A modification to the Indian practice of scour depth prediction around bridge piers

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Estimation of the expected maximum scour depth is very crucial at the design stage of a bridge. In India, for design of a bridge pier, the guidelines given by the Indian Road Congress (IRC) code is generally followed. The IRC equation is generally used to estimate the mean scour depth with a factor of safety of two to obtain the maximum design scour depth. The IRC equation comprises of two factors pertaining to the flow characteristics and bed material characteristics respectively. However, pier geometry is one crucial attribute which contributes to the local scour, and it is not considered in the IRC equation. Due to this, the computed mean scour depth will be much less. In the present study, a modification to the IRC equation is suggested based on detailed experimental studies using different shapes of piers. The scour pattern is obtained on a sand bed for various shapes at different flow conditions. Based on the study, an empirical constant is introduced in the IRC equation, which attributes to the pier geometry. The proposed modified IRC (M-IRC) equation gives better scour prediction for all the shapes. The effectiveness of the proposed equation is verified by comparing with other equations available in literature. The proposed M-IRC equation will be very useful for the design of bridge foundation in Indian conditions.

Keywords: Bridge, equilibrium scour depth, IRC equation, local scour, modified IRC equation, pier geometry.

ASSESSMENT of the maximum expected scour depth is essential for the design of a safe bridge pier because one of the primary causes of failure of the bridge is local scour at the foundation. Several researchers have studied the various parameters affecting the formation of scour^{1–8}. Based on these studies, local scour at piers can be expressed as a function of the following: (1) bed material characteristics like granular or non-granular, cohesive or non-cohesive, erodible or non-erodible; (2) bed configuration such as flatbed or with bed forms, clear water scour or live bed scour; (3) flow characteristics such as velocity and depth just u/s of the pier, angle of attack, free surface or pressure flow; (4) fluid properties such as density, viscosity and surface tension; and (5) geometry of the pier such as type, dimensions and shape^{1–8}.

Neill equation (modified Laursen equation)¹¹

The Neill equation is given as follows

$$\frac{d_{\rm s}}{h} = 1.35 \left(\frac{y}{h}\right)^{0.3}.$$
 (1)

Here d_s is the equilibrium scour depth, b the width of pier and y is the depth of flow. In this equation, the factors considered are only approach flow depth and geometry of the pier. The flow velocity is not considered in this equation

Shen et al. formula¹¹

In this equation to predict the scour depth, the Froude number (F_r) is also incorporated, which represents the flow velocity, in addition to the pier width and approach flow depth is given as

$$\frac{d_{\rm s}}{h} = 3.4(F_{\rm r})^{2/3} \left(\frac{y}{h}\right)^{1/3}.$$
 (2)

Kothyari et al. formula¹²

In this formula in addition to the pier width, approach flow depth (d) and Froude number, an additional factor called open ratio (β) of the channel is considered. The open ratio of the channel $\beta = ((B-b)/B)$, where B is the centre to centre spacing between the piers and b is the width of the pier. The formula is as follows

$$\frac{d_{\rm s}}{b} = 0.66 \left(\frac{b}{d}\right)^{-0.25} \left(\frac{y}{b}\right)^{0.16} (F_{\rm r})^{0.4} \beta^{-0.3}.$$
 (3)

*Melville Sutherland equation*⁴

Melville and Sutherland⁴ developed a pier-scour equation based upon envelop curves drawn to experimental data

Several empirical equations are available to estimate the local scour depth at the upstream nose of the pier^{5–10}. Some of these are discussed below.

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obtained by many laboratory experiments. This expression is known as Melville and Sutherland equation as given below

$$\frac{d_{\rm s}}{h} = k_{\rm s} k_{\alpha} k_{\rm l} k_{\rm y} k_{\eta} k_{\sigma}. \tag{4}$$

Here k_s represents a pier shape factor, k_α a factor representing the angle of attack of flow, k_l represents a factor of flow intensity, k_v a factor of flow depth, k_η a factor for sediments size and k_σ represents a factor for sediment gradation.

Melville equation¹³

Melville¹³ modified Melville and Sutherland's equation as follows

$$\frac{d_{\rm s}}{h} = k_{\rm s} k_{\alpha} k_{\rm l} k_{\rm yb} k_{\eta} k_{\sigma}. \tag{5}$$

All the factors are the same as that of eq. (4) except k_{yb} , which is a composite factor of flow intensity and pier width.

HEC-18 equation

HEC-18 equation is used in a widespread manner around the world for estimation of scour depth⁷

$$\frac{d_{\rm s}}{b} = 2.0K_1K_2K_3 \left(\frac{y}{b}\right)^{0.65} (F_{\rm r})^{0.43}.$$
 (6)

 K_1 is a correction factor for pier nose shape, K_2 a correction factor for the angle of attack of flow and K_3 is a correction factor for bed condition.

65-2 equation⁸

The 65-2 equation was proposed by the Ministry of Railways of the People's Republic of China. It was developed by considering the influence of flow velocity, sediment size and pier width and is given as follows

$$d_{\rm s} = k_{\eta} k_{\rm s} b_{\rm e}^{0.6} h_{\rm p}^{0.15} \left(\frac{V - V_0'}{V_{\rm c}} \right) \qquad V \le V_{\rm c}, \tag{7}$$

$$d_{\rm s} = k_{\eta} k_{\rm s} b_{\rm e}^{0.6} h_{\rm p}^{0.15} \left(\frac{V - V_0'}{V_{\rm c}} \right)^{\eta_2} \qquad V > V_{\rm c}.$$
 (8)

Here k_{η} and k_{s} are factors representing sediment size and pier shape respectively, b_{e} the projected width of the pier,

 $h_{\rm p}$ the flow depth after contraction scour, V the depth-averaged flow velocity, $V_{\rm c}$ the critical velocity for sediment entrainment, V_0' is the incipient velocity for local scour at a pier.

IRC equation 14,15

In India, the Indian Road Congress (IRC) code is followed to get the guidelines required for construction of road bridges^{14,15}. The IRC equation is used for the design of bridges in India^{15,16}. IRC equation is developed from the Lacey's formula. The mean scour depth is obtained using the IRC formula as follows

$$d_{\rm sm} = 1.34 \left(\frac{D_{\rm b}^2}{K_{\rm sf}}\right)^{1/3}.$$
 (9)

The maximum scour depth is obtained by applying a factor of safety of 2. The maximum scour depth is given by

$$d_{\rm smax} = 2d_{\rm sm}. (10)$$

Here, D_b is discharge/unit width, $K_{\rm sf}$ the silt factor of the bed material given by $K_{\rm sf} = 1.76 (d_{\rm m})^{1/2}$ in which $d_{\rm m}$, is the weighted mean diameter of the sediment. Here $d_{\rm sm}$ is the sum of flow depth and scour depth.

In India, pioneering studies on formation of scour and its parameters were made by Kothyari *et al.* and others ^{17–22}. These studies highlighted the various parameters which affect the bridge pier scour and gave insight into issues pertaining to bridge pier failures. The scour is caused by an acceleration of flow around the bridge foundation that accompanies a rise in water levels during flood events. Local scour can be either clear-water or live-bed. Accurate determination of the equilibrium depths of local scour around bridge piers is a vital issue in the design of bridges concerned with the hydraulics engineering. The size of the scour holes depends on the shape of the pier that affects the flow. It is the shape of the pier that generates a turbulent flow, which, in turn, conditions the occurrence of the structures that affect bed scour process^{20–22}.

The experiments carried out in IIT Bombay revealed that the scour depth obtained using the IRC formula was much less in comparison to the scour depth observed from the experiments¹⁸. The IRC formula contains only two factors; discharge per unit width and the silt factor, which corresponds to the flow and soil characteristics respectively.

Circular piers are the most commonly used pier shape for bridges around the world. If the road/railway track above is wide with multiple lanes, elongated piers or multiple piers would be needed to support the superstructure^{18–20,23}. Depending on the width of the bridge, single or multiple circular piers are provided. When multiple piers are used, the scour pattern around each pier is

affected by the mutual interference in addition to the formation of the horseshoe vortex system around each pier^{20,21,24–26}. According to requirement at the field, engineers select the type of pier required. It is to be noted that the IRC equation does not have a factor attributing to the pier geometry.

In the present study, an attempt is made to understand, through experimental studies, whether a modification to the existing IRC equation considering the pier geometry parameter as a factor would improve its ability to give a better prediction of scour depth. Further, based on extensive experimental studies, the IRC equation is modified based on the pier geometry. The effectiveness of the modified IRC (M-IRC) equation is verified for further experimental data and in comparison with other equations available in the literature.

Experimental details

The experiments were carried out at the Hydraulics Laboratory, Indian Institute of Technology, Bombay. The initial experiments were run in a rectangular flume 8.5 m long, 0.305 m wide and 0.6 m deep (flume I) with a recirculating facility²². The walls and bottom of the flume are made of plexiglass. The rectangular flume has three sections: inlet section, outlet section and test section in the middle. There is a mild steel tank at the upstream end of the flume. Water is pumped into the tank from the sump. This tank is connected to the inlet section of the flume using a bell mouth arrangement. A flow straightener is provided at the end of the bell mouth. A tailgate is fixed at the end of the outlet section. The water flowing through the flume is collected in the outlet tank. The outlet tank is connected to the sump using a PVC pipe, enabling re-circulation. The water gets recirculated continuously through the flume. The test section of the flume is 4.75 m long. The test section was filled with the bed material for scour experiments. The depth of the sand bed was 0.2 m.

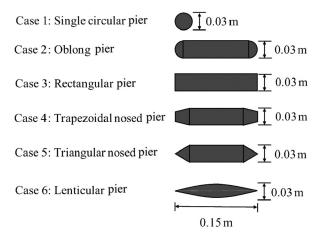


Figure 1. Shapes of piers used for this study.

Six cases of piers were used for this study. The various cases studied are as follows: single circular pier; oblong pier; rectangular pier; trapezoidal nosed pier; triangular nosed pier, and lenticular pier (Figure 1). The diameter of the single circular pier in case 1 and the width of piers in the rest of the cases were 0.03 m. The length of the piers for case 2 to case 6 was 0.15 m. Among these, cases 1, 2 and 3 were blunt-nosed piers, and cases 4, 5 and 6 are sharp-nosed piers.

For the experimental purpose, a measuring section at the flume was chosen at 3 m downstream from the beginning of the test section. The pier was placed at that location. The flow discharge was chosen in such a way that the dimensionless bottom shear stress was below the threshold value for the initiation of sediment movement at the undisturbed plane sand bed of grain size $d_{50} = 0.80$ mm, i.e. when there was no sand transport at the bed. Water from the tank entered into the flume so slowly that there would not be any disturbance to the sand bed. The outlet section was at the end of the test section, where the dislodged sand particles, if any, were deposited.

For each case, experiments were carried out for five different discharges ranging from 0.006 to 0.018 m³s⁻¹, with the flow parameters as given in Table 1. The bed material used was alluvial quartz sand of $d_{50} = 0.8$ mm. Each experiment was carried out for 8 h until the scour reached such a stage that the change in scour depth with respect to time was minimal.

Experimental results and modified IRC equation

The observations from all experiments were analysed. The scour depth obtained from all the experiments was compared with that obtained using the IRC equation ^{14,15}. It was observed that the scour depths obtained by IRC equation were much less than the experimental findings. The values obtained from the experiments²² appear to shift from the IRC equation by a constant value.

Attempts were made to find out the magnitude of that shift for each case by considering 50% of the data. A multiplying factor was obtained for each case, which when multiplied with the IRC equation gave a value closer to the observed scour. Blunt-nosed pier and sharp-nosed pier were considered separately. Fifty per cent of

Table 1. Flow parameters for experiments in flume I

Discharge $Q (m^3 s^{-1})$	Flow depth h (m)	Average velocity $U_{\rm av}~({\rm ms}^{-1})$	Reynolds number (Re)	Froude number (Fr)
0.018	0.165	0.36	59055	0.28
0.015	0.153	0.32	49212	0.26
0.012	0.143	0.28	39370	0.24
0.009	0.126	0.23	28980	0.21
0.006	0.111	0.18	19980	0.17

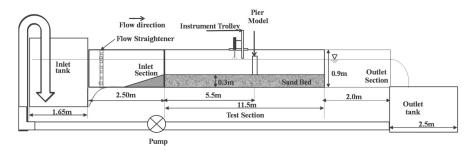


Figure 2. Schematic diagram of flume used for validation experiments (flume II).

Table 2. Shape factor corresponding to different shapes

Shape of pier	Shape factor (C)	
Single circular pier	1.43	
Rectangular pier	1.44	
Oblong pier	1.42	
Trapezoidal pier	1.15	
Triangular pier	1.15	
Lenticular pier	1.15	

Table 3. Flow parameters for experiments in flume II

Discharge Q (m ³ s ⁻¹)	Flow depth h (m)	Average velocity $U_{\rm av}({\rm ms}^{-1})$	Reynolds number (Re)	Froude number (Fr)
0.02	0.165	0.24	39286	0.19
0.018	0.159	0.22	35183	0.175
0.015	0.152	0.19	29469	0.158

the data was used for obtaining the multiplying factor. The multiplying factors obtained for various shapes of piers are given in Table 2. The average value of the multiplying factor obtained for each case was calculated and is proposed as the multiplying factor C. Based on the obtained results, the value of C for the blunt-nosed pier is proposed as 1.43 (though a small variation observed among circular, rectangular and oblong shapes), and for the sharp-nosed pier, it is recommended as 1.15. Analysis of the scour values obtained indicated that by introducing a multiplying constant, the IRC equation gives a much better prediction of scour. Hence modification of the IRC equation is proposed by introducing an empirical constant C, which attributes to the shape of the pier. The M-IRC equation is proposed as follows

$$d_{\rm sm} = 1.34C \left(\frac{D_{\rm b}^2}{K_{\rm sf}}\right)^{1/3}.$$
 (11)

Among all the cases, scour depth was the least around lenticular piers²².

Validation of the equation

To understand whether the modification constant is affected by sediments size, duration of the experiment, the size of the flume and to validate the proposed equation, further experiments were carried out on selected cases (case 1, case 2 and case 6) in a bigger flume, flume II. Flume II is a rectangular flume, 15 m long, 0.51 m wide and 0.90 m deep used for experiments (Figure 2). The bottom of the flume is made of mild steel plate. The walls are made of glass sheets. The rectangular flume is made into three sections: inlet section, outlet section and test section in the middle. There is a mild steel tank at the upstream end of the flume. Water gets recirculated into the flume from the sump using a pump. The tank is connected to the inlet section of the flume using a bell mouth arrangement. A flow straightener is provided at the end of the bell mouth. A tailgate is provided at the end of the outlet section. The test section of the flume is 11.5 m long. The test section was filled with the bed material. The depth of the sand bed was 0.3 m. The schematic diagram of the flume is shown in Figure 2. For experimental purpose, a measuring section at the flume was chosen at 5.5 m downstream from the beginning of the test section. The flow discharge was chosen in such a way that the dimensionless bottom shear stress was below the threshold value for the initiation of sediment movement at the undisturbed plane sand bed of grain size $d_{50} = 0.50$ mm, i.e. when there was no sand transport at the bed. Water from the tank entered into the flume so slowly that there would not be any disturbance to the sand bed. The outlet section was at the end of the test section, where the dislodged sand particles, if any, were deposited. These experiments were carried out for three different discharges. The bed material used was alluvial quartz sand of $d_{50} = 0.5$ mm. The flow parameters for these experiments are shown in Table 3. The duration of each experiment was 18 h.

The results obtained in flume II experiments were analysed and compared with the results from IRC equation and M-IRC equation, to understand the validity of the M-IRC equation. The comparison between scour depths obtained from the experiments in flume I and flume II as well as by IRC equation and M-IRC equation is shown in

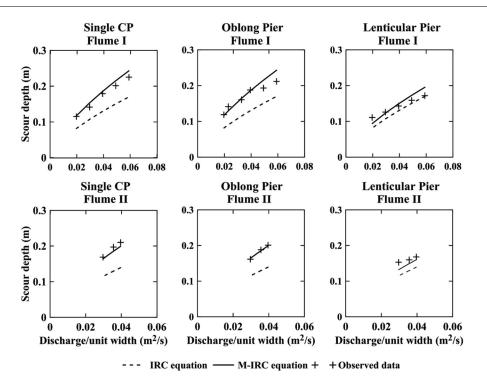


Figure 3. Comparison between the scour computed using the Indian Road Congress (IRC) equation and modified IRC (M-IRC) equation with the observed scour.

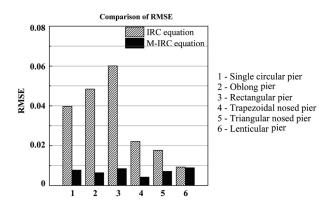
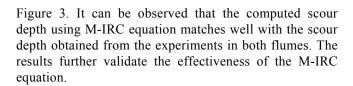


Figure 4. Root mean square error (RMSE) values obtained for IRC equation and M-IRC equation for experiments in flume I.





To comprehend how well the M-IRC equation is behaving, the root mean square errors (RMSEs) obtained using M-IRC equation, for the remaining 50% of the data, with respect to the observed scour for each of these cases were analysed and discussed in the following paragraphs.

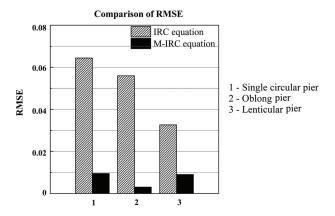


Figure 5. RMSE values obtained for IRC equation and M-IRC equation for experiments in flume II.

The root mean square error (RMSE) is given by the following relationship

RMSE =
$$\sqrt{\frac{\sum (Scour \ depth_{computed} - Scour \ depth_{observed})^2}{n}}$$
. (12)

Here n is the number of observations.

Comparison of RMSEs obtained using the IRC equation and the M-IRC equation in comparison with the observed value of scour for flume I experiments is shown in Figure 4 and that from the flume II experiments is

shown in Figure 5. The RMSE reduces significantly by introducing the modification constant as can be observed from both the figures.

The RMSE values for both equations for flume I experiment for all the cases are shown in Table 4. The lower values of RMSE indicate a better agreement between the observed and computed values. There is a significant reduction in RMSE values for M-IRC equation in comparison with the IRC equation. It can be inferred from the Table 4 that for most of the cases the reduction is in the

Table 4. Percentage reduction in RMSE for M-IRC (flume I)

RMSE				
Case	IRC	M-IRC	Percentage reduction for M-IRC	
1	0.039699	0.007726	80.54	
2	0.048433	0.006398	86.78	
3	0.060053	0.008474	85.89	
4	0.022129	0.004219	80.94	
5	0.017683	0.007107	59.81	
6	0.009252	0.0089	3.81	

RMSE, Root mean square error; IRC, Indian Road Congress; M-IRC, modified Indian Road Congress.

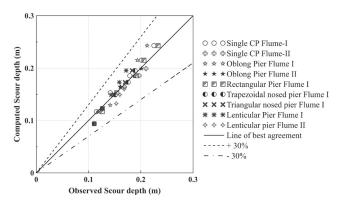


Figure 6. Comparison of observed and predicted values of scour depth computed using M-IRC equation.

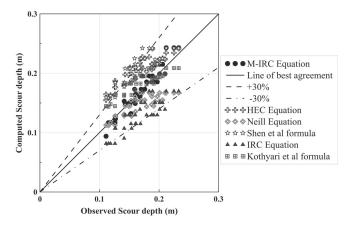


Figure 7. Comparison of computed scour obtained from various equations with the observed scour.

range of 60-87% except for Lenticular pier, the reduction is 3.81%. Lenticular pier is a sharp nosed pier with a curved body, owing to which the streamlines pass smoothly along the sides of the pier. Hence, the horse shoe vortex formation is minimized and therefore scour is much less. The effect of horseshoe vortex on scour is thus minimal for lenticular pier, owing to the reduction in scour as well as the multiplying factor C.

The comparison between the observed scour from experiments and calculated values with M-IRC equation is shown in Figure 6. It can be observed from the figure that the scour computed using the M-IRC equation agrees well with the observed scour within the range of –30% to +30% error values. Further to verify the effectiveness of the proposed M-IRC equation, a comparison between the results of various equations of scour calculations such as Neill equation¹¹, Shen *et al.* formula¹¹, HEC-18 equation⁷, Kothyari *et al.* formula¹², IRC equation^{14,15} and M-IRC equation is carried out. Figure 7 shows the comparison between M-IRC equation and other equations. As can be seen, the M-IRC gives a better agreement with the observed scour values compared to the IRC equation and other available equations.

As can be observed from the present study, the shape of a pier is an important parameter in the scouring process. The scour around the respective pier mostly depended on the shape of the bridge pier^{21,22}. The occurrence of the location of maximum scour depth also depended on the geometry of the pier shape. It is important to note that the trapezoidal-nosed, triangular-nosed and lenticular shapes were effective in reducing the scour depths for all discharges. The scour depth at the upstream nose of the pier was reduced with respect to the reduction in the frontal area. Thus, it is important to modify the scour calculation equation based on the shape of the pier. For the Indian conditions, as IRC equation is mostly used for scour calculation, the M-IRC equation will be useful for practical scour estimation for bridge pier designs.

Conclusions

For the bridge pier design, estimation of the expected maximum scour depth is essential. For bridge pier design in India, the IRC equation is generally used to estimate the mean scour depth around the bridge pier. In this study, a modification for the existing IRC equation is proposed based on the experiments carried out on different combinations of circular piers and different shapes of elongated piers, in laboratory flumes. A modifying factor is introduced having a value of 1.43 for blunt-nosed piers and 1.15 for sharp-nosed piers respectively. The M-IRC equation agrees well with the observed scour from experiments and falls within an error bar of 30%. The RMSE value reduced considerably for M-IRC equation. The modified equation can be used for the design of bridge

piers with an appropriate factor of safety as given in the IRC code so that the bridge foundation would be safe against scour.

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