



Gauge Fields, Knots and Gravity (Series on Knots and Everything – Vol. 4), John C. Baez and Javier P. Muniain. World Scientific Publishing Co Pte Ltd, 5 Toh Tuck Link, Singapore 596224. 2011. xii + 465 pp. Price: £44.00/US\$ 67.00.

The aim of this book is to introduce concepts of modern differential geometry and topology with their applications to gauge theory and general relativity to physicists. It also aims to provide an introduction to recent developments in gauge theory and general relativity to mathematicians. What distinguishes this book from several others that are available in this subject, like *Topology and Geometry for Physicists* by C. Nash and S. Sen, is a slight focus on developments in applications of Chern–Simons theory to knot theory and also its appearance in the loop gravity programme of Ashtekar and others. The target audience of this book are undergraduates who are familiar with electrodynamics, special relativity, linear algebra and vector analysis. These pre-requisites are covered in all standard undergraduate programmes both in physics and mathematics. Of course it is assumed that a student at this level is mathematically motivated to read and work through the book.

The book is divided into three parts. These are organized in increasing levels of difficulty. The first part deals with the formulation of electromagnetism in terms of differential forms. What is quite appealing in this part is that Maxwell's

equations are first stated in familiar form and then gradually the authors develop relevant mathematical concepts to reformulate Maxwell's equations in terms of exterior forms. The whole discussion in this chapter is centred around these equations, including several exercises and examples which makes it coherent to read. There are good examples from physics to illustrate the mathematical concepts like the discussion of the Aharonov–Bhm effect in De Rham theory. An important omission in this part is a discussion of the Hodge decomposition theorem. This could have been easily motivated and introduced using the Helmholtz decomposition theorem of vector analysis, which will be familiar to the target audience.

The second part of the book deals with its main content, nonabelian gauge fields and knot theory. This part begins with the introduction to Lie groups, Lie algebras and matrix groups and then moves on to a discussion of vector bundles. Here examples from physics are scarce, the one exception being the discussion of the isospin. Then the Yang–Mills field is introduced followed by Chern–Simons theory. Again the number of examples discussed is reduced, but some of the results are explicitly worked out like the construction of the Yang–Mills field strength and the Bianchi identity. One surprise in the organization of concepts is that the action principle is introduced in the section on Chern–Simons theory, whereas it could have been introduced earlier in the part dealing with electromagnetism. The relationship of knot theory with Chern–Simons theory is sketched only briefly towards the end of this part. Since the uniqueness of this book is its claimed emphasis on this connection, this section could have been more detailed.

The last chapter deals with gravity. Most of the discussion is quite conventional and overlaps many existing books, like that in *General Relativity* by R. M. Wald or even the short discussion given in *Field Theory* by P. Ramond. The uniqueness in the presentation of the authors is the introduction of the new variables in gravity put forward by Ashtekar and others. But again the discussion is

very short and is confined to the last chapter. It is nice that the authors end with a discussion where the Chern–Simons action and knots again make a brief appearance.

Overall the book is a good introduction to modern geometry, topology and how mathematical structures in these subjects are used in present-day physics. Its level is on par with the book by Nash and Sen mentioned earlier, but lower than that of *Geometry, Topology and Physics* by M. Nakahara. However, there are several books available in the market with the same agenda. As mentioned earlier, the uniqueness of the presentation by the authors is the slight emphasis on the theory of knots and Ashtekar variables in quantum gravity. The notes for each of the parts are another useful feature of the book. They contain a brief history of the topics covered in the book and useful references, including original ones for future study. This will certainly be useful for a serious reader or for an instructor who is planning to design a course around this book. The brief snippets of history and quotes of famous personalities add charm to the book and makes pursuing this subject a romantic quest. This is certainly an attractive feature for motivating the target audience of the book. However, it can perhaps be a distraction for advanced and serious users of the book. This book is certainly an important addition for a library of any scientific institute. However, for an undergraduate, the target audience of this book, the monetary investment to purchase the book is certainly prohibitive. In the Indian context the book on *Symmetries, Gauge Fields, Strings and Fundamental Interactions: Mathematical Techniques in Gauge and String Theories, Vol. 1* by Tulsī Dass is more affordable, and does a competent and succinct job of introducing most of the subject matter of this book.

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