\rangle$ formulas. Eight *D* and eight *X* are also obtained from the 28 left unary $\langle z| = \langle x|[L]$ formulas. *T* or *F* vector inputs in a binary formula $z = \langle x|[L]|y\rangle$ give only the scalar 1 or 0 outputs, i.e. the *D* and *X* vector outputs obtained from the *T* or *F* vector inputs in the first step with the 14 connectives (above) are reduced to scalar *T* or *F* outputs after the second step. Suppression of intermediate *D* and *X* outputs in the second step of a binary formula can be viewed as a transition from *T* to *F* via *X* or *D* (Figure 2, right). Algebraically, it is due to the complementation assumed in the inputs of the connective matrices.

D and *X* conserve information

With the exception of equivalence and negation, truth tables of the other fourteen PBL connectives are not reversible[2]. In the two sets of truth tables for *L* (= OR, AND) in Table 4, 1 and 0 outputs from binary $xL^{xy}y = z$ (left) are used as inputs for the formula $zL^{zy}y = x$ (middle) or $xL^{xy}z = y$ (right). In $z = x\mathrm{OR}^{xy}y$, $z = 0$ if $x = y = 0$, and $z = 1$ for the other three pairs (left). If the *T* and *F* outputs for *z* (left) are used as inputs for $x = z\mathrm{OR}^{zy}y$ (middle) or $y = x\mathrm{OR}^{xz}z$ (right), the output may be *D* (*T* or *F*) of *X* (neither *T* nor *F*) in certain rows.

Reference Tables 5 and 6 are useful for programing. Complementation of *T* with not-*T* as *F* makes the Boolean functions non-invertible or irreversible. With this insight the truth tables for binary connectives can be recast as *U* for unanimity for OR, and as *V* for *Vidya* (knowledge) for AND (Table 5)[1]. In these tables the outputs for $z = xL^{xy}y$ are obtained with *T*, *F*, *D* or *X* inputs for *x* and *y*. In both cases the *T* and *F* outputs for $z = xL^{xy}y$ are invertible (reversible), and also for $y = xL^{xz}z$ or $x = zL^{zy}y$ (not shown). The $z1$ and $z2$ components of the output vector are obtained separately from the first and second components of the input vectors as Boolean sums for $z = xUy$ and Boolean products for $z = xVy$:

For *U*: $z1 = x1 \;\mathrm{OR}\; y1 = x1 + y1$;
$z2 = x2 \;\mathrm{OR}\; y2 = x2 + y2$

For *V*: $z1 = x1 \;\mathrm{AND}\; y1 = x1 \otimes y1$;
$z2 = x2 \;\mathrm{AND}\; y2 = x2 \otimes y2$

The addition in *U* is for unanimity of *x* with *y* such that *TUT* (*T* with *T*) gives *T*, and *FUF* gives *F*. On the other hand, *TUF*, *FUT* and *FUD* = *D*. Note that *TUD* = *T*, where *D* is resolved in unanimity with *T*. In the last row or column, additive interactions of *X* are without any additional information and therefore, for *U* the outputs remain the same as the inputs. Multiplication in *V* provides a check on the consistency of *x* and *y*, such that *TVT* = *T* and *FVF* = *F*. *TVF* or *FVT* = *X* or indeterminate as expected for the contradictory inputs. Also, *XVT* or *TVX* or *FVX* or *TVX* or *DVX* or *DVX* = *X* because the *X* input in the product nullifies the *T*, *D* or *F* inputs, i.e. a contradictory proposition in a set makes the compound proposition contradictory.

Tables for *U* and *V* impose *D* whenever different choices of *D* inputs do not cancel the uncertainty about *T* or *F*. Such reference tables with *D* can be programmed and implemented as the logic gates or filters between input and output variables in logic circuits. As a step towards reversible logic, *D* permits determining a map of inverse operations from that of the direct operations. The operating principle is that if the inputs are not adequate to independently 'know' *T* and *F*, information is conserved as *D* to be recycled and resolved by introducing additional axiom, hypothesis or criteria.

D as a quantifier state within the logic space of *T* and *F* has been interpreted[4,31] to generate fuzzy, intuitionistic or modal logics. Many-valued logics with 0, 1, –1, 2, *i* or ½ basis vectors have also been described[2,28,31–33]. In such interpretations the total number of possible matrix functions increases exponentially with the number of independent assertions or truth values. For example, a total of 512 (=2^9) two-valued, 3×3 matrix functions are possible for two assertions, whereas 19,683 (=3^9) three-valued, 3×3 matrix functions result from three vectors for a 3-valued logic.

A well known limitation of PBL is encountered in the measures of the complementary variables of elementary particles. Simultaneous measurements of their position and momentum do not apparently conform to the distributive law[34]:

$$x \text{ AND } (y \text{ OR } z) = (x \text{ AND } y) \text{ OR } (x \text{ AND } z),$$

where *x* represents that the particle is moving to the right, *y* the particle is in the interval, and *z* the particle is not in the interval.

For a particle moving in a line, the proposition '*y* OR *z*' is true, and the truth value of *x* AND (*y* OR *z*) is determined by the truth value of *x*. According to the Heisenberg uncertainty principle, the position and momentum of a particle cannot be measured simultaneously, which makes both (*x* AND *y*) and (*x* AND *z*) on the right-hand side always false. By acknowledging such limitations of PBL and by postulating superposed or undecided states (*D*), the quantum theory has made rigorous and testable predictions about the observed and measured behaviours of the atomic particles.

Quantum logic and computing

The logic of quantum mechanics[34] is a set of mathematical rules for reasoning about the quantum behaviours. It projects measurements (propositions) as probabilities in Hilbert spaces. It has been elaborated with scores of mathematical formalisms, including the VM algebra of the quantum states as orthogonal vectors. The classical computing bit has either *F* (0) or *T* (1) scalar state. Quantum logic is implemented with qubits (quantum bits) of quantum states. A qubit of two basis vectors (0 1) and (1 0) also includes their linear combination by quantum superposition (1 1) and interference (0 0). Thus, a qubit with *n* vectors can simultaneously maintain 2^n states, which cuts down the number of memory swaps during a computing operation. In principle, each additional vector in a qubit increases the computing speed 2-fold.

Superposition of the basis vectors permits reversibility of quantum logic operations that conserve information. A family of reversible and conservative gates are generated from *n* + 1 bit inputs for *n* valued logics[35,36]. In such logic circuits, forward operation is simultaneously checked against the reverse operation during the course of computation without storing it in the memory. Also as a part of programming strategy, knowledge of the rules of forward and reverse inferences permits deduction driven by facts or by questions. Toffoli–Fredkin (TF) gates with three qubit inputs have been implemented as binary-coded adders[37], and for reversible logic operations with optical[38] and ion trap[39] quantum devices.

The sub-matrices of the 3×8 bitmap in Table 6 implement Boolean connectives and more complex functions. In the gating functions the first four rows retain input information, whereas the last four rows are processed for output. One of the truth tables in such a circuit is the TF gate[26,40]. TF gate is a complex matrix to map 0 or 1 valued Boolean functions (O_1, O_2) from three bit inputs (C, I_1, I_2) onto three bit outputs (C, O_1, O_2). *C* input mapped directly as *C* output serves as a control. No swap is performed with $C = 0$ and the companion signal I_1 maps to O_1, and I_2 maps to O_2. If $C = 1$, outputs in at least two rows are swapped so that I_1 maps to O_2, and I_2 maps to O_1. Thus a 3×8 matrix partitioned into two 2×2 matrices retains *D* and permits simultaneous implementation of logic of *T* and *F* vectors.

TF gates are universal, that is, a network of such gates can produce any binary function. The set of binary connectives (NOT, OR, AND, implication) can be implemented in almost all of these three input/three output reversible logic gates with suitable choices of the filtering function for the output channel and modification of the input channel. A gate is said to be conservative if outputs are permutations of inputs. Reversible TF gate computes invertible mapping, i.e. injection of outputs as new inputs returns the original inputs. No information is lost in reversible TF gate because it conditionally routes information bits to move around the states during computation. It retraces itself backwards because the bits can be moved but the total number of bits remains intact during computation. Another requirement for a reversible gate is that for each possible truth-table output there is only one input which will produce it. A 3×8 logic gate with three inputs and three outputs might have any one of 8^8 (≈ 16 million) possible truth tables. The requirement of reversibility reduces the number of possibilities down to 8! (≈ 40,000). Omitting duplicates these are reduced to 8000, which requires additional filtering criteria.

Doubt is a necessity

Cognized awareness of an object of concern and its behaviour consequences is the basis for its description as a function of the actor–spectator mind. In such thought abstractions intelligence formalizes the states that are well within the awareness and distinguishable through language. Tools of observation and measurement improve the criteria-based awareness, and philosophy provides meaningful boundaries for the interpretation and representation. Logic seeks valid relations for concept formation within the bounds of the psychological and physical interactions with the identified parts of an object of concern. It is like setting up a problem for solution with available information about the variables, evidence and assumptions. Questionable assumption or interpretation leads to questionable inference. The logic of doubt wards against make-beliefs and irreversible actions while addressing emotive (meaning and desires), existential (values) or skeptic concerns.

Brute logic of doubtful states dictates that for survival with incomplete information it is prudent to retain options and conserve information, howsoever tentative. A hallmark of natural languages is the processing continuum of possibilities to resolve layers of meaning that impregnate words. Fine-grained awareness of probable states and their relations provides a cognitive basis for reasoning with assertions. Partitioning of equivocation is the first step towards its resolution with suitable evidence. Assertions affirmed by independent evidence prune equivocation and enhance the degree of belief in a proposition. A response to real-time inputs requires extrapolation of outcomes and consequences to weigh plausible options.

Reasoning built on orthogonal assertions is remarkably isomorphous with the contemporary scientific reasoning to arrive at a conclusion on the basis of inferences each supported by independent evidence. The two-step Saptbhangi *Nay* syllogism for validity of a proposition within bounds of its affirmed assertions contains kernel of a theory of inference in terms of the interactions of the assumptions and evidence with the logic status of assertions as the basis of the logic states of a proposition. The logic space of orthonormal assertion vectors can be formalized in Hilbert space. Such descriptions with suitable assumptions and boundary conditions for complementation and closure can be reduced to wide-ranging logics. The challenge of their machine implementation remains.

Finally, reasoning with a matrix of assertions affirmed by cognized sense inputs and experience, as an intuitive basis of contemplation, contains not only kernel of a theory of inference, but also permits speculation about a theory of mind in which such inputs are structured to be interpreted within the framework of speech, memory and recall. It is tempting to suggest that a net of inputs configured as a multidimensional orthogonal neural qubit (nubit) could tentatively retain plausible inputs in real time to filter and gate outputs. A very large nubit will not only have the efficiency of a reversible conservative device, but a set of external inputs may lead to unique outputs modulated by the granularity of the internal inputs from an individual.

1. Ramachandran, G. N., Syad Nyaya System (SNS) – a new formulation of sentential logic and its isomorphism with Boolean algebra of genus 2. *Curr. Sci.*, 1982, **51**, 625–636.
2. Ramachandran, G. N., Vector-matrix representation of Boolean algebras and application to extended predicate logic (Parts I and II). *Curr. Sci.*, 1983, **52**, 292–302; 335–341.
3. Suppes, P., *Introduction to Logic*, D. Van Nostrand Co, Inc, Princeton, 1957, p. 312.
4. Ramachandran, G. N., Common origin of all paradoxes and incompleteness in logic. [Last of the 79 reports that formulate vector-matrix algebra of logic.] Available at Part IX at http://www.hira-pub.org/nay/index.html; Matphil Report, Indian Institute of Science, Bangalore, 1990, vol. 79, p. 61.
5. Nagel, E. and Newman, J. R., *Godel's Proof*, New York University Press, NY, 1960.
6. Rogers, J. M. and Jain, M. K., Inference and successful behavior. *Q. Rev. Biol.*, 1993, **68**, 387–397.
7. Jain, M. K., Nay: English translation of the seminal works, 1999; http://www.hira-pub.org/nay/index.html
8. Hiraiynna, M., *Outlines of Indian Philosophy*, George Allen & Unwin Ltd, London, 1921, p. 420.
9. Raju, P. T., The principle of four-cornered negation in Indian philosophy. *Rev. Metaphys.*, 1954, **7**, 694–713.
10. Haldane, J. B. S., A logical analysis of learning, conditioning, and related processes. *Behavior*, 1954, **6**, 256–270.
11. Haldane, J. B. S., The Syadavada system of predication. *Sankhya: Indian J. Stat.*, 1957, **18**, 195–200.
12. Bharucha, F., *Role of Space–Time in Jaina's Syadvada and Quantum Theory*, Sri Satguru Publications, Delhi, 1993, p. 67.
13. Suri, G. and Bal, H. S., *A Certain Ambiguity*, Princeton University Press, Princeton, 2007, p. 281.
14. Mahalanobis, P. S., The foundations of statistics. *Dialectica*, 1954, **8**, 95–111.
15. Williamson, J., *In Defense of Objective Bayesianism*, Oxford University Press, Oxford, 2010, p. 185.
16. Binmore, K., *Game Theory: A Very Short Introduction*, Oxford University Press, Oxford, 2007, p. 184.
17. Rutherford, D. E., *Introduction to Lattice Theory*, Hafner Publishing Co, New York, 1965, p. 117.
18. Kuhn, T., *The Structure of Scientific Revolutions*, Chicago University Press, Chicago, 1962.
19. Cohen, D. W., *An Introduction to Hilbert Space and Quantum Logic*, Springer-Verlag, New York, 1989, p. 145.
20. Dirac, P. A. M., *The Principles of Quantum Mechanics*, In The International Series of Monographs on Physics 27 (ed. Marshall, D. H. W. W.), The Clarendon Press, Oxford, 1958, 4th edn, p. 314.
21. Albert, D. Z., *Quantum Mechanics and Experience*, Harvard University, Cambridge, 1992, p. 206.
22. MIttelstaedt, P., *Philosophische Probleme der Modernen Physik*, Bibliographices Institute, Mannheim, 1968.

23. Stern, A., *Matrix Logic*, North-Holland, Amsterdam, 1988, p. 215.
24. Stern, A., *Matrix: Logic and Mind*, Elsevier, Amsterdam, 1992, p. 290.
25. Mizraji, E., Vector logics: the matrix-vector representation of logical calculus. *Fuzzy Sets Syst.*, 1992, **50**, 179–185.
26. Mizraji, E., The operators of vector logic. *Math. Logic Q.*, 1996, **42**, 27–40.
27. Westphal, J. and Hardy, J., Logic as a vector system. *J. Logic Comput.*, 2005, **15**, 751–765.
28. Stern, A., *The Quantum Brain: Theory and Implications*, North-Holland Elsevier, Amsterdam, 1994, p. 213.
29. Stern, A., *Quantum Theoretic Machine: What is Thought from the Point of View of Physics*, North-Holland Elsevier, Amsterdam, 2000, p. 588.
30. Qiu, D., Automata theory based on quantum logic: reversibilities and pushdown automata. *Theor. Comput. Sci.*, 2007, **386**, 38–56.
31. Mizraji, E., Vector logic: a natural algebraic representation of the fundamental logical gates. *J. Logic Comput.*, 2008, **18**, 97–121.
32. Malinowski, G., *Many-Valued Logics. Oxford Logic Guides* (eds Gabbay, D., Macintyre, A. and Scott, D.), Clarendon Press, Oxford, 1993, vol. 25, p. 131.
33. Rescher, N., *Many-Valued Logic*, McGraw Hill, New York, 1969.
34. Birkoff, G. and Von Newman, J., The logic of quantum mechanics. *Ann. Math.*, 1936, **37**, 823–843.
35. Cattaneo, G., Leporati, A. and Leporini, R., Quantum conservative gates for finite-valued logics. *Int. J. Theor. Phys*., 2004, **43**, 1769–1791.
36. Toffoli, T., Computation and construction universality of reversible cellular automata. *J. Comput. Syst. Sci*., 1977, **15**, 213–231.
37. James, R. K., Jacob, K. P. and Sasi, S., Reversible binary coded decimal adders using Topffoli gates. In *Advances in Computational Algorithms and Data Analysis. Lecture Notes in Electrical Engineering* (ed. S.I.A.e. al.), Springer, New York, 2009, pp. 117–131.
38. Lanyon, B. P. *et al.*, Simplifying quantum logic using higher-dimensional Hilbert space. *Nature Phys.*, 2009, **5**, 134–140.
39. Montz, T. *et al.*, Realization of the quantum Toffoli gate with trapped ions. *Phys. Rev. Lett.*, 2009, **103**(20), 200503.
40. Fredkin, E. and Toffoli, T., Conservative logic. *Int. J. Theor. Phys*., 1982, **21**, 219–253.

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