Quantum general relativity

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After a brief historical introduction on quantum gravity as a whole, the current status of loop quantum gravity is discussed. Because of space limitation, I could only illustrate recent advances through one example – cosmology of the very early universe – and provide references for results in other main areas.

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Introduction

EINSTEIN'S reservations on foundations of quantum mechanics are well-known. However, being a founding father of that subject, he was well-aware of the limitation of classical theories and emphasized[1], already in 1916, that *quantum theory would have to modify not only Maxwellian electrodynamics but also general relativity*. Three decades later he was even more explicit saying, in the context of cosmology[1]

> 'One may not assume the validity of field equations at very high density of field and matter and one may not conclude that the beginning of the expansion should be a singularity in the mathematical sense.'

By now, we know that classical physics cannot always be trusted even in the astronomical world because quantum phenomena are not limited just to tiny, microscopic systems. For example, neutron stars owe their very existence to a quintessentially quantum effect: the Fermi degeneracy pressure. At the nuclear density of $\sim 10^{15}$ g/cm^3 encountered in neutron stars, this pressure becomes strong enough to counterbalance the mighty gravitational pull and halt the collapse. The Planck density is some *eighty* orders of magnitude higher! Astonishing as the reach of GR is, it cannot be stretched into the Planck regime; here one needs a grander theory that unifies the principles underlying both general relativity and quantum physics.

Early developments

Serious attempts at constructing such a theory date back to the 1930s with papers on the quantization of the linearized gravitational field by Rosenfeld[2] and Bronstein[3]. Bronstein's papers are particularly prescient in that he gave a formulation in terms of the electric and magnetic parts of the Weyl tensor and his equations have been periodically rediscovered all the way to 2002 (ref. 4)! Analysis of interactions between gravitons began only in the 1960s when Feynman extended his calculational tools from QED to general relativity[5]. Soon after, DeWitt completed this analysis by systematically formulating the Feynman rules for calculating the scattering amplitudes among gravitons and between gravitons and matter quanta. He showed that the theory is unitary order by order in perturbation theory (for summary, see, e.g. ref. 6). In 1974, 't Hooft and Veltman[7] used elegant symmetry arguments to show that pure general relativity is renormalizable to 1 loop but they also found that this feature is destroyed when gravity is coupled to even a single scalar field. For pure gravity, there was a potential divergence at two loops because of a counter term that is cubic in the Riemann tensor. However there was no general argument to say that its coefficient is necessarily non-zero. A heroic calculation by Goroff and Sagnotti[8] settled this issue by showing that the coefficient is $(209/2880(4\pi)^2)$! Thus in perturbation theory off Minkowski space, pure gravity fails to be renormalizable at 2 loops, and when coupled to a scalar field, already at 1-loop.

The question then arose whether one should modify Einstein gravity at short distances and/or add astutely chosen matter which would improve its ultra-violet behaviour. The first avenue led to higher derivative theories. Stelle, Tomboulis and others showed that such a theory can be not only renormalizable but asymptotically free[9]. But it soon turned out that the theory fails to be unitary and its Hamiltonian is unbounded below. The discovery of supersymmetry suggested another avenue: with a suitable combination of fermions and bosons, perturbative infinities in the bosonic sector could be cancelled by those in the fermionic sector, improving the ultraviolet behaviour. This hope was shown to be realized to 2 loops by a number of authors[10]. However, by the late 1980s a consensus emerged that all supergravity theories would diverge by 3 loops and are therefore not viable (see, e.g. ref. 11; note 1).

A series of parallel developments was sparked in the canonical approach by Dirac's analysis of constrained Hamiltonian systems. In the 1960s, this framework was applied to general relativity by Dirac, Bergmann, Arnowitt, Deser, Misner and others[12–16]. The basic canonical variable was the 3-metric on a spatial slice and general relativity could be interpreted as a dynamical theory of

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3-geometries. Wheeler therefore baptized it *geometrodynamics*[17]. A distinguishing feature of the canonical approach is that in contrast to perturbative treatments it does not split the metric into a kinematic background and a dynamical fluctuation. As a result, a number of conceptual problems were brought to the forefront which revealed the deep structural differences between general relativity and more familiar field theories in Minkowski space-time. By now there is a near universal appreciation of the importance of background independence and of the necessity of facing the ensuing complications. However, this very feature made it difficult to use the standard techniques from QED to face the mathematical difficulties associated with the infinite number of degrees of freedom of the gravitational field. Consequently, most of the work in full quantum geometrodynamics remained rather formal. The program also faced a sociological limitation in that the ideas that had been so successful in QED played no role: in a non-perturbative, background independent approach, it is hard to see gravitons, calculate scattering matrices and use virtual processes to obtain radiative corrections. Therefore, after an initial burst of activity, the quantum geometrodynamics program became rather stagnant.

A third avenue was opened in the mid 1950s: explorations of the effects of a *classical* gravitational field on quantum matter fields. Early work by Parker explored quantum fields in the Friedman Lemaitre Robertson Walker (FLRW) space-times[18]. As recent successes of inflationary scenarios show, this choice was prescient. Indeed, this is the arena where we are most likely to first test the interface of gravity and quantum physics through observations. But this general area did not draw much attention until Hawking's seminal discovery in 1974 that quantum field theory (QFT) on a black hole background predicts that black holes emit quantum radiation and resemble black bodies when seen from infinity. Not only did the entire area of QFT in curved space-time experience an explosion of activity but this discovery has served as a focal point for a great deal of research in all areas of quantum gravity over the last four decades.

Current status

Ideas developed in QFT in curved space-times have had a number of fascinating applications, ranging from the study of diverse aspects of the Casimir effect[19] to the feasibility studies of creating time machines by exploiting the violations of local energy conditions that are allowed in QFT[20]. On the mathematical side, investigations led to the algebraic approach, which is by now fully-developed[21]. Because it respects general covariance, this framework provides a deeper understanding of the essence of the conceptual structure of QFT.

In quantum gravity proper, while both the perturbative and the canonical approaches reached an impasse by the early 1980s, they provided seeds for most of the subsequent developments. Although GR is perturbatively non-renormalizable, an effective field theory was developed systematically[22] and has had remarkable successes in the low energy regime, e.g. in the treatment of dynamics of compact binaries in classical GR[23]. Therefore the problem of finding a viable quantum gravity theory can be rephrased as that of obtaining an appropriate completion of the effective theory in which the outstanding conceptual issues – such as the fate of the classical singularities of GR, the statistical mechanical accounting of black hole entropy, and the final stages of the black hole evaporation – can be analysed systematically.

This quest has been undertaken in a number of directions[21]. In broad terms, approaches that came from particle physicists led first to *perturbative string theory* and then to the *AdS/CFT conjecture*, which has dominated the field over the past two decades or so. The canonical approach, pursued by the general relativity community has led to *loop quantum gravity* (LQG), the space-time version of which goes under the name *spin-foams*. A set of new ideas, motivated by the interface of general relativity and thermodynamics and first discussed by Bekenstein and Jacobson, is also being pursued. Each program adopts a different point of departure, treating certain aspects of the problem as more fundamental, and hoping that the remaining aspects can be handled successfully once there is a resolution of the key difficulties. Consequently, there is healthy diversity in directions of research that different approaches discussed in this issue currently emphasize.

The general motivation behind the gravity/thermodynamics approaches is rather similar to that used in the discussion of black holes in LQG. However, since these approaches are more recent, the central issues of quantum gravity are yet to be addressed systematically, at the mathematical level that other approaches have achieved, thanks to a steady stream of developments over the past two decades. The AdS/CFT conjecture in string theory captures deep mathematical features of a gravity/gauge-theory duality. The greatest successes use general relativity techniques – particularly from the black hole sector – to problems in *non-gravitational* physics. These results have a great deal of appeal as they have extended the reach of general relativity well outside its traditional domain. However, because the conjecture makes heavy use of a *negative* cosmological constant and involves higher dimensions whose radius is as huge, *equal to the cosmological radius*, from gravity perspective the paradigm remains far removed from the physical universe we inhabit. Because of this, and because one does not have adequate tools to provide a *space-time* descriptions of Planck scale processes, this approach has not shed much light on the age-old conceptual issues at the heart of quantum gravity, such as the fate of the cosmological singularities, space-time physics absence of a background geometry, and the issue of time in Planck scale physics. These issues are at the forefront in the general relativity-based approaches such as LQG (briefly discussed below) and other initiatives such as asymptotic safety (that could not be covered because of space limitation). However, because their primary focus is on gravity, they do not have adequate tools to address particle physics issues, related to unification.

Thus, the strengths and limitations of various approaches are complementary. For sustained progress, it is important not only to continue to make advances within each approach but also keep in mind their basic limitations. In areas of science that have raced ahead of technology, making direct observations difficult, there is an unfortunate tendency to mix what one believes and what one knows. In quantum gravity, this tendency has led to bouts of euphoria that the 'final theory' is close at hand which then turned out to be unfounded. It is vitally important to avoid the misplaced sense of certitude. Only then can there be communication *between* various approaches that has, unfortunately, become increasingly rare; only then can one hope to exploit their complementary strengths.

Loop quantum gravity

I mentioned in the previous section that already in 1916 Einstein realized that general relativity would have to be modified to incorporate the principles of quantum mechanics. Yet, a century later the goal still evades us. Why is the problem so difficult? An obvious response is that this is because there are no observations to guide us. However, this cannot be the whole story because, if there are no observational constraints, one would expect an overabundance of theories, not scarcity!

The viewpoint in LQG is that the primary obstacle is rather that, among fundamental forces of Nature, gravity is unique: it is encoded in the very geometry of space-time. This is a central feature of GR, the elegant crystallization of the well-tested equivalence principle. Therefore, one argues, it should be incorporated at a fundamental level in a viable quantum theory. The strategy is to free oneself of the background space-time that has seemed indispensable for formulating and addressing physical questions; the goal is to lift the anchor and learn to sail the open seas. To achieve this, one has to start afresh and introduce novel conceptual frameworks and mathematical techniques. From the LQG perspective, we do not yet have a complete quantum gravity theory primarily because serious attempts to meet these challenges squarely are relatively recent. However, the community *has* made notable advances towards this goal in recent years.

Main ideas

In LQG one begins with the premise that, as Einstein taught us, the physical and dynamical nature of space-time geometry is fundamental. However, this does not imply a conventional quantization of GR. In LQG, the fundamental quanta of geometry are one dimensional, polymer-like excitations over nothing, rather than gravitons – the wavy undulations over a continuum background. In particular, classical general relativity is recovered only in an appropriate coarse-grained limit. In classical GR, the encoding of gravity in geometry opened entirely new possibilities that dominate contemporary physics and astronomy: black holes, a dynamical universe, and gravitational waves. The expectation is that the encoding of quantum gravity in quantum Riemannian geometry will similarly open new vistas, successfully resolving singularities of general relativity and curing the ultraviolet divergences of QFTs based on continuum background space-times. The hope that *quantum* space-times will have novel features and rich physics has been realized through a number of detailed and concrete calculations.

These results are based on a specific quantum theory of Riemannian geometry that was constructed in the 1990s, paying close attention to all the functional-analysis issues related to infinite dimensional spaces. The starting point was a reformulation of GR, where the emphasis is shifted from metrics to spin-connections, bringing general relativity closer to the non-Abelian Yang-Mills theory[24]. The quantum Riemannian geometry uses well-defined measures and integrals on an appropriately defined infinite dimensional space of spin connections. This framework led to a striking result: Geometrical operators such as areas of surfaces and volumes of regions have purely discrete eigenvalues, showing that geometry is quantized in a precise sense. In particular, there is *an area gap* Δ_A – the smallest non-zero eigenvalue of the area operator. In classical GR, curvature can be defined by evaluating the holonomy of the spin connection along a loop, dividing it by the area enclosed by the loop and then taking the limit as this area shrinks to zero. In LQG, because of fundamental discreteness, one can only shrink the loop until its area becomes Δ_A. As curvature dictates dynamics, the area gap serves as the fundamental microscopic parameter in quantum dynamics. Because there are no degrees of freedom below Δ_A, ultraviolet infinities can naturally tamed both in the Hamiltonian formulation that is well-tailored to cosmology and in the path integral framework of spin-foams used to explore full, non-perturbative dynamics. (For details, see, e.g. refs 25, 26).

However, LQG is still incomplete and significant open issues remain. Nonetheless, notable advances could be made by first truncating the theory to the physical problem/process of interest and then exploring the consequences of the *qualitative* changes brought about by the quantum Riemannian geometry (see note 2). In particular, recently significant results have been obtained in the black hole sector of GR, in the low energy regime, and in the cosmology of the very early universe. In the black hole sector, Gambini, Pullin and others have shown that, in the spherically symmetric case, the singularity is resolved and one can discuss Hawking radiation on the resulting *quantum* geometry[27]. In the low energy regime, using spin foams, the graviton propagator has been systematically derived by Bianchi and others[28]. This development is conceptually non-trivial because a priori it is not clear why a diffeomorphism invariant theory can have *any* non-trivial *n-point* functions, let alone if they agree with those of perturbative treatments. Also, this calculation shows that, contrary to a common belief in the string theory community, LQG does have the correct Feynman propagator in the low energy limit. However, open issues remain. The issue of information loss is yet to be faced squarely in the black hole sector and the detailed relation between spin foams and the effective theory remains elusive beyond the leading order.

The very early universe

I will now provide a flavor of the current status of LQG by describing some of the recent advances in loop quantum cosmology (LQC) – the application of LQG to the very early universe. However, even this discussion has to be brief. For details, see, e.g., the review article[29] that summarizes a very large body of literature on the singularity resolution in LQC and a more recent review[30] that summarizes the phenomenological applications.

Thanks to the spectacular advances on the observational front over the last two decades, the very early universe now provides a fertile arena to test quantum gravity theories. The success of the inflationary paradigm in accounting for the observed inhomogeneities in the cosmic microwave background already illustrates this point because the analysis is based on QFT on the *curved* cosmological space-times. However, it excludes the Planck era because one assumes that the background space-time is well described by the FLRW space-times of classical GR all the way back to the big bang singularity. Any viable quantum gravity theory has to provide a successful extension of this paradigm to include the Planck regime.

Specifically, there is a long-standing expectation that quantum gravity effects will resolve the big bang singularity of GR. This would require *very large* quantum corrections to the underlying geometry, resulting in a paradigm shift at the Planck scale. One is therefore led to ask: Will inflation arise naturally in this deeper theory? Or, more modestly, can one at least obtain a consistent quantum gravity extension of the inflationary scenario? Can one meaningfully specify initial conditions in the Planck regime? In a viable quantum gravity theory, this should be possible because there would be no singularity and the Planck scale physics would be well-controlled. Would the resulting systematic evolution from the Planck epoch again lead to the correlation functions and the spectral index that are compatible with observations? If not, that quantum gravity approach would be *ruled out* at least in the cosmological sector. If these CMB features are consistent with observations, are there new predictions for future missions which keep memory of the pre-inflationary dynamics? If so, one would be able to directly confront that quantum gravity theory with observations. Thus, attempts to overcome conceptual incompleteness of the inflationary scenario can provide novel ways to test and guide candidate quantum gravity theories.

Singularity resolution in LQC

Thanks to systematic studies over the past decade, loop quantum cosmology (LQC) is sufficiently developed to address these issues[29–32]. First, the big-bang singularity is indeed naturally resolved as a direct result of the underlying quantum geometry. Quantum geometry effects create an effective repulsive force that is negligible until a curvature scalar approaches about a thousandth of the Planck scale. But then the new repulsive force rises *very* quickly, overwhelms the attractive force of classical gravity, dilutes the curvature scalar, preventing the formation of a singularity that would have resulted in classical GR.

In the simplest FLRW models, there is only one independent curvature scalar which diverges at the big bang of classical GR. In LQC, by contrast, once it reaches the Planck scale, it gets diluted and the universe bounces. One can show that there is a self-adjoint matter-density operator $\hat{\rho}$, whose spectrum is bounded above by $\rho_{\text{sup}} \approx 0.41\rho_{\text{Pl}}$ on the *physical* Hilbert space of LQC. Thus, matter density or curvature cannot diverge on *any* physical state. In any solution to the Hamiltonian constraint – the LQC analog of the Wheeler–DeWitt equation – the expectation value of $\hat{\rho}$ achieves the maximum value at the bounce. If the state is sharply peaked, the maximum value is indistinguishable from ρ_{sup}. Since in LQG curvature is naturally regulated because of the area gap Δ_A, it is Δ_A that determines the value of ρ_{sup} : $\rho_{\text{sup}} = \text{const}/(\Delta_A)^3$. Note that there is an interesting analogy with another quintessentially quantum phenomenon, that of superconductivity. In the theory of superconductivity, the energy-gap Δ_E – the energy needed to break a cooper pair – serves as the *microscopic* parameter and determines the values of *macroscopic* parameters such as the critical temperature below which superconductivity sets in: $T_{\text{crit}} = (\text{const})\, \Delta_E$. As the energy gap Δ_E goes to zero, the critical temperature goes to zero and we no longer have the novel phenomenon of superconductivity. Similarly, the microscopic parameter Δ_A of LQG determines the macroscopic parameters such as ρ_{sup} associated with a large scale quantum behaviour. If we let the area-gap Δ to go to zero – i.e. ignore the quantum nature of geometry underlying LQG – ρ_{sup} diverges, quantum effects disappear, and we are led back to the big bang of GR.

In LQC, then, the big bang singularity is resolved in the following precise sense: physical observables, such as energy density and curvature which diverge at the big bang in GR, have a *finite* upper bound on the entire Hilbert space of states ψ_o of the FLRW quantum geometry of LQC. (This resolution has also been understood in detail in the 'consistent histories' framework.) By now, a large number of cosmological models have been studied in detail, including the closed and open FLRW models, models with a cosmological constant, inflationary models with the quadratic and Starobinsky potentials, the Bianchi models and the Gowdy models which incorporate the simplest types of inhomogeneities in full GR. Detailed studies were carried out using Hamiltonian methods and canonical quantization, complemented by a sum over histories analysis à la Feynman for FLRW and Bianchi I models. In all cases, the singularity is resolved. (This is notable already for Bianchi models where, because the anisotropic shear terms grow as $1/a^6$ (with a the scale factor) near the big bang in GR, singularity resolution has been difficult in other approaches.)

LQC and observations

In the inflationary paradigm one focuses on the sector of GR consisting of the FLRW space-times (sourced by scalar fields with suitable potentials) *and* first order perturbations thereon. For phenomenological applications of LQC, one focuses on the same sector. However, now, the classical FLRW space-time in the background is replaced by a *quantum* FLRW geometry, represented by a wave function ψ_o. The quantum fields $(\hat{R}, \hat{T}_1, \hat{T}_2)$ representing the scalar and two tensor modes of perturbations now propagate on this quantum FLRW background ψ_o rather than on a classical Friedmann metric. This shift in the paradigm enables one to squarely face all the 'trans-Planckian issues'. The analysis required several new ingredients: Dynamics of quantum fields on cosmological quantum space-times; regularization and renormalization of the stress-energy tensor in this theory; accurate numerical evolutions in the pre-inflationary phase; and, matching the physics in the deep Planck regime at the bounce with the CMB observations. The final results show that the predictions of LQC are compatible with current observations but there are also rather surprising new effects that carry imprints of LQC that are absent in standard inflation (see e.g. refs 30–32 and references therein).

The origin of these effects lies in the fact that, as is common in physics, a more fundamental analysis introduces a new scale at which novel phenomena can occur. In FLRW models, the curvature at the bounce is universal in LQC and introduces a new length scale $\ell_{\rm LQC}$ which, as one would expect, is determined by the area gap $\Delta_{\rm A}$. The key new phenomenon is the following: Pre-inflationary LQC dynamics modifies the standard inflationary predictions in a universal way for modes whose wavelength at the bounce is larger than $\ell_{\rm LQC}$. Detailed analysis shows that these correspond either to the longest wavelength modes observable today[30] and/or modes whose wavelength is larger than the radius of the observable universe but which can couple to the observable modes[31]. Therefore the pre-inflationary dynamics of LQC can have interesting ramifications for the ~3σ anomalies in the Planck-satellite data associated with the largest angular scales.

At first reading, this assertion may seem counter-intuitive on two accounts. First, one generally expects quantum gravity effects to modify only the short-distance behaviour. How could they have any implications to predictions for the longest wavelength modes? Second, it is often claimed that while quantum gravity effects may be conceptually interesting, they will not be relevant for cosmological observations because they will all be 'diluted away' during inflation. I will conclude by explaining why these expectations are *not* borne out.

The belief that the pre-inflationary dynamics does not matter stems from the following argument (see the left panel of Figure 1). If one evolves the modes that are seen in the CMB *back* in time using GR, their physical wavelengths $\lambda_{\rm phy}$ continue to remain smaller than the curvature radius $R_{\rm curv}$ all the way to the big bang. The equations governing the evolution of these modes then imply that they propagate as though they were in flat space-time and cannot get excited in the pre-inflationary stage. Therefore, the argument goes, they will be in the Bunch-Davies (BD) vacuum at the onset of inflation. But in the pre-inflationary calculations, dynamical equations of GR cannot be trusted in the Planck regime; we must use instead a candidate quantum gravity theory. In LQC, if a mode has $\lambda_{\rm phy} > \ell_{\rm LQC}$ at the bounce, it *does* experience curvature during pre-inflationary dynamics and can get excited (see the right panel of Figure 1). For suitable choices of initial conditions at the bounce, these modes correspond to the largest angular scales seen in the CMB, roughly to $\ell \lesssim 30$ in the spherical harmonics decomposition of correlation functions. Thus, the *ultraviolet* modifications of the *background dynamics* that cure the big bang singularity can directly influence the *infrared* behaviour of perturbations. These longest wavelength modes, then, will not be in the BD vacuum at the onset of inflation. But why will this fact alter the observable predictions of inflation? Will not these excitations just get washed away during inflation? The answer is in the negative because of the accompanying *stimulated* emission: if one were to start with a candidate non-BD vacuum at the onset of inflation, the stimulated particle creation would result in certain departures from the standard predictions based on the BD vacuum[33]. The pre-inflationary dynamics of LQC provides specific non-BD initial states at the onset of inflation, thereby streamlining the possibilities and leading to an interplay between the Planck scale physics and observations[32].

To summarize then, thanks to the underlying quantum geometry, LQC has provided a viable extension of

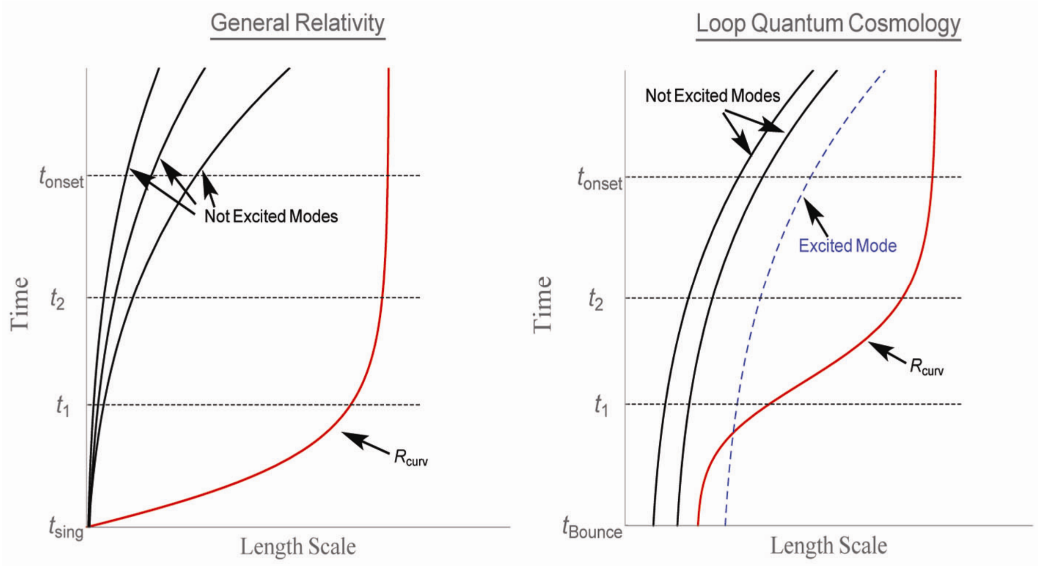

Figure 1. Time evolution of the curvature radius and three wave-lengths of interest. *Left Panel*: General relativity. The modes of interest have physical wave lengths less than the curvature radius ($\sqrt{6/R}$, with R, the scalar curvature) all the way from the big bang until after the onset of slow roll. All three wave-lengths remain well inside the curvature radius until they exit the Hubble horizon during inflation and are in the Bunch–Davies vacuum at the onset of inflation. *Right panel*: LQC. The dashed line shows the evolution of a mode whose physical wavelength exceeds the curvature radius at the bounce. It experiences curvature near the bounce, is excited and is not in the Bunch–Davies vacuum at the onset of inflation.

standard inflation over the 11–12 orders of magnitude that separate the Planck scale from the onset of inflation. Furthermore, because of the unforeseen interplay between the ultraviolet and infrared, pre-inflationary dynamics of LQC can leave signatures on the longest wavelength modes seen in the CMB. Specifically, LQC offers an avenue to account for the 3σ-anomalies seen by the Planck mission – the hemispherical anisotropy and power suppression at the largest angular scale – from fundamental quantum gravity considerations[30,31]. Thus, LQG has now evolved considerably beyond the mathematical physics domain of quantum gravity to the world of observational predictions and checks.

Notes

1. Interestingly, recent developments due to Bern and others have opened up the possibility that supergravity may in fact be perturbatively finite, and work by Anishetty and others suggests that at 1-loop the non-unitarity problem can be pushed to energies beyond the Planck scale.
2. This strategy is common in other areas of physics. For example, in any calculation with Feynman diagrams of low energy QED, one truncates the theory by allowing only a *finite* number of virtual particles.

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