

# ON LATIN AND HYPER-GRAECO-LATIN CUBES AND HYPER-CUBES

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**L**ATIN and Hyper-Graeco-Latin squares were first introduced by Euler<sup>1</sup> in 1782 and have since been extensively studied by a number of mathematicians like Gunther,<sup>2</sup> Cayley,<sup>3</sup> Maillet,<sup>4</sup> Cocozz,<sup>5</sup> Akar,<sup>6</sup> Brocard,<sup>7</sup> Tarry,<sup>8,9,10</sup> Macmahon,<sup>11</sup> MacNeish,<sup>12</sup> Margossian,<sup>13,14</sup> Fisher and Yates,<sup>15,16</sup> Fisher,<sup>17</sup> Bose,<sup>18</sup> Stevens<sup>19</sup> and Norton.<sup>20</sup> To Fisher is due the credit of pointing out their uses in the design of experiments; and with the realisation of their fundamental importance in the theory of this branch of statistics, much attention has been devoted to their study by statisticians in recent years.

2. Fisher introduced the idea of confounding of interactions in symmetrical factorial arrangements, which was extended by Yates to agronomic tests, involving a number of varieties equal to a prime positive integer or a power of prime, in order to increase the 'efficiency' (or accuracy) of the experiments. But it is no disparagement of their work to say that there is a lack of a unified general solution. Nair's work,<sup>21,22</sup> done subsequent to this, was an advance over our then existing knowledge of factorial arrangements in that he developed a method of constructing confounded arrangements in an  $s^m$  design, ( $s$  a prime positive integer or a power of a prime) in  $s^2$ -plot blocks, based on his theory of interchanges derivable from the associated Hyper-Graeco-Latin squares.

But a more complete solution in the case of the general symmetrical factorial arrangement was given by Bose and Kishen,<sup>23</sup> whose investigations achieved the unification and systematization which were lacking in previous work on the subject. Besides succeeding in giving a general method for the formation of confounded arrangements in an  $s^m$  design in blocks of  $s^{m-k}$  plots and the identification of the confounded degrees of freedom, the authors were able to enunciate the important principle of generalized interaction which enables the best sets of treatment comparisons which may profitably be confounded in any given case to be set down easily and elegantly. It is hoped that the concept of Latin and completely orthogonalized Hyper-Graeco-Latin cubes and

hyper-cubes which is now being introduced may be helpful for a fuller understanding of the theory of the general symmetrical factorial arrangement.

3. A Latin cube of the *first order* of side  $s$  may be defined as a cube arrangement of  $s^3$  letters,  $s^2$  of each of  $s$  kinds, such that each letter occurs exactly  $s$  times in each of its three sets of  $s$  planes, parallel to the three co-ordinate planes  $OX_1X_2$ ,  $OX_1X_3$  and  $OX_2X_3$ . A Latin cube of the *second order* of side  $s$  may be defined as a cube arrangement of  $s^3$  letters,  $s$  of each of  $s^2$  kinds, such that each letter occurs exactly once in each of its three sets of  $s$  planes parallel to the co-ordinate planes. Thus for  $s = 3$ , Latin cubes of the first and second orders may be diagrammatically represented as under in Figs. 1 and 2 respectively.

4. If an  $s$ -sided Latin cube of the *first order* is superimposed on another  $s$ -sided Latin cube of the *first order* such that every letter of one cube occurs exactly  $s$  times with every letter of the other cube, the two Latin cubes may be said to be orthogonal to each other. When the letters of the first

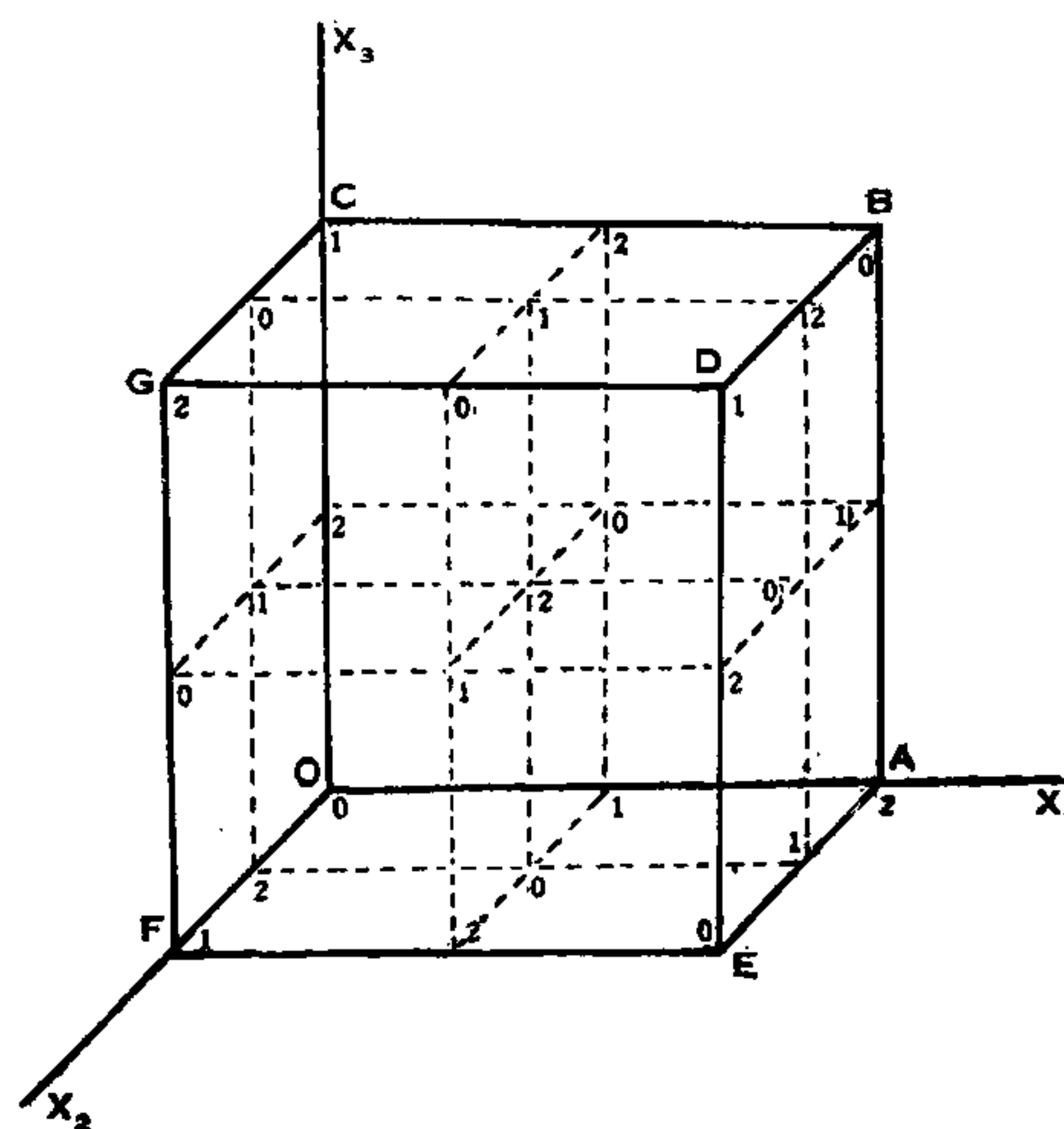


FIG. 1  
2x2x2. Latin Cube of First Order

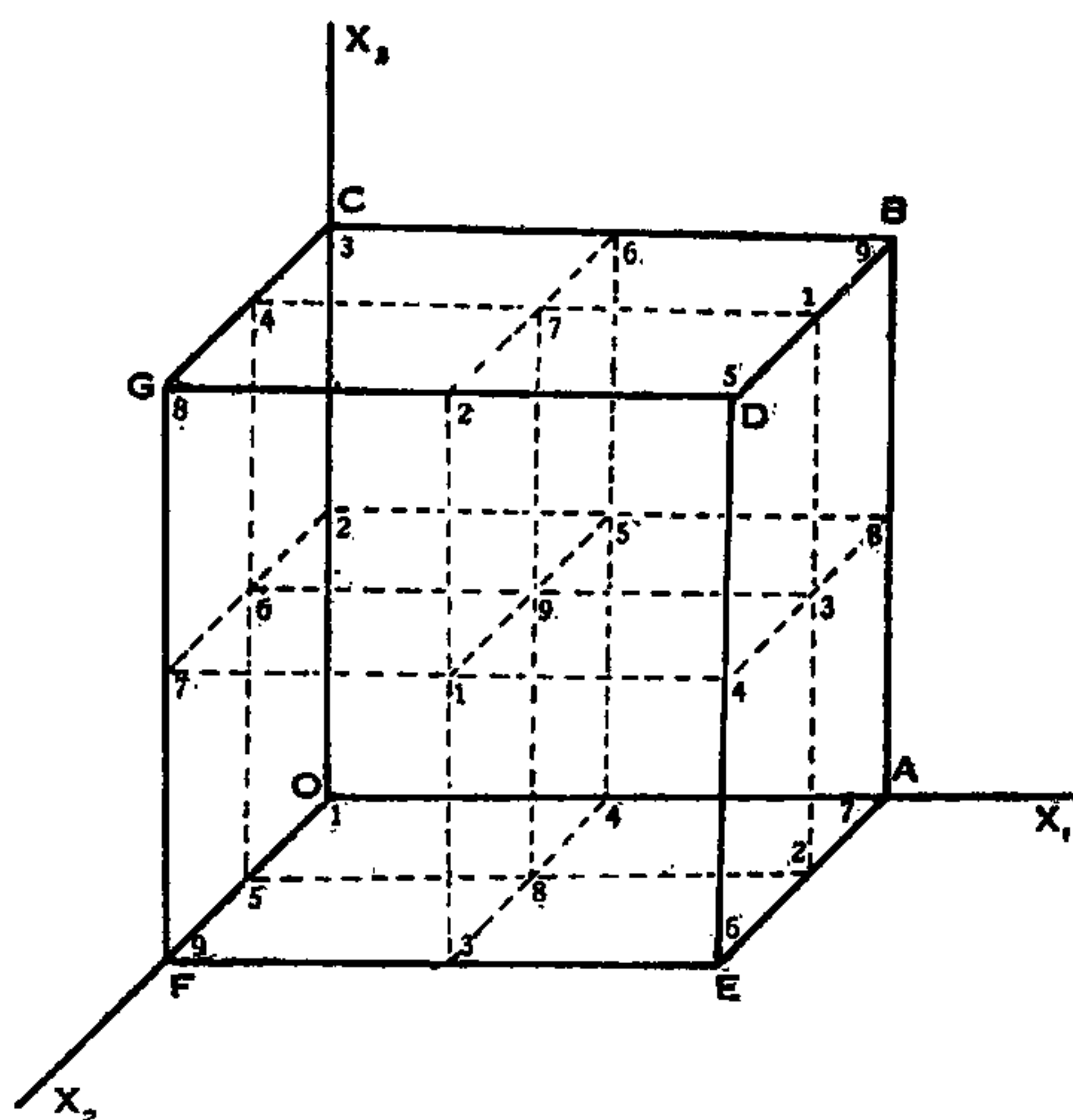


FIG. 2

2x2x2. Latin Cube of Second Order

cube are denoted by Latin letters and those of the second cube by Greek letters, and the second is superimposed on the first, the two together may be said to constitute a Graeco-Latin cube of the first order. The number of Latin cubes of the first order constituting a completely orthogonalized Hyper-Graeco-Latin cube of the first order has been found to be  $s^2 + s - 2$ .

5. In general, we may define an  $s$ -sided  $m$ -fold Latin hyper-cube of the  $r$ -th order as an  $m$ -fold hyper-cube arrangement of  $s^m$  letters,  $s^{m-r}$  of each of  $s^r$  kinds, such that each letter occurs exactly  $s^{m-r-1}$  times in each of its  $m$  sets of  $s$   $(m-1)$ -flats, parallel to the  $m$  co-ordinate  $(m-1)$ -flats  $OX_1X_2\cdots X_{m-1}$ ,  $OX_1X_2\cdots X_{m-2}X_m\cdots$ ,  $OX_1X_2\cdots X_{i-1}X_{i+1}\cdots X_m$ ,  $\cdots$   $OX_2X_3\cdots X_m$ . Two such Latin hyper-cubes, one superimposed on the other, such that every letter of the one occurs exactly  $s^{m-2r}$  times with every letter of the other, may be said to be orthogonal to each other. Denoting, as before, letters of the first hyper-cube by Latin letters and those of the second hyper-cube by Greek letters, the composite hyper-cube may be said to constitute an  $m$ -fold Graeco-Latin hyper-cube of the  $r$ -th order and it is obvious that the highest possible value for  $r$  is  $\frac{m-1}{2}$ , when  $m$  is odd, and is  $\frac{m}{2}$ , when  $m$  is even.

6. I have been able to establish that Latin cubes and hyper-cubes of the first order of any side exist and that  $s$ -sided  $m$ -fold Latin hyper-cubes of the  $r$ th order [ $r \leq (m-1)$ , the sign of equality not holding in certain cases] also exist,  $s$  being a prime positive integer or a power of a prime. I have also been able to demonstrate that the existence of an  $s$ -sided  $m$ -fold Hyper-Graeco-Latin hyper-cube of the first order is exactly equivalent to the existence of the finite hyper-dimensional projective geometry  $PG(m, s)$ , whence it follows that the total number of  $m$ -fold Latin hyper-cubes of the first order constituting an  $s$ -sided  $m$ -fold completely orthogonalized Hyper-Graeco-Latin-hyper-cube of the first order is  $s^{m-1} + s^{m-2} + \cdots + s^2 + s - (m-1)$ . For full details the interested reader is referred to the author's paper on the subject to be published shortly elsewhere.

- <sup>1</sup> L. Euler, *Wetenschappen, Vlissingen*, 1782, 9, 85.
- <sup>2</sup> S. Gunther, *Z. Math. Phys.*, 1876, 21, 61.
- <sup>3</sup> A. Cayley, *Messeng. Math.*, 1890, 19, 135.
- <sup>4</sup> E. Maillet, *Encyclopédie des sciences mathématiques pures et appliquées*, 1906 Paris. Leipzig.
- <sup>5</sup> V. Coccoz, *C. R. Ass. franc. Av. Sci.*, 1894, 23 (2), 163.
- <sup>6</sup> A. Akar, *Intermed. Math.*, 1895, 2, 79.
- <sup>7</sup> H. Brocard, *ibid.*, 1896 3, 90.
- <sup>8</sup> G. Tarry, *Mathesis*, 1900, 10, Supp. 23.
- <sup>9</sup> —, *Ass. franc. Av. Sci.*, 1904, 33, 95.
- <sup>10</sup> —, *Intermed. Math.*, 1905, 12, 174.
- <sup>11</sup> P. A. Macmahon, *Combinatory Analysis*, Camb. Univ. Press, 1915.
- <sup>12</sup> H. F. MacNeish, *Ann. Math.*, 1922, 23, 221.
- <sup>13</sup> A. Margossian, *Enseign. Math.*, 1931, 30, 41.
- <sup>14</sup> —, *ibid.*, 1935, 34, 365.
- <sup>15</sup> R. A. Fisher and F. Yates, *Proc. Camb. Phil. Soc.*, 1934, 30, 492.
- <sup>16</sup> — —, *Statistical Tables*, Edinburgh; Oliver and Boyd, 1938.
- <sup>17</sup> R. A. Fisher, *Design of Experiments*, 1937, 2nd ed., Edinburgh: Oliver and Boyd.
- <sup>18</sup> R. C. Bose, *Sankhya*, 1938, 3, 323.
- <sup>19</sup> W. L. Stevens, *Ann. Eugen.*, Lond., 1939, 9, 82.
- <sup>20</sup> H. W. Norton, *ibid.*, 1939, 9.
- <sup>21</sup> K. R. Nair, *Sankhya*, 1938, 4, 121.
- <sup>22</sup> —, *ibid.*, 1940, 5, 57.
- <sup>23</sup> R. C. Bose and K. Kishen, *ibid.*, 1940, 5, 21.



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## CRYSTAL ORIENTATION AND THE RAMAN SPECTRUM OF CALCITE

NEDUNGADI<sup>1</sup> and Bhagavantam<sup>2</sup> studied the effect of crystal orientation on the Raman spectra of sodium nitrate and calcite respectively. It has been observed in both the cases that the low frequency lines are very weak while the line due to the total symmetric oscillation comes out quite strongly when the incident light vector lies in the plane of the nitrate or the carbonate ion and the scattered light is along the optic axis. The reverse is the case when the incident light vector is parallel to the optic axis and the direction of the scattered light lies in the plane of the nitrate or carbonate ion. The above observation clearly indicates that the various tensor components, especially in the case of the total symmetric oscillation, are unequal among themselves as  $\alpha_{xx} = \alpha_{yy} \neq \alpha_{zz}$ . Bhagavantam has postulated that in the case of the 1085 line in calcite,  $\alpha_{xx}$  and  $\alpha_{yy}$  are each about three times as large as  $\alpha_{zz}$ . Intensities involving  $\alpha_{xx}$  or  $\alpha_{yy}$  will correspondingly be about ten times as large as those containing  $\alpha_{zz}$ .

The author has obtained the Raman spectra of a well polished one-inch cube of calcite,

cut in such a manner that its optic axis is parallel to one of the edges, for both the above-mentioned orientations on the same photographic plate. Great care has been taken to see that the time of exposure and the intensity of the source remained the same while photographing the two spectra. A set of intensity marks obtained by the method of varying slit widths is recorded on the same plate and the relative intensities are obtained from the density-log intensity curves in the usual manner. The ratio of the intensities of the 1085 line, in the two cases, is obtained as 7.94 for the 4358 excitation and 7.38 for the 4047 excitation. Hence, it may be concluded that the corresponding tensor components are related as  $\alpha_{xx} = \alpha_{yy} = 2.8 \alpha_{zz}$ .

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<sup>1</sup> Nedungadi, *Proc. Ind. Acad. Sci.*, 1939, 10, 197.

<sup>2</sup> Bhagavantam, *Ibid.*, 1940, 11, 62.