# Kalyanapuram Rangachari Parthasarathy (1936–2023)*

Professor K. R. Parthasarathy ('Partha' to his western friends and collaborators and KRP to the Indian Mathematical Community) passed away on 14 June 2023. One of the handful of the legendary creators of schools of excellence of modern Mathematics in India, KRP was an expositor–par–excellence (both in lectures and in written articles) – a 'great teacher'. Like many very distinguished mathematicians, he was often 'terse', he would sometimes say that 'Mathematics is about the economy of thought and the economy of expression'. His absence will be felt for a long time in the Indian Mathematical scene.

KRP was born in Chennai, Tamil Nadu, had his early education (including B.Sc. honours in Mathematics) in Chennai before joining the 3-year 'advanced professional statisticians' training course in the Indian Statistical Institute (ISI), Kolkata in 1956. ISI was established in the 1930s by Prasanta Chandra Mahalanobis and for a long time developed and nurtured by him and C. R. Rao, the eminent statistician to grow into a world–class institution for study and research in Statistics, Probability theory, related areas of Mathematics and applications. The academic climate in ISI in 1950s and 1960s was somewhat unstructured and liberal and three bright young budding mathematicians (V. S. Varadarajan, R. Ranga Rao and KRP), to be expanded a few years later to include S. R. S. Varadhan as well and named for posterity 'the Famous Four', taught each other much of modern mathematics. Nearly two decades later when I met KRP for the first time in ISI, Delhi, he would often regale us with many stories of his time in Kolkata and talk about the 'underlying unity in Mathematics', for example, among the Kolmogorov's decomposition theorem, the Gelfand–Naimark–Segal construction and Bochner's theorem on positive definite functions. This point of view had a major influence on many of us, including myself. However, this expansive (wide–canvas) view of Mathematics needed a demanding and relatively mature mathematical background, which was often missing in average graduate students at that time, and many of them had some difficulty in keeping up with KRP's zeal and intensity.

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Under the supervision of C. R. Rao, KRP completed his Ph.D. dissertation entitled 'Some problems of ergodic theory and information theory' in 1962 and started a career in ISI itself. After a tradition instituted by the visionary founder of ISI, P. C. Mahalanobis, many distinguished Statisticians and Mathematicians from the world

over visited ISI. Professor A. N. Kolmogorov, the founder of modern theory of probability and a great mathematician, visited ISI in 1962 and KRP was deputed to accompany and guide him during his stay in India. This led to KRP visiting Kolmogorov for a year during 1962–63 in the Steklov Mathematics Institute in Moscow. Though life in Moscow at that time was quite difficult for a vegetarian KRP from a warm tropical country, he was very happy about having the opportunity of attending the seminars of great minds like those of E. B. Dynkin, I. M. Gelfand and others. When KRP returned from Moscow in 1963, Varadarajan had also returned from the United States. Though Varadhan left for the United States in 1963, this period (1962–65) saw a burst of academic activities among the 'famous four', resulting in 5–6 top-class publications, culminating with the 'Representations of complex semi-simple Lie groups and Lie algebras' by KRP, Ranga Rao and Varadarajan (*Ann. Math.*, 1967, **2**, 25). It was during this period, 1964–65, that Varadarajan started giving a course of lectures on 'Mathematics of Quantum Mechanics' (now available as a book by Springer-Verlag publications, 1984), which KRP attended and this may have sown the germ of interest in this area in KRP's mind. This interest stayed with KRP till his last days and may have been instrumental in my getting together with him.

Another landmark event happened to KRP: he got married to Shyamala (Shyama to friends) in 1965 and left for the United Kingdom. After several years of teaching at the Universities of Sheffield (1965–68) and of Manchester (1968–1970), he decided to return to India for good and following a few years in the University of Bombay (1970–73), in the Indian Institute of Technology, Delhi (1973–76), he settled in the new campus of the Delhi Centre of ISI from which he superannuated in 1996. There he continued as Professor Emeritus till his last days. His celebrated book *Probability Measures on Metric Spaces* (AMS 1967, reprinted 2005) was written during this period as was also the beautifully presented more basic book *Introduction to Probability and Measures* (Macmillan 1977, Hindustan Book Agency, 2005). I learnt my basics in Probability theory from that book and sometimes used theorems from this book to prove results in Operator theory.

The study of Central Limit theorems and of infinitely divisible probability distributions on topological spaces or groups were of great interest to the probabilists during the 60–70s and KRP was no exception; he wrote several important articles on these topics. In structures admitting some kind of continuous convolution as in the case of groups (much later quantum groups as well), it was known from KRP's (as well as many other probabilists') work that infinitely divisible distributions can be canonically associated with a one-parameter semigroups of maps on a suitable space of functions on the structures. On the other hand, KRP's studies and investigations in the underlying structures arising in Quantum Mechanics (including his mastery of Mackey's theory of induced representations and the systems of imprimitivity) led to the natural query of the possibility of dropping the underlying space/groups in the space of functions and replace them by some kind of *–algebras in a Hilbert space which is often the vessel carrying the description of quantum systems. This, 'large canvas', more formally the construction of Markov Processes over *–algebras in a Hilbert space occupied KRP and many of his collaborators for nearly four decades. This process of replacing classical objects like 'the commutative family of functions on a suitable set' by a non-commutative family of 'operators in a suitable Hilbert space' would become (at the hands of KRP and Robin L. Hudson), the central tool in creating a new non-commutative family of stochastic processes, driving the action of a one-parameter semigroup of (completely positive and unital) maps on a *–algebra in a Hilbert space. As is often the case, KRP first looked at the finite dimensional case, in which classical observables (random variables) were mapped into the set of real diagonal matrices there, while the quantum ones went into the set of all Hermitian matrices. In this setup, the classical processes driven by classical Markov semigroups were constructed naturally by Fokker–Planck type equation associated with stochastic matrices. This point of view allows a quick natural generalization to the more interesting infinite–dimensional case, reinforcing the thought, held strongly by KRP and many others, that the Quantum theory is fundamentally a theory of probability, albeit of non-Kolmogorovian variety.

This gradual, yet striking change in the point of view in the research canvas of KRP took place over the years (1972–onwards) via the paper on the representation of current groups and the Araki–Woods imbedding theorem (coauthored with K. Schmidt, *Acta Math*., 1972, 128) and the lecture note on positive definite kernels, continuous tensor products and central limit theorems of probability theory (with K. Schmidt. LNM 272, Springer, 1972). The continuous tensor product of an indexed family of Hilbert spaces $\{H_{s,t} | 0 \leq S \leq t < \infty\}$, turned out to be the right vehicle to incorporate, the semigroups driven by continuous–time quantum stochastic processes. After some interesting interludes including attempts by KRP (with R. L. Hudson and P. D. F. Ion, *Comm. Math. Phys*., 1982, 83) on Feynman–Kac-like formulae, as time–orthogonal product integral, KRP along with Hudson realized that the Fock–space representation of the unitary Weyl–Segal system on $L^2(R_+)$ provides an ideal setup to implement the continuous tensor product structure explicitly and develop a nice kind of stochastic calculus, and the rest is history.

At about this time in January 1982, as a part of the Golden Jubilee celebrations of the ISI, a conference on the 'Theory and Applications of Random Fields' was organized in the Bangalore Centre of the ISI, and a large number of luminaries in Probability theory including E. B. Dynkin, T. Hida, P. A. Meyer, S. Watanabe, K. Bichteler, J. Jacod lectured there. Both KRP and Hudson, of course, were there and they talked about their nascent theory and the quantum Ito table (with the annihilation and creation processes only) made its appearance, implicitly, there for the first time.

This non-commutative calculus led to the beautiful paper 'Quantum Ito's formula and stochastic evolutions' (*Commun. Math. Phys*., 1984, 93) and to a more elaborate and abstract presentation of the theory in the lectures of KRP in 1988 in the Delhi Centre of ISI. These lectures of KRP appeared in 1992 as a monograph 'An Introduction to Quantum Stochastic Calculus' (Birkhauser, 1992) and are presented in a delectable style with the right kind of mixture of abstract ideas and their concrete manifestations. Suitable linear combinations of the mutually conjugate position and momentum processes lead to the annihilation and creation processes and the now familiar complete Quantum Ito table appears for the first time, which was incomplete in earlier attempts in the absence of the 'Conservation or number process'. In fact, this process makes its appearance in the simple example of the 'second quantization' of an operator of multiplication by an 'adapted family' of function $\{f\chi_{[0,t]} | 0 \leq t < \infty, f \varepsilon L^2(R_+)\}$. These names led KRP, in his book to compare them with the trilogy of Gods in Hindu scriptures: Shiva (the annihilator), Brahma (the creator), and Vishnu (the conservator). This 'completion' allowed them to formulate and solve quantum stochastic differential equation (qsde) with bounded operator-valued coefficient in the Hilbert tensor-product of the initial Hilbert space (in which the operator-coefficients act) and the Fock space mentioned earlier. The unitarity of the solution, giving the quantum stochastic evolution of the Hilbert space vectors is of great importance (just as in the so–called 'Schrödinger picture' of ordinary quantum mechanics) and the authors solved this problem completely for the case of constant bounded operator coefficients. The book *Quantum Probability for Probabilists* by P. A. Meyer appeared soon after in 1992 and was written in a style perhaps closer to that of classical probabilities. Curiously, Robin Hudson earlier and Accardi, Frigerio and Lewis in their article 'Quantum Stochastic Processes' (Publ. RIMS, Kyoto Univ., 1982, 18) had emphasized the corresponding 'Heisenberg picture', obtained by conjugating an observable (or any bounded operator) in the initial Hilbert space by these unitaries mentioned above. This looked like an aesthetically satisfactory situation in the development of the theory.

However, for the theory to be more useful to serve as a possible model to describe 'dissipative phenomena in Quantum Physical systems', two further generalizations were needed: (i) a suitable class of unbounded operator coefficients, mentioned above, needs to be admitted in the analysis of the qsde and (ii) 'the stochastic or the noise' part, as modelled in the Fock space over $L^2(R_+)$, needed to be expanded in variety.

In the same paper (CMP, 1984), KRP and Hudson also showed that the two abelian *– algebras generated by functions of classical Brownian motion and by classical Poisson processes are sub-algebras of the non-abelian algebra of all bounded operators in the Fock space and classical notions of 'independence' in each of these cases are subsumed by the tensor-independence in the new structure. Thus, in a sense, the tensor-independence used in the so-called Hudson–Parthasarathy (or HP for short) stochastic calculus is the simplest generalization of the concept of independence in classical probability; the one farthest from the classical independence is that of 'free independence' introduced by Voiculescu. Thus, the germ of idea in KRP's mind that quantum theory is a new kind of theory of Probability grew in four decades into a mature tree having more than one 'colour' of probability theories, giving rise to multiple possible choices of 'noises', relating to the question (ii) raised above. The partial resolution of the mainly analytical question in (i) took another decade or so.

In this way, the Delhi Centre of the ISI during 1980–2000 (2 decades) became one of the worlds' major centres (with KRP as the driving force) for research and dissemination of quantum probability theory. Just as it happened during the younger days of KRP in ISI, Kolkata, many international researchers visited ISI, Delhi regularly. The list included Professors T. Hida, Robin Hudson, Wilhelm von Waldenfels, P. A. Meyer, L. Accardi, and Professors V. P. Belavkin, Martin Lindsay, F. Fagnola, S. Attal, N. Obata, M. Schurmann, U. Franz, and many others of the younger generation.

All these visits, back and forth, fostered all-round collaborations not only with KRP, but also amongst the whole group of researchers including students; the world of quantum probability, at the end of the millennium, grew to be of significant strength. It was at the beginning of this period of excitement and growth that Prof. Accardi, along with a few others formed an association, which came to be named 'Association for QPIDA (Quantum Probability and Infinite Dimensional Analysis)', of which KRP was the first president. Furthermore, this association became quite active in holding annual conferences all over the world; several in the universities of Heidelberg and of Greifswald (Germany), Oberwolfach Institute (Germany), Levico (Italy), Nottingham (UK), Marseilles (France), Universities of Columbia, NY, and Ohio State University (USA), Oxaca (Mexico), Chungbuk and Yeosu (South Korea), Kuantan (Malaysia) and of course, Delhi and Bangalore Centres of the ISI. Of these, the one in 1990 in ISI Delhi had KRP in his prime, both in academic and organizational fronts, and in all these conferences the research-talks on the HP-calculus formed a major component.

As one would expect for such a distinguished personality, KRP won many laurels and awards in his career – the Shanti Swarup Bhatnagar Award (1977), the Ramanujan medal of the Indian National Science Academy (INSA, 2013) for lifetime achievement in Mathematical Sciences, TWAS award in Mathematics (1996), and the fellowships of INSA, TWAS and the Indian Academy of Sciences, Bangalore. Furthermore, he received the prestigious Hardy Lectureship in 1995 from the London Mathematical Society and was awarded Doctorate Honoris–causa from the Nottingham Trent University, UK and from the ISI.

I came to the Delhi Centre of ISI in late 1978 and till mid-1985, was mostly occupied with pursuing my earlier interests in 'Spectral Theory of Schrödinger Operators', though I had already collaborated with KRP in three articles, in one of which entitled 'A Random Trotter–Kato Product Formula' (in *Statistics and Probability, Essays in honor of C. R. Rao*, North-Holland, 1982), the question of unitary stochastic evolutions was discussed with only classical Brownian 'noise'. Our offices were essentially next door to each other and KRP would often drop in to pose one problem after another – his enthusiasm was infectious and I was soon drawn into a dance of ideas–computations–discussions, leading to many jointly authored articles of which I shall mention only two. The earlier attempts by Hudson and Lindsay for a martingale representation theorem in non-Fock case (in which the conservation process does not appear) led KRP and me (*J. Funct. Anal*., 1986, 67) to investigate a general class of bounded regular martingales in Fock space which admits quantum–stochastic integral representation in terms of all three processes, viz. annihilation, creation and conservation processes, Another idea of Hudson to characterize a 'quantum stop time' as a non-negative self-adjoint operator in Fock space whose spectral family $\{S(t)\}_{t\geq 0}$ is adapted with respect to the Fock space tensor–filtration, led KRP and me (*Prob. Theory and Related Field*, 1987, 75) to create a theory of stopped Weyl processes and prove the strong Markov property of the Fock space. By the end of the millennium, my interests moved more towards non-commutative geometry and my move to Kolkata and then to Bangalore reduced my collaboration with KRP though we would often discuss over the phone and exchange newly written articles.

After more than two decades of development in the stochastic calculus, there was a bit of lull in this area. At the turn of the new millennium, the restless creativity of KRP made him turn to his existing knowledge of classical information theory and use it in the newly emerging fields of the quantum information theory. As was often the case with KRP, after he (by himself alone or along with few others) has established a substantial body of mathematical knowledge in an area, he would write a book on the subject and this new area was no exception. His book *Coding Theorems of Classical and Quantum Information Theory* (Hind. Book Agency, 2013) was written in his inimitable style.

For more than two decades, KRP and I lived in the campus of ISI Delhi as (vertical) neighbours and I had seen at first hand his ideal of 'simple living and high thinking', his sense of discipline, and his dedication to Mathematics. He was also a scholar – would often recite Sanskrit verses, learnt in childhood. KRP had an impish sense of humour as well – at the 'drop of a hat', he could imitate the oratory styles of P. C. Mahalanobis and of some of the political leaders of former Soviet union.

In the last five years or so, KRP started losing effectively his eyesight and yet, true to his nature, took up detailed study and analysis of the well-investigated area of quantum Gaussian states. But his 'large–canvas vision' brought a great sense of clarity and completeness to this whole area, leading to several beautiful articles (in collaboration with much younger collaborators).

I guess it is fair to say that KRP could achieve so much not only because of his intellectual heights, but also because of the immense, patient support he received generously from his better half, Shyama. They had two sons Ramesh and Harish; and we have fond memories of many get-togethers and of the wonderful musical renderings by Ramesh in the Indian flute.

Unlike many other mathematicians – some ex-colleagues of mine from the ISI, some my professional friends – KRP's breadth of mathematical interest was staggeringly large and that suited me very well. In my view, he was temperamentally a bit of a mathematical physicist like me – impatient in solving a mathematical problem before jumping onto another one, both possibly having origin in some distant physics question and then bringing to bear on it whatever 'imagination and mathematical artillery' that may be needed. It is a unique privilege – for me to have known and worked closely with such a mathematical giant, and for the generations of Indian mathematical community to have him walk amongst them.

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