of these principles that the continuity of

$$-g^{\alpha 4}\sqrt{-g}\,A^{\gamma}_{\mu\alpha} + \tfrac{1}{2}\, g^{4}_{\mu}\, g^{\alpha\beta}\, \sqrt{-g}\,A^{\gamma}_{\alpha\beta} \quad \mu=1,2,3,4;\ \gamma=1,2,3 \quad (10)$$

"On the boundary is necessary. Here

$$A^{\gamma}_{\alpha\beta} = -\Gamma^{\gamma}_{\alpha\beta} + \tfrac{1}{2} g^{\gamma}_{\alpha}\, \Gamma^{p}_{\beta p} + \tfrac{1}{2} g^{\gamma}_{\beta}\, \Gamma^{p}_{\alpha p} \qquad (11)$$

This continuity ensures the conservation of energy and momentum provided

$$\frac{\partial}{\partial x^4} \iiint \Big(-g^{\alpha 4}\sqrt{-g}\,A^{4}_{\mu\alpha} + \tfrac{1}{2}\, g^{4}_{\mu}\, g^{\alpha\beta}\sqrt{-g}\,A^{4}_{\alpha\beta} \Big)\, dx^1\, dx^2\, dx^3 = 0, \quad (12)$$

the integral being taken over the whole space for $\mu = 1, 2, 3, 4$.

It is believed that the results (7), (8) and (9) are new. We are publishing elsewhere the full implications of Tolman's tacit assumption regarding the continuity of (12) which leads to (7). He seems to be unaware of (7).[3]

Lastly, another new point that has struck us in this connection may be noted. For the geodesies on the boundary surface we have

$$g_{\mu\nu}\frac{dx^{\mu}}{ds}\,\frac{dx^{\gamma}}{ds} = \pm 1, 0, \qquad (13)$$

$$f_{\mu}\,\frac{dx^{\mu}}{ds} = 0, \qquad (14)$$

$$\Big(-\frac{\partial f}{\partial x^{\mu}}\,\Gamma^{\mu}_{\alpha\beta} + \frac{\partial^2 f}{\partial x^{\alpha}\partial x^{\beta}} \Big)\, \frac{dx^{\alpha}}{ds}\,\frac{dx^{\beta}}{ds} = 0 \quad (15)$$

along with (1). If there is to be no ambiguity about these geodesies (15) must be the same whether it is couched in terms of $g_{\mu\nu}$ or $g'_{\mu\nu}$. That this is true has been verified for spherical distributions.

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September 12, 1942.

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[1] Eddington, Sir A. S., *The Mathematical Theory of Relativity*, 1924, 93.

[2] Tolman, R. C., *Relativity, Thermodynamics and Cosmology*, 1934, 232.

[3] —, *Phys. Rev.*, 1939, **55**, 364.

ON A LACUNA IN THE TREATMENT OF INTERNAL SOLUTIONS IN GENERAL RELATIVITY

WE give here an example to show how the usual treatment of internal solutions is inadequate and sometimes even wrong. Consider a line-element,

$$ds^2 = -e^{\mu}\,(dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2) + e^{\nu} dt^2, \quad (1)$$

which satisfies the internal field equations for $r \leqslant a$ and is continuous with the isotropic line-element of the external field for a mass m. A new solution of the equations is given by

$$e^{\mu} = \Big(1 + \frac{m}{2a}\Big)^4 \Big(\frac{r}{a}\Big)^{\lambda}, \qquad (2)$$

$$e^{\nu/2} = \frac{1 - \frac{m}{2a}}{1 + \frac{m}{2a}}\,\frac{1}{2d}\,\Big(\frac{r}{a}\Big)^{1+\frac{\lambda}{2}-n} \Big[(c+d) - (c-d)\Big(\frac{r}{a}\Big)^{2n}\Big], \quad (3)$$

$$8\pi\rho = -\frac{a^{\lambda}}{\Big(1+\frac{m}{2a}\Big)^4}\, r^{-\lambda-2}\Big(\lambda + \frac{1}{4}\lambda^2\Big), \qquad (4)$$

$$8\pi p = \frac{a^{\lambda}\,(c^2 - d^2)}{\Big(1+\frac{m}{2a}\Big)^4}\, r^{-\lambda-2} \Big[\frac{a^{2n} - r^{2n}}{(c+d)\, a^{2n} - (c-d)\, r^{2n}}\Big], \quad (5)$$

where $c = 2 + 3\lambda + 3\lambda^2/4$, $d = n\,(\lambda + 2)$,
$n = (1 + 2\lambda + \lambda^2/2)^{1/2}$

For the pressure and density to be non-negative it is necessary that

$$0 \geqslant \lambda \geqslant -2 + \sqrt{2} \qquad (6)$$

when n is real. For imaginary values of n the solution can be similarly discussed. We take up now the question whether the distribution so obtained behaves like a particle of mass m at infinity. In the customary treatment this question is not examined and solutions like the one given above are understood as correct. Having discussed the equations of fit at $r = a$ we find that an additional condition, *viz.*,

$$\frac{m}{a} = -\frac{2\lambda}{\lambda + 4} \qquad (7)$$

has to be satisfied if the internal distribution is to have the same field at infinity as a particle of mass m. The equations of fit have been considered by two of us (V. V. N. and P. C. V.) in another communication to *Current Science*. The above solution is valid only if (7) is fulfilled.

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