

## Hidden treasures of the past

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*In a space of four years, three Nobel Prizes in physics have been awarded in area of relativity, relativistic astrophysics and cosmology. This note will limit itself to Nobel Prizes in 2017 and 2020 to which India has made significant contributions in the past. The 2020 Nobel Prize in Physics was shared among Roger Penrose, Andrea Ghez and Reinhard Genzel. Penrose received the prize for establishing that black holes are a robust prediction of general relativity. The 2017 Nobel Prize went to Rainer Weiss, Kip Thorne and Barry Barish for decisive contributions to the LIGO detector and the observation of gravitational waves. From India there were significant contributions to these discoveries towards the understanding of black holes, formation of singularities and direct detection of gravitational waves. This note describes and also highlights these contributions.*

The general theory of relativity (GTR) was founded by Albert Einstein in 1915, little more than a century ago. It is really a theory of gravitation which encompasses Newton's theory of gravitation and the special theory of relativity<sup>1</sup>. In the weak-field slow motion limit, Einstein's theory reduces to Newton's theory. We first briefly explain the two theories and the relationship between them and then consider their spectacular predictions and further describe the Indian role in these endeavours.

The theory of gravitation one usually learns at first is Newton's theory of gravity and the related inverse square law. Newton's theory not only explained terrestrial gravity – the legendary falling of an apple – but also the motions of astronomical objects such as planets orbiting the Sun and the Moon orbiting the Earth. From Newton's inverse square law and his laws of motion, one could derive the Kepler's laws for planetary orbits. Newton's theory came to be known as the universal theory of gravitation because it unified terrestrial gravity with gravity in space. It was a resounding success. So then why do we need another theory of gravity?

### A primer on Einstein's gravitation

In 1905, Einstein presented to the world his special theory of relativity<sup>2</sup>. This essentially deals with measurements of distance, time, mass, etc. when the experimenter is moving with respect to the system of objects he is measuring. Special relativity does not concern itself with any specific physical law, but requires all physical laws to conform to it. But Newton's theory of gravity does not conform to the special theory of relativity.

This inadequacy is corrected in Einstein's theory of gravity; it incorporates the special theory of relativity. As history bears out, GTR has come out in flying colours in all gravitation experiments conducted so far. Instead of modifying Newton's theory, Einstein formulated a conceptually completely different theory – GTR which is also a theory of gravitation.

We describe here GTR in a prescriptive manner. Matter and energy (described by the energy momentum stress tensor) curve the spacetime in their vicinity. Gravitation is the manifestation of the curvature of spacetime. If we consider our solar system with the Sun as a central body producing the gravitational field and planets responding to this field, in Einstein's theory the picture is that the Sun curves the spacetime around it and the planets move along the straightest possible paths they can in this curved geometry of spacetime. The orbit of the planet appears curved because the spacetime is curved. The planet strives to follow the 'straightest' possible path, but since the spacetime itself is curved, the 'straight' path appears curved. Compare the situation with a sphere, which is an example of the simplest curved space. On the sphere the great circles are the 'straightest' possible paths – but they are markedly different from the straight lines of Euclid's geometry. Such 'straight' paths in general are called geodesics.

Analogous to the Newton's inverse square law (or Poisson's equation given below), we need to prescribe how matter distribution curves spacetime<sup>3</sup>. This is accomplished by Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}, \quad (1)$$

where on the left-hand side (LHS) we have terms describing curvature as the Riemann tensor and the metric, and on the right-hand side we have the stress tensor of matter distribution. The constants  $G$  and  $c$  denote respectively, Newton's gravitation constant and the speed of light. On the LHS appear the Ricci tensor  $R_{\mu\nu}$  and the scalar curvature  $R$ , which are derived from the Riemann tensor  $R_{\mu\nu\lambda\sigma}$ . These equations are the analogue of Newton's equation

$$\nabla^2\phi = 4\pi G\rho, \quad (2)$$

where  $\phi$  is the Newtonian gravitational potential and  $\rho$  is the mass density of matter. But Einstein's equations are far more mathematically complicated. They are ten coupled, nonlinear, second-order partial differential equations for ten components of the metric tensor  $g_{\mu\nu}$  (the metric tensor is symmetric and so has ten independent components in four dimensions). The situation is far more complex than just one equation eq. (2) of Newton. Therefore, Einstein's equations are extremely difficult to solve in general. One way is to assume enough symmetries, which then effectively reduces their complexity, but even then solutions are hard to come by. For example, no exact analytic solution exists so far for the two-body problem in GTR. (This is in fact a textbook problem in Newton's theory and easily solved.) It is only after years of clever hard work and only recently, that some reasonable progress has been possible.

Further GTR reduces to Newton's theory of gravitation in the limit of weak fields and slow motion as it must, because a new theory must certainly explain phenomena described by the old

theory in its regime of validity. However, when velocities are comparable to the speed of light and when the fields are strong as in the vicinity of black holes, Newton's theory can no longer describe gravitational phenomena accurately and reliably – GTR must be used. The successes and predictions of GTR are spectacular. GTR predicts the expanding universe, black holes and gravitational waves among many other physical phenomena.

### Black holes and singularities

#### *The lull before the storm*

Although Einstein's equations are extremely difficult to solve, surprisingly, the first solution came in 1916, just about a year after Einstein gave his field equations. This is the famous Schwarzschild solution<sup>4</sup>. It describes a non-rotating, spherically symmetric, electrically neutral black hole. The solution was obtained under the assumption of spherical symmetry and in vacuum. This solution has a singularity at its centre. Not much later the cosmological solutions – the Friedmann–Lemaître–Robertson–Walker models – were found, which also assumed high degree of symmetry and had a singularity in the past.

Apart from these early successes, the subject lay quiescent for several decades. The basic reason seems to be that there were very few experiments/observations to guide further progress. It should be realized that the root cause for the lack of experiments is that gravity is a weak force – it is the weakest among the four fundamental forces that we know of. Other interactions overwhelm gravity in the laboratory. Also, it is difficult to sift gravity from the other forces. Therefore such experiments were hard to design and perform in the laboratory (although, in the astronomical scale, it is gravity that dominates). Thus, till the early 1950s several of the world's leading scientists considered GTR to be an inactive field. It is only in the past few decades, with precision technology taking great strides that such experiments have become possible, although still very difficult.

For a different reason, even Einstein did not take the Schwarzschild solution seriously as a physical solution. The solution described a black hole – we will

describe a black hole more precisely later in the text. It is an object from which not even light can escape from its interior. Moreover, the solution has a singularity at its centre, where the gravitational tidal forces become infinite and physics breaks down. Then some other theory must take over, such as the quantum theory of gravity. Were these features artifacts of the symmetry assumed? In the real universe it would be impossible to find such a perfect symmetry. So the question arises: would such a black hole form at all, or even if it did form somehow, would it survive the asymmetry perpetrated by its asymmetric environment? The solutions in fact raised more questions than they answered. What is in fact a black hole? How does one define a singularity? Are the black holes and singularity artifacts of the assumed spherical symmetry?

#### *The storm*

The storm first arrived in 1955 with Amal Kumar Raychaudhuri from India. He gave his famous equation – the Raychaudhuri equation in 1955 (ref. 5) – Einstein expired the same year. The Russians, Lifschitz (well-known to many from the famous Landau–Lifschitz series) and Khalatnikov<sup>6</sup> around that same time tried to introduce rotation into a collapsing star to check whether it can in some way halt the singularity. In 1963, the solution for a rotating black hole was given by Roy Kerr<sup>7</sup> from New Zealand. In 1965, Roger Penrose<sup>8</sup> from the UK proved the first singularity theorem for which he has received the Nobel Prize 2020. Later in 1970, Stephen Hawking, Penrose and Geroch went on to establish more singularity theorems<sup>9</sup>.

These theorems showed that singularities were inevitable in GTR if the matter causing them behaved in some reasonable way – if it obeyed the so-called energy conditions. However, in establishing these theorems the Raychaudhuri equation is vital; it, in fact, paved the way for these theorems. Furthermore, these theorems hold generically, regardless of symmetries. Also black holes could form regardless of any assumption of symmetry, that they were a robust prediction of GTR. C. V. Vishveshwara (Vishu for his friends) gave a clear description of the black hole or event horizon. Then he investigated the stability of the Schwarz-

schild black hole against small perturbations and showed for the first time that a black hole rings like a bell – the black hole emits quasi-normal modes as it settles into its stationary state<sup>10</sup>. This is now called the ring down of black holes and has been observed in the recent observations of black holes by LIGO and Virgo in their data runs O1, O2 and O3 (ref. 11).

In what follows, we will describe in some detail the contributions of Raychaudhuri and Vishveshwara to these endeavours.

### Understanding black holes

Vishu's salient work will be described in this context. We list these works below:

- The distinction between the infinite red-shift surface and the event horizon of a black hole<sup>12</sup>.
- Investigating the stability of the Schwarzschild black hole<sup>13</sup>.
- Quasi-normal modes of black holes<sup>10</sup>.

For the first time, the distinction between the infinite red-shift surface and the event horizon – which are not the same in general – was clearly pointed out by Vishu. The Schwarzschild metric in the usual coordinates is given by

$$ds^2 = \left(1 - \frac{2m}{r}\right) c^2 dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (3)$$

where  $m = GM/c^2$ ,  $M$  being the mass of the black hole. Here  $m$  has the dimensions of length. In order to get some idea of the numbers involved, if our Sun whose mass  $M \sim 2 \times 10^{33}$  g were to become a black hole, then  $m \sim 1.5$  km. To understand the concept of the infinite red-shift surface, consider the following situation. Let a source situated at  $r = r_0 > 2m$  emit light in the outward radial direction (for simplicity). If this light is observed by an observer very far from the central mass, it is easy to show that the light is red-shifted. In the Newtonian picture, the photons must do work against gravity of the central mass and hence must lose energy. Now if the source of light at  $r_0$  is moved towards the  $r = 2m$  surface, the red-shift increases and tends to infinity as  $r_0 \rightarrow 2m$ .

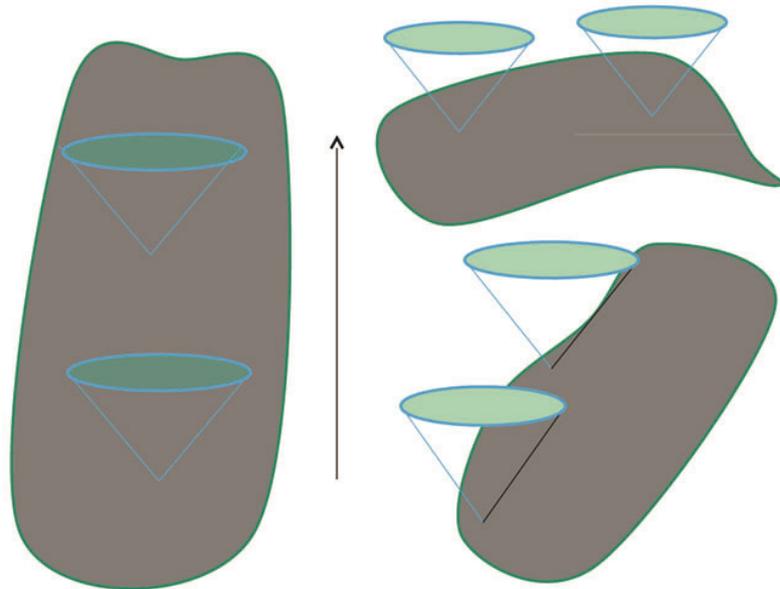
Therefore, the surface  $r = 2m$  may be called the infinite red-shift surface. But as we will see,  $r = 2m$  happens also to be the event horizon – a one way membrane for causal propagation. Due to this coincidence, there was no clear understanding.

While in the case of the rotating black hole – the Kerr solution – the two are different. The infinite red-shift surface is defined by  $r = m + \sqrt{m^2 - a^2 \cos^2 \theta}$  (in the Boyer–Lindquist coordinates), while the event horizon is given by  $r = m + \sqrt{m^2 - a^2}$ , which distinctly lies inside the infinite red-shift surface. Here  $a$  represents the angular momentum of the black hole.

For the first time the event horizon was explained by Vishu in a simple way for black holes which have reached their final equilibrium state, or what is known as the stationary state – it is a one-way membrane for causal propagation. This was elegantly demonstrated by Vishu as described below.

One can envisage three kinds of hypersurfaces in a spacetime: time-like, null and space-like. For a time-like hypersurface, the future lightcone (from now for brevity we will use the term lightcone for future lightcone) lies on both sides of the hypersurface – the hypersurface cuts through the lightcone (Figure 1, left). The particles are allowed to cross the hypersurface in both directions. A time-like hypersurface is a two-way membrane, like a door in a wall. However, for null and space-like hypersurfaces the lightcones lie on only one side, which implies that particles can cross the hyper-surface in only one direction – these are called one-way membranes (Figure 1, right-top and bottom).

Even in the spacetime of special relativity (no gravity), there are space-like and null hypersurfaces. For example,  $z = ct$  is a null hypersurface. It is easy to see that the lightcones lie on one side just touching the surface, and therefore it is a one-way membrane. However, it is not anything out of the ordinary, because it extends to infinity and is a part of everyday life. It can be taken to represent a plane wavefront of light travelling along the  $z$ -axis. The wavefront can only cross a particle/observer once, after which the particle/observer cannot catch up with it, let alone overtake the wavefront to cross it again, because that would be tantamount to the particle/observer travelling faster than light, which is not allowed.



**Figure 1.** (Left) Time-like hypersurface. The (future) lightcone extends on both sides of the hypersurface and so particles can cross the surface from either side. (Right, top and bottom) Hypersurfaces that are spacelike and null respectively, and so have lightcones only on one side – particles can cross in one direction only. Hence these are one-way membranes. The vertical arrow at the centre depicts the direction of time. One space dimension has been suppressed in the figure.

In a curved spacetime however, it is possible to have closed null and space-like hypersurfaces which do not extend out to infinity. Such surfaces are one-way membranes and even light cannot escape from them. Since far away from the central mass we do not encounter such surfaces, by invoking continuity arguments (of the metric), one may argue that the outermost one-way membrane is a null hypersurface. This is the event horizon. For the Schwarzschild solution,  $r = 2m$  is the event horizon. Thus it describes a black hole solution.

The second piece of work concerned the stability of the Schwarzschild black hole. This is an important question to address from the point of view of astrophysics. It had been shown earlier that a gravitational collapse of a spherically symmetric configuration leads to a black hole – the Schwarzschild solution. Granted that a Schwarzschild black hole can be formed in nature, the question arises whether such an object can continue to exist in nature. A priori, there is no reason to believe that the Schwarzschild spacetime represents a stable configuration.

The stability problem was first studied by Regge and Wheeler in 1957, but it remained unsolved. Although they devised a method of using tensor spherical

harmonics to investigate the perturbations, they still used the Schwarzschild coordinates in which the metric has a coordinate singularity at  $r = 2m$ . Due to this it was impossible to judge whether any divergence shown by the perturbation at  $r = 2m$  was physical or only a spurious effect caused by improper choice of the coordinate system.

In 1960, a coordinate system was furnished for the Schwarzschild spacetime in which the metric was non-singular at  $r = 2m$  (ref. 14). Vishu used the Kruskal–Szekeres coordinates for addressing the stability problem. This separated the physical effects from the spurious – any observed divergence of perturbations could be interpreted unequivocally as physical. Vishu showed that if one initially (at  $t = 0$ ) started with modes which were regular in the spatial region exterior to the horizon  $r = 2m$  and out to infinity, then they must have real frequencies if they are not to diverge at the horizon. He showed that modes with real frequencies were stable, although he pointed out that issues could arise in building up a generic perturbation from these modes. Such were the great strides taken by Vishu in the context of the stability problem of the Schwarzschild black hole. These issues were later dealt with by Zerilli, and Chandrasekhar and Detweiler<sup>15</sup>. For a

complete proof of the linear stability of the Schwarzschild black hole, see Dafermos *et al.*<sup>16</sup>.

The third important observation was discovering the quasi-normal modes of the Schwarzschild black hole. If a black hole is perturbed for an instant say, it rings like a bell – the black hole oscillates at certain characteristic frequencies which depend on its mass with the oscillations decaying in amplitude with time.

The problem Vishu considered was the scattering of gravitational waves by a Schwarzschild black hole. He used the formalism developed by Regge and Wheeler. He sent in a Gaussian wave packet towards a Schwarzschild black hole and looked for the reflected wave. What he found was an outgoing wave at infinity which not only oscillates, but the oscillations decay within a characteristic time. The frequency of oscillation and amplitude decay rate are dependent on the mass of the black hole and therefore bring information about the mass of the scattering object. In a more general setting of say a rotating black hole, the quasi-normal modes carry information about the mass and angular momentum of the black hole. In the current detections of gravitational waves by LIGO and Virgo, the quasi normal modes were clearly observed in the binary black hole waveforms.

Vishu predicted the quasi-normal modes of a black hole for the first time in 1970 and recently they have been seen by the LIGO and Virgo in their observations.

### The Raychaudhuri equation

As remarked before, GTR was considered a difficult and inactive field in the 1950s, because there were no experiments to speak of to guide the theory. So even globally, only a handful of scientists pursued GTR. It was at such an inopportune time Raychaudhuri made his important contribution – the famous Raychaudhuri equation – which has had such a far-reaching impact – recently, culminating in the Nobel Prize in physics to Penrose in 2020.

A lot of leading scientists, Einstein included, surmised that singularities and the Schwarzschild black hole, in particular, were just artifacts of high symmetry. It was thought, for example, that if one

introduced a little rotation, say, in a collapsing scenario, it would not lead to a singularity – the particles would just swirl around each other without collapsing to a singularity. Raychaudhuri was basically concerned with the cosmological singularity occurring in the Friedmann–Lemaître–Robertson–Walker models which were generally accepted because they explained the observed redshifts of galaxies. He asked the following question: is the initial singularity in these models due to the strong symmetry assumptions made in setting up these models?

With this question in mind, Raychaudhuri, without assuming any symmetries on the geometry, derived an equation – the famous Raychaudhuri equation<sup>5</sup> – whose clear consequence was the occurrence of singularities for which matter obeyed reasonable physical conditions. This was in the year 1955 and it incidentally coincided with the demise of Einstein. Later in 1965–70, the famed singularity theorems were rigorously proved by Hawking, Penrose, Geroch and Carter for which Raychaudhuri’s results were vital. In fact Raychaudhuri’s equation paved the way for these theorems. These theorems used very general differential topological methods and causality arguments to prove rigorously that singularities are inevitable in GTR under very general conditions imposed on the matter distribution<sup>9</sup>.

We now state the Raychaudhuri equation. Actually there are Raychaudhuri equations which hold under different situations<sup>17</sup>. Here we will consider just two situations. In the first case, consider a flow of freely falling material particles which are described by a congruence of time-like geodesics described by a velocity field  $v^\mu$ . In this case, the Raychaudhuri equation in its usual notation is given by

$$\frac{d\Theta}{d\tau} + \frac{1}{3}\Theta^2 + \sigma^2 - \omega^2 + R_{\mu\nu}v^\mu v^\nu = 0, \tag{4}$$

for the expansion of  $\Theta$ . Here  $\sigma$  denotes the shear and  $\omega$  the vorticity or rotation of the congruence. We have chosen the proper time  $\tau$  to parametrize the geodesics – it is the time measured by an observer following a geodesic curve. There are similar equations for  $\sigma$  and  $\omega$  as well, but we will restrict ourselves to  $\Theta$  only. Our interest lies in  $\Theta$  which is the fractional rate of expansion of the area taken

orthogonal to the bundle of geodesics. The first thing to note here is that eq. (4) describes the evolution of  $\Theta$  along the flow and is essentially a geometric statement independent of Einstein’s equations.

If the flow does not rotate or has no vorticity,  $\omega = 0$  (it can be shown that if  $\omega_{\mu\nu} = 0$  at one instant of time, then it continues to remain zero). Further, if we demand that  $R_{\mu\nu}v^\mu v^\nu \geq 0$ , then  $d\Theta/d\tau \leq 0$ . This condition on  $\Theta$  implies the focusing of the geodesic congruence. This is in fact the focusing theorem. Introducing Einstein’s equations in the condition  $R_{\mu\nu}v^\mu v^\nu \geq 0$  leads to the condition  $(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)v^\mu v^\nu \geq 1$ , where  $T$  is the trace of  $T_{\mu\nu}$ . This is known in the literature as the strong energy condition. If the energy–momentum tensor has a diagonal form, with energy density and pressure, then this puts reasonable conditions on the energy density and pressure of matter. Assuming the strong energy condition leads to the result that in a curved space-time manifold, generically there are world-lines of free particles which are necessarily incomplete. That is, they inevitably focus into singularities in a finite proper time  $\tau$ .

Besides the above-mentioned important consequence, there are many other applications of the Raychaudhuri equation. We will just describe one more here from which follows the area theorem for black-hole horizons. We now consider the flow to be of null geodesics described by a null vector field  $k^\mu$  and also which is irrotational ( $\omega = 0$ ). Since there is no proper time defined along a null geodesic, it is replaced by an affine parameter  $\lambda$  which marks the points along the geodesic. The Raychaudhuri equation now reads

$$\frac{d\Theta}{d\tau} + \frac{1}{2}\Theta^2 + \sigma^2 + R_{\mu\nu}k^\mu k^\nu = 0. \tag{5}$$

The only change here is that the coefficient of  $\Theta^2$  which was  $1/3$  in eq. (4) is replaced by  $1/2$ . This is because we are dealing with one dimension less. This change, however, does not affect the fundamental conclusions that were drawn from eq. (4). If  $R_{\mu\nu}k^\mu k^\nu \geq 0$ , then this results in general to the focusing of the geodesics. From Einstein’s equations, we have  $R_{\mu\nu}k^\mu k^\nu = T_{\mu\nu}k^\mu k^\nu \geq 0$ . In the literature, this is called the weak energy condition on the matter distribution

described by  $T_{\mu\nu}$ ; it physically means that every observer in his rest frame measures the energy density of matter to be non-negative. This is a reasonable condition for matter to obey.

We may apply these results to the event horizon of a black hole which is generated by null geodesics. If  $\Theta < 0$  at some point  $\lambda = \lambda_0$  on the geodesics, then it leads to a singularity within finite values of  $\lambda$ . Since the event horizon cannot end with a singularity in the future<sup>9</sup>, we must have  $\Theta \geq 0$ . However, if we consider a narrow bundle of geodesics with cross-sectional area  $\mathcal{A}$ , then since  $\Theta$  is just the fractional rate of change of this area, namely,  $\Theta = (1/\mathcal{A}) \times (d\mathcal{A}/d\lambda)$ , it follows that  $d\mathcal{A}/d\lambda \geq 0$ . These results imply that the area of the event horizon can never decrease in future. This is called the second law of black-hole thermodynamics. This behaviour is analogous to that of entropy in usual thermodynamics which never decreases in a thermodynamical process. Here the area of the black hole plays the role of entropy.

We have just mentioned two spectacular consequences of the Raychaudhuri equation. The equation has various other applications in diverse contexts. We refer the interested readers to Kar and Sen Gupta<sup>17</sup>.

## Gravitational waves

In 2017, the Nobel Prize Physics was awarded ‘For decisive contributions to the LIGO detector and the observation of gravitational waves’ to Rainer Weiss, Kip Thorne and Barry Barish. Gravitational waves were observed for the very first time by the LIGO detectors in 2015. Einstein predicted in 1916 that gravitational waves (GW) exist, but realized that they were too weak to be detected by the technology of those times. However, by the 1960s, the technology of lasers had arrived and at the same time, theoretical studies made extreme astrophysical phenomena such as black holes a reality, as we have described here – due to the work of several scientists with important contributions from India. Compact objects such as binary black holes are strong emitters of GW and their detection became a distinct possibility with the advent of laser interferometric detectors.

The current detectors of GWs are giant laser interferometers with armlengths

extending to a few kilometres – the LIGO detectors have 4 km armlength. As mentioned before, gravity is a weak force, and so are the effects of GWs on matter; extremely sensitive instruments are required in order to observe them. In spite of technological advances, a typical GW signal is buried deeply inside the detector noise; it requires advanced mathematical and statistical analysis of detector data to extract the GW signal from detector noise and glean key astrophysical information from them. It is in this aspect of GW observation that Inter-University Centre for Astronomy and Astrophysics (IUCAA) made significant contributions in the last three decades. IUCAA made foundational contributions to the theoretical underpinnings of GW detection, especially in data analysis. This effort also led to developing a GW community which has in turn led to Ligo-India – building an interferometric detector on Indian soil<sup>18</sup>. We briefly list the salient contributions below.

The stationary phase approximation of the black-hole merger waveform was derived, which has been the mainstay in field for the last three decades. Almost immediately following this, a general procedure using matched filtering<sup>19,20</sup> with a bank of templates was set up in searching for binary black-hole/neutron star GW signals<sup>21,22</sup>. It now lies at the heart of the currently used algorithms by Ligo and Virgo. One of the key ideas used in the construction of a template bank is a geometrical framework for doing GW data analysis. The parameter space is viewed as a manifold and a suitable metric is defined on it<sup>23</sup>. This was done for the first time in GW data analysis, and allows templates to be placed in the parameter space in an uniform and efficient manner.

In searching for GWs from merging compact binaries, a hierarchical scheme was put forward, whose goal was to speed up the search and reduce computational resources<sup>24,25</sup>. For black-hole binary GW signals, we showed how data can be combined from multiple detectors in a phase-coherent manner which effectively treats the network of detectors as a single detector<sup>26</sup>. We developed the radio-metric search first for stochastic GW<sup>27</sup> – the main idea emanating from Weiss and Albert Lazzarini. It was later suitably modified and extended for periodic sources, namely, asymmetric spinning neutron stars<sup>28</sup>. It has been found recently

that for periodic sources this method performs best among all other search methods and is being applied to the recently concluded Ligo–Virgo data-taking run O3.

Turning to space-based detectors which necessarily involve unequal interferometer arms, time-delay interferometry must be employed to cancel the laser frequency noise in order that the detectors achieve requisite sensitivity. The basic idea of time-delay interferometry is to combine data streams with appropriate time-delays so that the laser frequency noise is cancelled. We showed how abstract mathematics such as commutative algebra and algebraic geometry could be used to cancel laser frequency noise<sup>29</sup>. The cancellation method is based on finding laser paths of equal length and this problem, in principle, is the same as finding the lowest common multiple (LCM) of two integers, familiar from school arithmetic. In school arithmetic, one is dealing with a mathematical structure – the ring of integers – which is much simpler. Such a problem can be handled by even school children. Here we have a far more complex structure of general polynomial rings and consequently, the procedure is enormously complex involving advanced mathematics.

Parallely, an effort towards waveform modelling of binary inspirals was launched in the Raman Research Institute (RRI), Bengaluru by Bala Iyer in collaboration with the French<sup>30–32</sup> (also see Blanchet<sup>33</sup> for more references). As mentioned earlier, there is no exact analytical solution to the two-body problem in GTR, and therefore approximate solutions and numerical relativity must be resorted to. Both these endeavours require high level of analytical and computational skills. The French and Indian effort involved computing the inspiraling binary waveform using iterative approximation methods carried to several orders (called post-Newtonian). This is important for obtaining sufficiently accurate phasing of the signal so that the matched filtering method can be successfully applied, and the detection and measurement of the GW signal can be carried out effectively.

These early efforts from IUCAA and RRI have led to the development of a competent GW community which has further led to LIGO-India – the building of a 4 km armlength interferometric

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detector on Indian soil. A detector located in this part of the world will supplement the global efforts in a major way. The former data analysis community has been augmented recently in an essential way by an experimental faction which is actively taking part in Ligo-India and the global effort.

### Concluding remarks

This note has been written with the purpose of bringing to light the early efforts of few Indian scientists who have contributed in an essential way to world science at the highest level in relativity and relativistic astrophysics. It is hoped that these world-class achievements will spur young researchers to scale even greater heights in the near future.

1. Einstein, A., *Ann. Phys.*, 1916, **354**, 769–822.
2. Einstein, A., *Ann. Phys.*, 1905, **322**, 891–921.
3. Narlikar, J., *An Introduction to Relativity*, Cambridge University Press, 2010.
4. Schwarzschild, K., *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)*, 1916, **1916**, 189–196; arXiv:physics/9905030.
5. Raychaudhuri, A. K., *Phys. Rev.*, 1955, **98**, 1123–1126.
6. Lifschitz, E. M. and Khalatnikov, I. M., *Adv. Phys.*, 1963, **12**, 185–249.
7. Kerr, R. P., *Phys. Rev. Lett.*, 1963, **11**, 237–238.
8. Penrose, R., *Phys. Rev. Lett.*, 1965, **14**, 57–59.
9. Hawking, S. W. and Ellis, G. F. R., *The Large Scale Structure of Space-Time*, Cambridge Monographs on Mathematical Physics, Cambridge University Press, 1973.
10. Vishveshwara, C. V., *Nature*, 1970, **227**, 936–938.
11. Abbott, B. P. *et al.*, *Phys. Rev.*, 2019, **X**, 9, 031040.
12. Vishveshwara, C. V., *J. Math. Phys.*, 1968, **9**, 1319–1322.
13. Vishveshwara, C. V., *Phys. Rev. D*, 1970, **1**, 287–2879.
14. Kruskal, M. D., *Phys. Rev.*, 1960, **119**, 1743–1745.
15. Chandrasekhar, S., *The Mathematical Theory of Black Holes*, Oxford Classic Texts in the Physical Sciences, Oxford University Press, Oxford, UK, 2002.
16. Dafermos, M., Holzegel, G. and Rodnianski, I., *Acta Math.*, 2019, **222**, 1214.
17. Kar, S. and Sengupta, S., *Pramana*, 2007, **69**, 49–76.
18. Iyer, B. *et al.*, LIGO-India, Proposal of the Consortium for Indian Initiative in Gravitational-wave Observations (IndIGO), Internal working note LIGO-M1100296, Laser Interferometer Gravitational Wave Observatory (LIGO), 2011.
19. Helstrom, C. W., *Statistical Theory of Signal Detection*, Pergamon, 1968.
20. Thorne, K. S., In *300 Years of Gravitation* (eds Hawking, S. and Israel, W.), Cambridge University Press, Cambridge, UK, 1987.
21. Sathyaprakash, B. S. and Dhurandhar, S. V., *Phys. Rev. D*, 1991, **44**, 3819–3834.
22. Dhurandhar, S. V. and Sathyaprakash, B. S., *Phys. Rev. D*, 1994, **49**, 1707–1722.
23. Balasubramanian, R., Sathyaprakash, B. S. and Dhurandhar, S. V., *Phys. Rev. D*, 1996, **53**, 3033–3055.
24. Mohanty, S. D. and Dhurandhar, S. V., *Phys. Rev. D*, 1996, **54**, 7108–7128.
25. Sengupta, A. S., Dhurandhar, S. and Lazzarini, A., *Phys. Rev. D*, 2003, **67**; 10.1103/physrevd.67.082004.
26. Pai, A., Dhurandhar, S. and Bose, S., *Phys. Rev. D*, 2001, **64**, 042004.
27. Mitra, S., Dhurandhar, S., Souradeep, T., Lazzarini, A., Mandic, V., Bose, S. and Ballmer, S., *Phys. Rev. D*, 2008, **77**; 10.1103/physrevd.77.042002.
28. Dhurandhar, S., Krishnan, B., Mukhopadhyay, H. and Whelan, J. T., *Phys. Rev. D*, 2008, **77**, 082001.
29. Dhurandhar, S. V., Rajesh Nayak, K. and Vinet, J.-Y., *Phys. Rev. D*, 2002, **65**, 102002; 10.1103/PhysRevD.65.102002.
30. Blanchet, L., Damour, T., Iyer, B. R., Will, C. M. and Wiseman, A. G., *Phys. Rev. Lett.*, 1995, **74**, 3515–3518.
31. Blanchet, L., Iyer, B. R., Will, C. M. and Wiseman, A. G., *Classical Quant. Grav.*, 1996, **13**, 575–584.
32. Blanchet, L., Iyer, B. R. and Joguet, B., *Phys. Rev. D*, 2002, **65**, 064005.
33. Blanchet, L., *Living Rev. Relativ.*, 2002, **5**; 10.12942/lrr-2002-3

ACKNOWLEDGEMENTS. I thank Dr. K. Kasturirangan for encouraging me to write an article highlighting the Indian role in relativity and gravitational waves at the highest level, and Prof. Jayant Narlikar for carefully reading through the manuscript and suggesting important improvements. I acknowledge the support of NASI Senior Scientist Platinum Jubilee Fellowship.

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