Sharad S. Shrikhande (1917–2020)

Professor Sharad Shankar Shrikhande, popularly known among his friends and colleagues as Shrik, one of the most well-known mathematicians and statisticians from India, passed away on 21 April 2020. By this time he had moved around the Sun 102 times. It is rare for a mathematician to live that long. He lived happily, all these years and till the very end was very devoted to his field Combinatorics or what is nowadays known as Discrete Mathematics. His cheerful and peaceful personality and silent style reflects perhaps the secret of his long life.

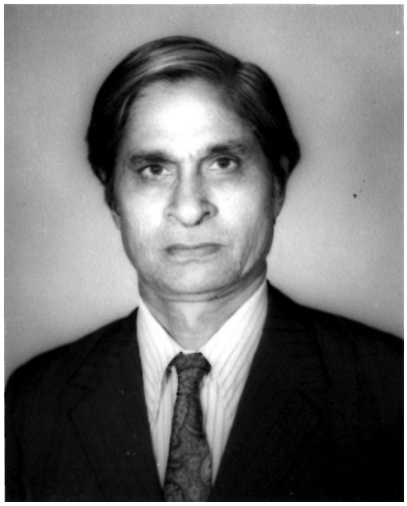
Shrikhande is one of those whose research contributions had a tremendous impact on the development of combinatorics in the second-half of this century. It is believed that one of the most important factors in the sudden spurt in the study of Designs and related areas in the 1960s and 1970s was the sensational result due to R. C. Bose, S. S. Shrikhande and E. T. Parker, on the falsity of a conjecture by Euler on the existence of orthogonal Latin Squares. This conjecture was fondly believed to be true for almost 176 years. The unexpected result created such an excitement that photos of these mathematicians and their discovery found a prominent place on the front page of the *New York Times* and attracted a large number of young mathematicians into this area.

Shrikhande was in NY city that day to give a lecture. Interestingly, he once recalled that the man at the reception of the hotel, where he stayed then, curiously showed him the newspaper and asked if he was the man on the front page, to which he had smiled and said, yes. The man replied: You must have done something big; *NYT*s front page cannot be bought for even millions of dollars.

What follows is a modest attempt to look at the work of this great researcher, which not only had high esthetic beauty, but also proved to be quite useful in diverse applications.

Shrikhande was born in a middle class family on 19 October 1917 at Sagar, a small town in Madhya Pradesh, close to the geographical centre of India. He received the Bachelor of Science (Honors) degree in Mathematics from Nagpur University in 1939. He was a rank holder, generally first ranker in practically all his exams in school and college. He had been awarded Gold Medal in his final year of B Sc too, as a first rank holder.

During the early 1940s on a fellowship he spent almost a year at Indian Statistical Institute at Kolkata. Mahalanobis had just started the institute. He was trying to collect young researchers in Mathematics and Statistics. This stay made Sharad interested in designs of experiments, a branch of Statistics. It remained as one of his prime interests, all his life.



He joined the Nagpur College of Science in 1942, teaching Mathematics and Statistics and continued there till 1958. This was a period of extreme difficulties as the country, under a colonial regime, was directing all its resources towards supporting the war efforts of the British. There was hardly any support for basic research. Young Shrikhande struggled against all odds, while working at the Science College. He continued to pursue his studies via his basic initial learning at the Statistical Institute. That one year stay at the institute was very fruitful for him. Perhaps that stay also played a part in his getting a fellowship for Ph D in University of North Carolina at Chapel Hill. He was very soon in USA. The Statistics Department in that university had already become an internationally reputed centre with many well-known researchers in its faculty.

Meanwhile R. C. Bose from Indian Statistical Institute, who had already become famous for his application of algebraic methods to create statistical designing schemes used for agricultural experiments, also joined that department, as a regular faculty member. Shrikhande worked with him, to complete his Ph D in 1950 and thus became the first doctoral student of Bose.

The close collaboration between them continued till the last years of Bose's life. During 1951–53, Shrikhande was an Assistant Professor of Statistics at the University of Kansas, Lawrence, USA and was Associate Professor at Chapel Hill during 1958–60. It was during the later period that the famous result on Eulers conjecture was settled.

Instead of staying on in USA, he decided to return to India to take up a Professorship at Banaras Hindu University where he worked till 1963. He then joined the University of Bombay as Professor and Head of the Department of Mathematics until his formal retirement in 1978. During this period, he was also Director of Centre for Advanced Study in Mathematics at Bombay. During 1983–86, he held the position of Director of Mehta Research Institute at Allahabad, which is a research centre for Mathematics and Mathematical Physics.

Shrikhande had been a Visiting Professor at various US Universities like University of Wisconsin, The Ohio State University, State University of New York, Stanford University and Colorado State University. He has been associated with the Indian Statistical Institute in various capacities. He was member of a number of learned societies: Indian National Science Academy, Indian Academy of Sciences, Institute of Mathematical Statistics and International Statistical Institute, to mention a few. He has also been on the editorial boards of a number of internationally reputed journals.

Combinatorics today has become a rather strange discipline attracting researchers not only from Mathematics and Statistics, but also from diverse areas like Computer Science, Electronics, Management, Biology, Sociology, indeed any application-oriented branch of Science or Technology seems to produce problems of a Combinatorial nature. The foundation for this development was laid in the 1940s and 1950s, when geometers, statisticians, group theorists, number theorists and engineers independently started posing in their own style, interesting combinatorial problems and developing

necessary language and tools to solve them. It was decades later, when common themes in these diverse developments were observed and attempts were made to unify them. Among the many early pioneers, some names stand out: Marshall Hall, Herbert Ryser, Alan Hoffman, R. H. Bruck, R. C. Bose and S. S. Shrikhande, to mention a few. Shrikhande was perhaps one of the youngest among them.

Shrikhande's first paper, published in 1950, in *Annals of Mathematical Statistics*, on the impossibility of certain symmetrical balanced incomplete block designs (SBIBD) is path breaking. He extended the style developed by Bruck and Ryser of using the quadratic form associated with a projective plane to all symmetric BIBDs. Results of this paper were independently obtained by M. Schutzenberger and by Chowla and Ryser. This was the first paper to systematically develop a method, using incidence matrix of a statistical design, to study non-existence of such designs. This helped to fill in gaps in the table of Fisher and Yates, created for statisticians. This method was later used to study similar questions for many other designs by Ogawa, Raghavarao, Vartak, Hall, Ryser, Hughes, etc. Shrikhande himself wrote several papers, jointly with others, applying the method to many different classes of designs group divisible designs, partially balanced designs, affine designs, cyclic designs and so on.

A pattern is visible in the work of Shrikhande. His main effort has been in the area of Combinatorial Designs and related topics. Several times, he developed path-breaking techniques, generating a lot of activity in the field. He himself continued to pursue them and enjoyed applying them to any new structure he studied. Often he successfully used them to solve well-known problems.

As mentioned earlier, the most well known among them is the result on historical Euler's conjecture on existence of orthogonal Latin Squares, that a pair of orthogonal Latin Squares of order $4t + 2$ does not exist. Shrikhande together with Bose disproved this conjecture for a particular case in 1959. Together with E. T. Parker, they showed later that Euler was wrong except for the orders 2 and 6. The methods developed by Bose and Shrikhande in construction of designs helped, not only in the solution of this problem, but gave a great impetus to the study of Design theory as such. A significant development, in terms of methods used, was the proof of embedding quasi-residual designs into symmetric BIBDs, jointly done with Bose and Singhi in 1974.

Rather than attempting an exhaustive survey of his work (his collected works are available), we explore how some of his methods led to these interesting developments. In 1959–60, he published two papers on characterization of association schemes. His paper on triangular association scheme $T(n)$ along with papers of Hoffman and Connor, characterized the scheme $T(n)$ by parameters, ($T(n)$ is the line graph of the complete graph K_n) for all n, $n \neq 8$. In the second paper, he characterized the association scheme $L_2(n)$ (or the line graph of the complete bipartite graph $K_{n,n}$ for all n, $n \neq 4$. These results show that the $T(n)$ and $L_2(n)$ graphs are characterized by their eigenvalues, the only exceptional cases being $T(8)$ and $L_2(4)$.

Seidel showed these exceptional cases are related respectively, to the $T(8)$ and $L_2(4)$ graphs by a process now referred to as Seidel switching. These were among the first papers in this direction. Such characterizations became an important activity in the 1960s and 1970s. Several papers appeared characterizing various graphs of balanced block designs and partially balanced block designs by Hoffman, Hall, Mesner, Connor, Bose, Ray-Chaudhuri, Seidel, Goethals, Bhagwandas, Singhi, etc., among others.

Hoffman and Ray-Chaudhuri characterized line graphs of projective spaces. In a landmark paper, R. H. Bruck characterized graphs arising out of a Net or affine designs. Bruck's paper developed the basic mathematical structure of graphs to study all such graphs. Bose generalized and unified all these characterizations in his famous paper on partial geometries and strongly regular graphs.

A decade later, Shrikhande together with Bose and Singhi extended the whole theory developed for graphs to multigraphs, by defining a very general structure called partial geometric designs.

In this context, Bose had mentioned the following incident to one of the authors. In 1961, Bruck showed to Bose an example (for $n = 4$) where $n - 3$ mutually orthogonal Latin squares (mols) cannot be extended into a complete set of mols. Shrikhande very soon showed that $n = 4$ is the only exception and that for all n, $n \neq 4$, any $n - 3$ mols can be completed into a full set of mols. When Bruck came to know of this, he might have been disappointed, he missed an interesting theorem. But he pursued the matter further resulting in his landmark paper on Nets generalizing Shrikhande's results to any set of $n - d$ mols, for all sufficiently large n, which in turn resulted in Bose's famous paper on partial geometries in 1963.

In 1960, Shrikhande published a simple proof of a well-known result of Hall and Connor on embedding quasi-residual designs with $\lambda = 2$ into symmetric BIBDs, using characterizations of triangular scheme. These methods were generalized in 1974 by Shrikhande and Singhi to show that embedding is possible for $\lambda = 3$ and for all such designs of block sizes more than 100. Finally Shrikhande, jointly with Bose and Singhi developed the general concept of partial geometric designs to prove the embedding results for any λ, and sufficiently large block sizes. An important aspect of this work is the generalization to multigraphs of the methods of Bruck and Hoffman in connection with the study of Nets using Claws and Cliques.

In 1959, Shrikhande, jointly with Bose devised an ingenious technique of using pairwise balanced designs (PBD) with varying block sizes to construct regular designs like BIBDs and latin squares, by pasting together smaller PBDs. Apart from its immediate application in disproving Euler's conjecture on Latin Squares, these techniques led to tremendous activity among design theorists. These developments led to the solution of Kirkman's school girl problem by Ray-Chaudhuri and Wilson, as well as to the proof of existence of t-designs for $t = 2$ (BIBDs) for sufficiently large number of points, by Wilson. Existence conjecture for t-designs for $t \geq 2$ was one of the central problems around which much of the theory of designs is developed. Some of those who contributed significantly to developing these techniques include Hanani, Ray-Chaudhuri, M. Hall, Wilson, Connor, Mullin and Stanton, Graver and Jurkat, Graham, Li, Li, Singhi. The conjecture was settled very recently by probabilistic methods by Keevash.

While working on the central areas of the theory of designs, Shrikhande also wrote several interesting papers on other problems. In late 1960s, he developed

with Bhat, a method of using good intersection patterns in Hadamard designs and affine designs to generate a large number of non-isomorphic solutions. Later Singhi extended their methods to classify all (19, 9, 4) designs. V. C. Mavron generalized these results to develop a complete theory to study matroids and groups related to affine designs.

Shrikhande's paper with Bhagwandas on Seidel equivalence of graphs is quite exceptional. They showed that all interesting examples of strongly regular graphs with the same parameters are equivalent in this sense. Seidel, Goethals, Hoffman and Ray-Chaudhuri extended their results considerably.

One of his well-known works is the joint paper with Singhi published in 1974 on the λ-design conjecture of Ryser. They showed that the conjecture is true for all primes. Their result has been only marginally improved so far.

Erdös used to like a joint paper of Shrikhande and Singhi published in 1986, in which they showed that if a conjecture of Erdös and Larson on pairwise balanced designs is true, then the projective plane conjecture is false.

A remarkable fact about Shrikhande, not usually mentioned, is that while in India, he did active research, although holding a good deal of administrative responsibilities. Head of a major department, Director of Centre for Advanced Study in Mathematics, member of several learned societies, member of governing council of the Indian Statistical Institute, member of various committees constituted by these bodies as well as many Indian Universities, it would have been quite tough for an ordinary person to give enough time to do his own research, let alone excel in his work. The Centre of Advanced Study in Mumbai University under his directorship was considered to be one of the best centres of research in Combinatorics in the world and attracted many experts from all over the world.

These were the days of license raj in India. Universities did not have enough funds. But Shrikhande with his simple but effective style, used meagre resources very well. He for example rarely used formal paper to write down his research papers. He used to open up used large envelopes, which he used to get in mail and use them to write. Most of his research was sent to typist on such envelopes.

One admirable quality of Shrikhande is that in spite of his great achievements he continued to be kind and courteous and genuinely helpful. We have often heard that among the famous people, Shrikhande may be one of the few with a large circle of friends and admirers. Many among Mathematicians in India working in Combinatorics, like V. N. Bhat Nayak, S. S. Sane, N. M. Singhi, K. S. Vijayan, S. B. Rao, A. R. Rao, Vijayakumar, R. Naik have sweet memories of help and guidance, Shrikhande had provided, at crucial moments of their careers. On the initiative of some of these, the Indian Statistical Institute organized an international seminar in his honour in 1982, on the occasion of his 65th birthday. Some of the best known names in Combinatorics from around the world were present on that occasion.

After the passing away of his wife Shakuntala almost a decade ago, Shrikhande for some years divided his time between India and USA, where his children Mohan, Asha and Anil are well settled. One of his sons Mohan Shrikhande himself is a famous Mathematician.

For last few years he has been mostly staying near Vijaywada in Andhra Pradesh in India. He had a large family of children, nephews, grandchildren, students, spread all over the world, who used to regularly visit him and keep in touch with him via phones, skype, etc. wherever they were. Till end at the age of 102, he had kept alive his interest in Mathematics. Whenever any of us called him, including his son Mohan, he will ask 'what are you doing in Mathematics' and keenly tried to understand problems and results.

He was an amazingly inspiring person. He maybe no more but his presence will be felt always via his Mathematics and in sweet memories of hundreds of researchers associated with him.

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Virender Lal Chopra (1936–2020)

Professor Virender Lal Chopra, an eminent agricultural geneticist and biotechnologist, passed away on 18 April 2020 at New Delhi. He was born on 9 August 1936 in Adhwal, a small village in the periphery of Rawalpindi in West Punjab that was part of British India. His father Harbans Lal and mother Sukhwanti moved to Delhi following partition of India in 1947 which also was the year of India's independence. His schooling was at Ramjas School, Delhi. He did B Sc (Honours) Agriculture from Central College of Agriculture (affiliated to University of Delhi), which was located at the Indian Agricultural Research Institute

(IARI), New Delhi in 1955. One of us (P.C.K.) joined the Central College of Agriculture two years later. During 1955–1957, Virender did his Associateship at the IARI with the other (M.S.S.), who had joined the Botany Division in 1954. Then he obtained the Senior Humboldt scholarship and studied at the Institute of Genetics, University of Cologne. On his return, he joined the cytogenetics group led by M. S. Swaminathan at the then Botany Division (which was subsequently changed as Genetics Division) of the IARI. The cytogenetic research during the 1950s and 1960s at the IARI was internationally acclaimed as among the