National Geospatial Policy: perspectives on height systems, vertical datums and gravimetric geoid modelling in India

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The National Geospatial Policy (NGP) and guidelines of India mention that the threshold value for vertical or elevation shall be 3 m. However, the terms 'height', 'elevation' or 'vertical' alone are not sufficiently self-explanatory. Therefore, this article provides an overview of India's height systems, vertical datums and vertical reference surfaces. The ellipsoidal heights obtained from GNSS have been discussed briefly, but the main focus remains on the physical height, commonly known as heights above mean sea level. This is because only the latter is used for large infrastructural projects and contouring in topographical maps. The geoid, a geopotential surface by definition, is the best candidate for a vertical reference surface. Some countries also employ quasigeoid, but India has always pursued geoid. Developing a geoid model is also one of the milestones for 2025 in the NGP. Geoid modelling has been studied in India for over a century but has never been discussed in detail. This article comprehensively discusses all the pertinent information on heights and vertical reference surfaces used in the country, which is important for various stakeholders and users of the NGP and guidelines. Some suggestions towards the successful implementation of the NGP in terms of maintaining consistency and avoiding duplication in densifying the national fundamental elevation dataset and a roadmap for developing the consistently precise national geoid model have also been provided for consideration by the national agencies and engineering surveyors.

Keywords: Geoid model, geospatial guidelines, height system, National Geospatial Policy, vertical datum.

THE threshold value for elevation has been provided as 3 m in the geospatial guidelines, i.e. elevation information equal to or greater than this value can be shared with anyone, nationally or internationally. Also, it is mentioned that the height/elevation information will be based on the Survey of India (SoI) topographical database. The term 'elevation' or 'height' must always be accompanied by information on the type of height and the associated vertical datum[1]. This is crucial to maintain consistency and avoid duplication, an essential mandate of India's National Geospatial Policy (NGP). Further, in the gazetted NGP, a few milestones have been set to be achieved by 2025, 2030 and 2035 for the realization of the visions of the Policy, one of which is the development of a geoid model[2]. It is important to note that this milestone is crucial to achieving milestones of the next targeted years. Development of a geoid model (goal of the year 2025, clause 2.2.5 of the NGP) is necessary for high-resolution topographical mapping, and developing precise and high-resolution digital elevation models (DEMs) (goals of the year 2030, clauses 2.2.7 and 2.2.8 of the NGP), which are further necessary for sea surface topography, mapping subsurface infrastructure and developing digital twins (goals of the year 2035, clauses 2.2.11, 2.2.12 and 2.2.13 of the NGP). However, limited information on the heights, vertical datums and geoid models in India is available to stakeholders and users in the public domain, which may hinder the successful implementation of the NGP[3]. This is because inconsistency and duplication in data collection are difficult to avoid with limited information on the metadata of the fundamental topographical dataset.

Therefore, the present article provides all the relevant information about height systems, vertical datums and geoids in India with respect to the NGP and geospatial guidelines. It will be important to all the stakeholders and users of elevation information in the country. Further, it may encourage users to request the discussed information/metadata while procuring the fundamental dataset from the concerned authority.

Height is generally perceived as the vertical distance between two points. However, one could ask: what is the direction of the vertical distance? Therefore, a more precise

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definition of height could be a vertical distance between two points on well-defined mathematical, physical or virtual surfaces along a specified direction. Scientifically, height is a coordinate seperating two points along a specified direction in a 3-dimensional (3D) space with the same 2-dimensional (2D) coordinates in one reference frame. Further, height must be accompanied by information on its vertical datum, a reference surface of zero elevation to which corresponding heights are referred. It could be a surface (e.g. geoid, quasigeoid, ellipsoid) or a set of specific points with known heights relative to the mean sea level (e.g. tide gauge bench-marks (TGBM)).

In the present satellite era, 2D coordinates of our position on the Earth's surface are obtained effortlessly using the Global Navigation Satellite System (GNSS). There are multiple options for the height coordinate, including using GNSS itself. These options primarily depend on the direction of the vertical distance and the reference surface from where the distance is being measured. Various researchers have discussed different types of heights and height-related terms[4–6]. Therefore, only those terms are discussed here, which are primarily used in the Indian context and the literature, i.e. orthometric height, normal height, normal–orthometric height, geodetic or ellipsoidal height, geoid undulation and height anomaly. Figure 1 shows a schematic diagram for the above-mentioned heights, and Table 1 explains/defines them.

The definitions in Table 1 can be read as '<u>Column 1</u> height is the height of a point on <u>column 2</u> from a corresponding point on <u>column 3</u> along <u>column 4</u>', e.g. '<u>Orthometric</u> height is the height of a point on the <u>Earth's surface</u> from a corresponding point on the <u>geoid</u> along the <u>curved and torsioned plumb line</u>'. In addition to the heights explained in Table 1, there is also dynamic height[7,8]. This is computed by dividing the geopotential number (difference between geopotential values at the geoid and the point under consideration on the Earth's topography) by some constant. The adopted constant for calculating dynamic heights is the normal gravity of the reference ellipsoid at 45° lat. (ref. 4). Also, the geodetic or ellipsoidal heights that are obtained from GNSS are geometric, i.e. they do not follow the waterflow criterion. Therefore, ellipsoidal heights are not used for terrestrial geodetic and engineering surveying measurements.

Figure 1 shows that a geometrical relationship exists between (i) geodetic height (h), orthometric height (H), and geoid undulation (N) given by eq. (1) as well as (ii) geodetic height, normal height (H^*) and height anomaly (ζ) given by eq. (2). Since no unique surface is defined for normal–orthometric height (H^{NO}), H or H^* in eqs (1) and (2) are sometimes replaced by H^{NO} to calculate geometric geoid undulation or geometric height anomalies respectively.

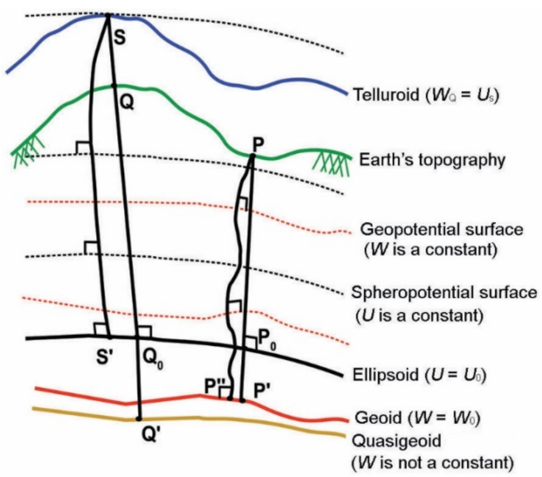
$$h = H + N, \tag{1}$$

$$h = H^* + \zeta. \tag{2}$$

It should be noted that a 'pure' orthometric height is impossible to be realized practically because it requires gravity and density information at every point on the curved and torsioned plumb line between the Earth's surface and the geoid (PP'' in Figure 1). Therefore, instead of using the integral mean value of the Earth's gravity along the plumb line, mean gravity is approximated using the Poincaré and Prey reduction, thus providing Helmert's orthometric height, which has geoid as the reference surface[4]. As an alternative, Molodensky *et al.*[9] proposed using normal heights, wherein the mean actual gravity is replaced by the mean normal gravity between the reference ellipsoid and the telluroid. The distance between the Earth's topographical surface and the telluroid is the height anomaly, and mapping of these height anomalies on the corresponding points on the ellipsoidal surface gives the quasigeoid. Although not a geopotential surface, the quasigeoid is a preferred choice of the vertical reference surface in many countries, including Australia and Sweden.

The main issue in determining orthometric heights is the computation of the integral mean gravity along the plumb line[10]. The 'rigorous' orthometric height involves calculating the mean gravity along the plumb line by considering the effect of second-order correction for normal gravity, the gravitational attraction of topographical (Bouguer shell and terrain roughness) and atmospheric masses, lateral variation of topographical mass density and gravity disturbance due to the masses below the geoid surface[11]. Santos *et al.*[12] derived the corrections to obtain the 'rigorous' orthometric height from Helmert's orthometric height.

However, despite recent advancements in height systems, Helmert's orthometric height is still in vogue in many countries, probably due to its relative ease of implementation. Also, many other countries, mainly in Eastern Europe,



Ellipsoidal height	:	P_0P and Q_0Q
Orthometric height	:	$P''P$ (and ~$P'P$)
Normal height	:	$S'S$ (and ~$Q'Q$)
Geoid undulation	:	P_0P'
Height anomaly	:	SQ and Q_0Q'

Figure 1. Different surfaces and heights.

Table 1. Definition of heights

Height	Height of a point on	From a corresponding point on	Along	Formula[5,6]
Orthometric	Earth's surface	Geoid	Curved and torsioned plumb line	$\frac{1}{\bar{g}}\int_0^H g\,dH;\ \bar{g}=\frac{1}{H}\int_0^H g\,dH$
Normal	Earth's surface	Quasigeoid	Curved normal plumb line	$\frac{1}{\bar{\gamma}}\int_0^{H^*} g\,dH^*;\ \bar{\gamma}=\frac{1}{H^*}\int_0^{H^*} g\,dH^*$
	Telluroid	Ellipsoid	Curved normal plumb line	
Normal–orthometric	Earth's surface	Geoid or quasigeoid*	Curved normal plumb line	$\frac{1}{\bar{\gamma}}\int_0^{H^{NO}} \gamma\,dH;\ \bar{\gamma}=\frac{1}{H^{NO}}\int_0^{H^{NO}} \gamma\,dH^{NO}$
Geodetic or ellipsoidal	Earth's surface	Ellipsoid	Ellipsoidal normal	h
Geoid undulation	Geoid	Ellipsoid	Ellipsoidal normal	$\frac{T_{P'}}{\gamma_{P_0}}+\frac{U_0-W_0}{\gamma_{P_0}}$
Height anomaly	Quasigeoid	Ellipsoid	Ellipsoidal normal	$\frac{T_Q}{\gamma_S}+\frac{U_0-W_0}{\gamma_{P_0}}$
	Earth's surface	Telluroid	Ellipsoidal normal	

*No unique reference surface is specified for the normal–orthometric height system[6,7]. g is the observed gravity, H the orthometric height, H^* the normal height, H^{NO} the normal–orthometric height, h the ellipsoidal or geodetic height, γ the normal gravity of reference ellipsoid, T the disturbing potential, W_0 the geopotential, U_0 the normal potential of reference ellipsoid, $\bar{g}$ the mean observed gravity along the curved and torsioned plumb line, $\bar{\gamma}$ is the mean normal gravity along the normal curved plumb line.

the former Union of Soviet Socialist Republics and South America, use normal heights.

Normal–orthometric heights are also used in several countries, like the United Kingdom, Australia, New Zealand and Sri Lanka[6,13–15]. This height system is defined when gravity observations are unavailable along the levelling lines and, therefore, spheropotential numbers are used instead of geopotential numbers[4–6]. The normal–orthometric correction is applied to the levelling height differences for calculating the normal–orthometric heights[16,17]. Moreover, unlike normal or orthometric heights, no unique reference surface is defined for normal orthometric heights, although the quasigeoid is sometimes preferred[7,14].

With so many height systems available, the discussion on the suitability of heights and geoid or quasigeoid as a reference surface for heights has remained group/country-specific[18,19].

In the next sections, we will discuss the height systems and reference surfaces chosen for defining the vertical datums in India.

Height systems in India

According to Burrard[20], the following four choices were considered and debated to establish the 'zero' surface/reference for the Indian Vertical Datum (IVD) defined in 1909 (IVD1909):

(i) Any one of the benchmarks established at Delhi, Jodhpur, Raichur, Sanichari or Naubatpahar.
(ii) Mean sea level (MSL) estimate determined at one tidal observatory.
(iii) MSL estimates determined at all the tidal observatories.
(iv) MSL from a few selected tidal observatories.

After considering all the merits and limitations of the above four options, it was decided to select a few tidal observatories to define the zero surface for the Indian levelling net[20]. To choose a set of tidal observatories from the then-maintained 42 observatories by the SoI, a simple rule was devised mentioning that the tidal observatory should be an open-coast station (not situated in any of the channels, estuaries, gulfs or rivers) at which successive annual determination of MSL should be consistent.

As such, only nine tidal observatories were selected that fulfilled the above-mentioned criteria: Karachi, Bombay (Apollo Bandar), Karwar, Beypore, Cochin, Negapatam, Madras, Vizagapatam and False Point (Figure 2). The first five lie in the Arabian Sea, while the last four are in the Bay of Bengal. Thus, the precise levelling net of India consisting of 86 main lines was terminated at the TGBM of the above nine tidal observatories. The heights of these TGBMs were transferred from the tidal observatories, considering that the MSL estimate at each of these nine stations is the same, i.e. zero. Thereafter, these 86 lines (including nine lines from the tidal observatory to TGBM) were adjusted using least squares (with the tide gauge MSL estimates constrained to zero) to define the first IVD, i.e. IVD1909 (ref. 20).

An important fact to note here is that though the sea surface in the Bay of Bengal and the Arabian Sea was considered to be equal, various observations (e.g. levelling from the east to the west coast, levelling from the east and west coasts to a centre location, etc.) suggested that there might be a difference of almost one Indian foot between the two. However, the difference (so-called error) in all the experiments was attributed to the possible levelling errors, and the difference of 1 ft was left for further confirmation by future successive levelling exercises. Later, it was confirmed that the Bay of Bengal is, on average, ~320 mm higher than the Arabian Sea (e.g. see the difference in mean dynamic topography (MDT) of the west and east coasts in Figure 2)[21]. It should be noted that some exercises for IVD1909 showed that even the sea surface along either the east or west coast was not the same, but this was also attributed to the levelling errors. There is a similar example of discrepancies in the mean sea surface (MSS) along and across North America's Atlantic and Pacific coastlines[22,23]. This approach of constraining the level net to multiple tide gauges is a possible cause of the north–south tilt that also seems to be present in India[24–26].

The precise levelling net for IVD1909 consisting of 86 main lines was observed from 1858 to 1909 and covered a total of ~28,922 km of double-line levelling, which was a practice of observing any given levelling line by two surveyors one after the other immediately. These main lines connected 15,981 benchmarks of different types, e.g., standard, embedded, inscribed, etc. In this half-a-century-long levelling exercise, 16 different levels (weighing from ~23 to ~12 kg) and four types of levelling staves (introduced in 1858, 1902, 1906 and 1907) were used[20].

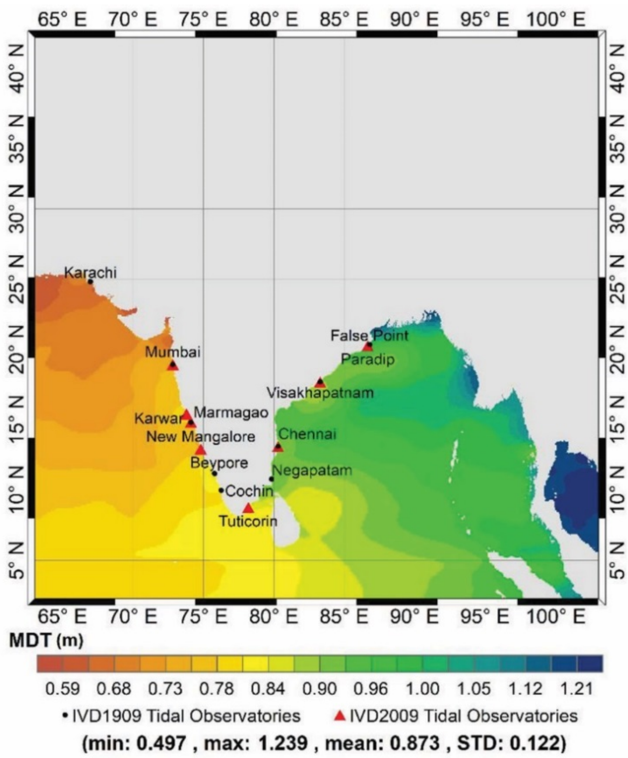


Figure 2. Mean dynamic topography (DTU19MDT) along with tidal observatories used in IVD1909 and IVD2009.

The spirit-levelling height differences were transformed to dynamic heights by applying a dynamic height correction using normal gravity instead of the observed gravity. This was because, until 1909, pendulum gravity observations were not taken at a sufficient number of benchmarks. These dynamic heights were used for the adjustment of the level net. The orthometric correction (also using normal gravity) was then applied to compute the so-called orthometric height. However, due to the use of normal gravity in place of the observed gravity, the resultant heights from IVD1909 were normal–orthometric heights (Table 1).

The IVD1909-based height information is sufficient for topographical mapping on scales 1 : 25,000 or 1 : 50,000, where the contour interval is 5 m/10 m in plain areas or 100 m/200 m in hilly regions (according to the SoI topographical maps). Now, with the demand of 0.5–1 m contours, the prevailing height information seems insufficient. In the past 100 years, most permanent benchmarks have been destroyed due to developmental activities like widening roads and railways, and constructing townships and industrial premises. The frequent seismic activities in various parts of the country and corresponding crustal movements have also necessitated the introduction of a new height system. Moreover, IVD1909 was defined as a suitable datum only for 50 years. It was recommended in the original report that the levelling should be revised without losing the values observed from 1858 to 1909, as they will be helpful for scientific studies[20].

Considering that height information was almost a century old and with the availability of precise and portable relative gravimeters, SoI started a project in around 2005 to redefine IVD and modernize the Indian height system. There were some improvements in the redefined IVD in 2018 (IVD2009) compared to IVD1909, such as using double-foresight backsight levelling lines with invar staves and observed gravity values[27]. Also, rather than fixing the MSL estimates to zero at the nine tidal observatories, the average of the local geopotential value computed at eight tidal observatories was constrained in IVD2009. Also, IVD2009 was defined with the best data available with SoI during that time (2017–18).

The eight tidal observatories were chosen such that each has data of at least 19 years (for 18.6 years nodal tidal cycle) without significant gaps in the data. We could not quantify the word 'significant' as no information is available. For this criterion, the following eight tidal observatories were chosen with their data from 1976 to 1994: Mumbai, Marmagao, Karwar and New Mangalore on the west coast, and Paradip, Vishakhapatnam, Chennai and Tuticorin on the east coast (Figure 2). The local geopotential value at each of the eight tidal observatories was calculated as the average value of six estimates using the same tidal data and the MDT models, but varying global geopotential models (GGMs).

The difference between the chart datum and MSS at these eight tidal observatories ranged from 0.62 to 2.56 m for the tidal data from 1976 to 1994. The average (of six) local geopotential value at these eight tidal observatories varied from 62,636,856.54 (at New Mangalore) to 62,636,861.80 $m^2 s^{-2}$ (at Karwar), with the final average value (of all eight observatories) as 62,636,859.40 $m^2 s^{-2}$, which was taken as the local geopotential value for IVD2009. Though differently but on similar lines of constraining MSL at the nine tidal observatories to zero in IVD1909, the local geopotential value was considered the same at the eight tidal observatories for IVD2009. Therefore, IVD2009 may also be prone to a north–south slope because the difference between the final geopotential value (62,636,859.40 $m^2 s^{-2}$) and its minimum (62,636,856.54 $m^2 s^{-2}$) and maximum (62,636,861.80 $m^2 s^{-2}$) values translates to a difference of approximately 0.29 and –0.26 m respectively. We are unable to discuss the reasons for choosing the average of the mean value for defining IVD2009 because they are not available in any publication.

The precise levelling net for IVD2009 is based on Helmert's orthometric height system that consists of 42 precise levelling lines (including eight lines between TGBM and tidal observatories) covering a distance of 19,450 km. The remarkable fact is that the distance of 19,450 km was covered in three years, i.e. from 2006 to 2008. The levelling net was adjusted using 41 observations (one was not included as a result of some trial-and-error exercises of adjustment) involving a total of 32 stations, including eight fixed TGBMs[27]. SoI is further densifying this levelling network to provide Helmert's orthometric height for geodetic surveys and infrastructural projects.

When a redefined IVD was proposed, a long-term goal was also set to develop a precise national gravimetric geoid model to be adopted as IVD, which is now also mentioned in the NGP. Since then, a few geoid-related studies involving terrestrial gravity data have been available in the literature. In the pre-geospatial policy era, an academia–SoI collaborative work involving gravity data could not have been possible because of the restrictions imposed by the archaic gravity data-sharing policy. However, with the new geospatial guidelines and policy, academia–research–industry–Government collaborative research involving geodetic data has begun, of which geoid modelling is one of the projects of interest to all stakeholders. Therefore, a detailed discussion on geoid modelling in India is given hereafter, briefly explaining the significance of the geoid model in the next section.

Significance of the geoid model

Geoid is an equipotential surface of the Earth's gravity field, which is best approximated by MSL. The orthometric heights refer to this surface. GNSS provides heights above the reference ellipsoid. The vertical difference between the geoid and the corresponding point on the ellipsoid is called geoid undulation. Hence, geoid undulation can be used to convert the ellipsoidal heights obtained from GNSS to physically meaningful orthometric heights. Thus, in layman's term, it can be said that the spatial representation of these geoid undulations is known as the geoid model, and the process of developing and implementing the mathematical algorithms for calculating this model is known as geoid modelling. The computation can be made at a local, regional or global level.

Geoid models are developed not only to be adopted as a vertical datum, but an immediate consequence of a precise geoid is also the conversion of the digital elevation models (e.g. national CartoDEM) and height observations in geodetic heights to orthometric heights effortlessly[28]. A freely available geoid model will allow surveyors to efficiently measure physical heights with GNSS positioning by replacing the costly and laborious differential levelling. Recently, the Indian Railways, Public Works Department, National Disaster Management Authority and Airports Authority of India suggested using a geoid model for their infrastructural projects. The Government can use the developed geoid model with the drone-acquired data to exploit several other applications in addition to village demarcations, e.g. creating regional high-resolution DEMs, floodplain mapping and irrigation. On a national level, the geoid model can also aid in the interlinking of rivers project. Since no Indian geoid model was available in the public domain, one had to rely on levelling, GNSS-levelling-based local geometric geoid model or comparatively less-accurate global geoid model, e.g. Earth Gravitational Model 2008 (EGM2008)[29,30].

Further, since the geoid is a physically meaningful surface, it responds to changes in the gravity field due to various geophysical and geodynamical phenomena, allowing us to study them as well[31]. Therefore, while benefitting several stakeholders, the national gravimetric geoid model (and its intermediate results, e.g. terrain corrections) will also invigorate sciences like geomorphometry, hydrography, oceanography and many other applications[31–34].

Geoid modelling in India

The first geoidal study in India started more than a century ago. Detailed information may probably be provided in the archaic SoI reports, which are unavailable in the public domain. However, here we provide concise introductory information gathered from different sources. The geoidal study was started in India around 1901 based on astrogeodetic observations with respect to the Everest 1830 ellipsoid[35]. Bomford[36] mentions that de Graaff Hunter compiled the first geoid map for India in 1922 based on astrogeodetic observations referred to an international spheroid. It was published in 1923, excluding the Himalayan region (information on where it was published is not available), and again in 1930 for the whole of India[37,38]. The geoid map from 1923 is also provided in Daly[39]. A geoid map for India and adjacent regions based on the astrogeodetic data referred to an international spheroid was also published in

1951 and 1957 (refs 40, 41). However, none of these is available in the public domain.

During the 1970s to mid-1980s, a few other gravimetric and astrogeodetic geoid-related studies were conducted in India with respect to both Everest and GRS67 ellipsoids[42–46]. The gravity data used were primarily from the geopotential coefficients and sometimes coarse (1° ×1°) observed mean gravity anomaly data. However, none of these models is available in the public domain.

After a significant gap of almost two decades, gravimetric geoid-related studies over India were again available in the literature from 2007 onwards[47–56]. This was probably because the idea of developing a gravimetric geoid model to be adopted as the new IVD was being discussed around 2005. The developed geoid models from the studies conducted in 2007–18 are not available in the public domain. However, the Indian geoid models developed between 2021 and 2022 are available in the public domain through the International Service for the Geoid[57–59]. It should be noted that these are not 'official' models and hence lack reliability in terms of accuracy due to the use of gravity data of unknown quality.

It is also important to note that the studies from 1901 to 1957 were all conducted by SoI; from 1973 to 1985 by SoI or the University of Roorkee in collaboration with SoI; and from 2007 to 2018 by SoI, Indian Institute of Technology (IIT) Roorkee (previously University of Roorkee) in collaboration with SoI, National Geophysical Research Institute (NGRI) in collaboration with SoI, and studies after 2018 by IIT Kanpur in collaboration with Curtin University with validation GNSS/levelling data (not gravity) from SoI. This shows that SoI is a major stakeholder in geoid modelling studies, and hence, the development of the Indian geoid model has been assigned to SoI under the NGP. Therefore, it is inevitable that the collaborative efforts initiated by SoI toward geoid-related studies in India should continue. Here, we are interested only in the gravimetric geoid model and will keep our further discussions limited to this alone.

Regional gravimetric geoid models in India

The gravimetric geoid modelling studies over India have been summarised in Table 2, followed by a discussion on individual studies[60–63]. Before discussing the Indian gravimetric geoid studies further, the following two points must be noted:

(i) The free-air gravity anomalies in India, either in SoI or NGRI database, are those on the geoid.

(ii) If RTM is used in GRAVSOFT, the resultant will be height anomalies for which free-air gravity anomalies are required on the Earth's topographical surface[64,65]. Thereafter, GRAVSOFT allows the computation of a geoid–quasigeoid separation term to calculate the geoid undulations from height anomalies[4].

All the methods explored in Table 2, i.e. GRAVSOFT, CUT, Stokes–Helmert (initially) and LSMSAC, use free-air gravity anomalies on the Earth's surface[65–68]. However, only limited studies have explained the computation of free-air anomalies on the Earth's surface. We assume that in the studies listed in Table 2: (i) free-air gravity anomaly on the geoid is assumed practically equivalent to that at the Earth's topographical surface (which is true only if it is assumed that the Earth's gravity gradient is equal to the normal gravity gradient, and normal height is equal to the orthometric height), and (ii) a few of these studies have used the terms 'geoid undulation' and 'height anomaly' synonymously. It should be noted that though free-air gravity anomalies at the Earth's topography and the geoid can be practically equivalent, the differences can be significant in view of the cm–precise geoid, primarily due to the mentioned assumptions[69]. Table 2 also mentions the study area and basic information on these gravimetric geoid modelling studies. Therefore, a few critical observations related to the adopted methodology in these studies are provided below.

(i) Singh[47]: The topography in the study area varies from 1 to 6918 m. A constant value of the atmospheric correction, i.e. 0.87 mGal is used for the entire study area. However, this value (0.87) is the maximum atmospheric correction that is obtained at sea level[70]. The atmospheric correction for the given height range will vary from 0.36 to 0.87 mGal (refs 70, 71). Further, the reported terrain corrections (TCs) vary from –3.38 to 36.69 mGal, with a mean and standard deviation of 0.598 mGal and ±3.871 mGal respectively. These values contrast with the non-negative planar TC computations over India and adjacent regions using freely available global DEM[54,72]. Since planar TCs are always positive, the most probable cause for the negative values can be the use of the fast Fourier transform (FFT) method for calculating planar TCs in regions with terrain slopes greater than 45° (ref. 72).

(ii) Singh *et al.*[48]: The flowchart and the discussed methodology in this study are two different methods of geoid calculation. The two methodologies and one set of results caused some confusion about the implemented methodology. However, since GRAVSOFT is used, we can safely assume that the method involving RTM has been followed in this study. A DEM has been prepared from the spot heights of a topographic map. It would have been useful for a relatively better comparison and understanding of the topographic corrections if some information had been provided regarding the scale of the map, the gridding method or the resolution of DEM. This is because some studies have already analysed the effect of DEM resolution on TCs and their effect on geoid modelling[73,74].

Table 2. Gravimetric geoidal studies in India

	Singh[47]	Singh *et al.*[48]	Carrion *et al.*[49]	Srinivas *et al.*[50]	Mishra and Ghosh[51]	Mishra and Ghosh[51]	Choudhary[52]	Singh and Srivastava[53]	Goyal *et al.*[54]	Goyal (ref. 55)[#4]	Goyal (ref. 55)[#4]	Goyal (ref. 55)[#4]
Region	24°–29°N; 76°–82°E	28°–29°N; 76°–77.30°E	6°–14°N; 74.5°–80.5°E	12.5°–18.5°N; 75.5°–79.5°E	Dehradun[#2] (30°19′N, 75°4′E)	Hyderabad[#2] (17.5°N, 78.5°E)	India	20°–31°N, 71°–79°E	7°–37°N, 68°–98°E	7°–37°N, 68°–98°E	7°–37°N, 68°–98°E	7°–37°N, 68°–98°E
Method	GRAVSOFT (FFT)	GRAVSOFT (FFT)	GRAVSOFT (Fast collocation)	GRAVSOFT (LSC)	GRAVSOFT (LSC)	GRAVSOFT (LSC)	NAv	GRAVSOFT (FFT)	CUT	CUT	Stokes–Helmert	LSMSAC
Type	Geoid	Geoid	Geoid	Geoid	Geoid	Geoid	Geoid	Geoid	Geoid	Geoid	Geoid	Geoid
Resolution	5′ × 5′	0.5 km × 0.5 km	2′ × 2′	NAv	NAv	NAv	NAv	15′ × 15′	0.02 × 0.02	0.02 × 0.02	0.02 × 0.02	0.02 × 0.02
Terrestrial gravity data	SoI database	SoI database	NGRI database	NGRI database	SoI database	SoI database	SoI database	SoI database	GETECH	GETECH	GETECH	GETECH
Global geopotential model	EIGEN-GL04C	EGM96	NAv	EGM2008	EGM2008	EGM2008	NAv	GGM05C	EIGEN-6C4	EIGEN-6C4	DIR-R5	DIR-R5
0 degree	NAv	NAv	NAv	NAv	NAv	NAv	NAv	NAv	Yes	Yes	Yes	Yes
Integration radius	2	0.5	NAp	NAp	NAp	NAp	NAv	NAv	1.5	1	1	1
Kernel modification	Wong and Gore[60]	Wong and Gore[60]	NAp	NAp	NAp	NAp	NAv	NAv	Featherstone *et al.*[61]	Featherstone *et al.*[61]	Vaníček and Kleusberg[62]	Sjoberg[63]
Topography treatment	TC and RTM	RTM	NAv	RTM	RTM	RTM	NAv	RTM	TC	TC	DTE, PITE, SITE	Nap
DEM/DSM	GLOBE 2′ × 2′, 4′ × 4′ and 6′ × 6′	Generated from 130 spot heights	NAv	GTOPO30	1 : 50000 topo map derived	SRTM 3″ × 3″	NAv	SRTM 30″ × 30″	MERIT 3″ × 3″	MERIT 3″ × 3″	MERIT 3″ × 3″	MERIT 3″ × 3″
Atmospheric correction	Constant 0.87 mGal	Yes	NAv	NAv	NAv	NAv	NAv	NAv	Yes	Yes	Yes	Yes
Ellipsoidal correction	NAv	NAv	NAv	NAv	NAv	NAv	NAv	NAv	NAp	NAp	Yes	Yes
No fit stats (min, max, mean, std)	–0.148, 0.304, 0.049, 0.089	–0.172, 0.189, –0.220, 0.083	NAv	–0.360, 0.170, –0.020, 0.090	0.023, 0.266, 0.175, 0.190[#3]	–0.130, 0.210, 0.070, 0.100[#3]	NAv	–0.346, 0.226, –0.005, 0.136	–0.881, 0.783, –0.176, 0.395	–0.973, 0.809, –0.149, 0.407	–1.266, 0.769, –0.328, 0.489	–1.050, 0.839, –0.140, 0.425
After fit stat (min, max, mean, std)[#0]	–0.102, 0.122, –0.001, 0.044	NAv	–1.510, 1.080, 0.000, 0.220[#1]	NAv	NAv	NAv	NAv	–0.064, 0.125, 0.039, 0.072	–0.475, 0.408, 0.000, 0.134	–0.475, 0.408, 0.000, 0.134	–0.503, 0.434, 0.000, 0.155	–0.464, 0.426, 0.000, 0.135

TC, Terrain corrections; RTM, residual terrain model; LSC, least squares collocation; FFT, fast Fourier transform; DEM, Digital elevation model; DSM, Digital surface model; DTE, direct topographical effect; PITE, primary indirect topographical effect; SITE, secondary indirect topographical effect; NAv, not available; NAp, Not applicable; CUT, Curtin University; LSMSAC, least squares modification of Stokes formula with additive corrections.

[#0]Before and after fit statistical values (min, max, mean, STD in m).

[#1]In Carrion *et al.*[49]; validation is done with respect to EGM2008-derived geoid undulations.

[#2]In Mishra and Ghosh[51]; only location is given in this article and not its extent.

[#3]In Mishra and Ghosh[51]; root mean square error (RMSE) is provided instead of standard deviation.

[#4]In all these studies, the statics is given after validation with all the ground truth at once and also cluster-wise. In the table, the values given are for validation with respect to the whole data used at once.

(iii) Carrion *et al.*[49]: We are unable to provide any observations on the computation strategy adopted in this study because not much information about the corrections/parameters listed in Table 2 has been discussed in this article.

(iv) Mishra and Ghosh[51]: The DEM for the Dehradun region was developed using a 1 : 50,000 topographical map, while for Hyderabad 3″ × 3″ SRTM DEM was used. The most plausible reason for using the topographical map in and around Dehradun is accuracy concerns of the height information. Dehradun is a relatively more undulating region than Hyderabad, and topographic maps would have provided more precise elevation information than the satellite-based DEM[75]. It would have been helpful in further understanding DEM and the calculated TCs if some information on the extent of the study area and resolution of the developed DEM had also been provided. It is essential because DEM is an accuracy-controlling input in geoid modelling[56].

Further, the discussed methodology sometimes deviates from the GRAVSOFT manual or other studies using the same software, making it difficult to understand the explained computational strategy. We cannot suggest reasons for the possible deviations, but there could be some typographical errors that might have changed the meaning/ flow of the explained approach for calculating the quasigeoid and then the geoid. A simple example could be that the Bouguer anomaly, and not a free-air anomaly, is used in the N2ZETA subroutine for calculating the geoid–quasigeoid separation (GQS) term, which is used to convert the height anomalies to geoid undulations.

(v) Choudhary[52]: This is based on news coverage of INDGEOID version 1.0 (https://www.geospatialworld.net/videos/survey-india-launches-geoid-model-country) developed by SoI. We cannot discuss this further because no information on the model and its computational methodology is available in the public domain.

(vi) Singh and Srivastava[53]: Limited information on the computational strategy is available in this study for replicability. A precise geoid model has been computed at a resolution of 15′ × 15′. Though the geoid is a smooth surface, the ~625 km^2 area (15′ × 15′) is too large for the geoid undulation to be almost constant. The geoid undulation can vary as large as 12 m in an area of ~625 km^2 (ref. 55). The limited and sparse gravity data in a larger study area could be a possible reason for this chosen resolution. SRTM 30″ × 30″ DEM has been used in the computation, which indicates a requirement for analysing topographical corrections (TCs) over India using DEM of different resolutions. The rationale is that a high-resolution topographical representation is necessary for precise topographical effects. However, obtaining a precise high-resolution DEM is challenging, especially in undulating regions.

(vii) Goyal[55]: Although the adopted geoid modelling methodologies are explained in detail, the major limitation of this study is the unavailability of information on the quality of the used gravity data. Also, it was suggested that geoid models can be validated with the components of vertical deflection. However, the conversion of vertical deflection from Everest to WGS84 ellipsoid was done using transformation parameters available in the public domain and not SoI parameters[76]. This is because of the non-availability of SoI transformation parameters (from Everest to WGS84) in the public domain. The GNSS-levelling data were used to validate the developed geoid models. However, the accuracy estimate of these GNSS-levelling data also remains uncertain. The author(s) showed that the generalized Brun's formula must be used in geoid computation. If neglected, it can cause a systematic bias of ~0.76 m for the International Height Reference System-adopted geopotential and GRS80 normal potential[56,77,78]. Although the developed geoid models in these studies are provided in the public domain, they need to be validated in the region of interest before being used for surveying applications because of the unknown quality gravity data used in the computations.

We summarize the discussion on gravimetric geoid studies in India by mentioning that the official and precise Indian gravimetric geoid model is still elusive after several efforts. It would be helpful for the Indian researchers if future geoid modelling studies report all the information in Table 2. It will facilitate a fair and objective comparison. The tilts and biases in the vertical datums are mostly eliminated when gravimetric geoid undulations are fitted on the geometric geoid undulations, providing hybrid geoid[56,79–81]. Therefore, it should be mentioned whether descriptive statistics is provided for validation of a gravimetric geoid or a hybrid geoid. Furthermore, different Indian stakeholders (e.g. Government, academia, research and industry) must collaborate if we aim to develop a precise gravimetric geoid model for India in the near future. This is advocated because all the previous geoid studies in Table 2 (before 2019) have some limitations in reporting the adopted geoid calculating strategy, while geoid studies after 2019 have limitations in data availability.

Roadmap for developing an Indian gravimetric geoid model

Here, we briefly discuss the requirements to achieve goal 2.2.5 of the NGP by 2025. However, it should be noted that further refinement in the data and geoid modelling methods must be pursued even after the year 2025 to refine the initial version of the geoid model. The following two aspects need to be considered for developing the Indian gravimetric geoid (IndGG) model:

(i) Dataset: For geoid modelling, we need gravity data, DEM and GGM, of which the latter two are freely available. It is suggested to use the latest high-resolution DEMs, e.g. Forest and Buildings removed Copernicus DEM (FABDEM)[82]. For the first version of IndGG within the time frame mentioned in the NGP, it is suggested to filter/clean the existing gravity data, densify the present gravity network (based on 'as-is' datum/methodology) and also plan for airborne gravity surveys for inaccessible areas. These can be transformed into the being-planned gravity datum later to develop a refined version of IndGG. Altimeter-derived gravity data can be used for the oceanic regions, and GGM and RTM-based fill-in gravity data can be used for trans-frontier regions. All these can then be merged to develop a regular grid of free-air gravity anomalies.
(ii) Methodology: Numerous methods of geoid modelling exist[77,83]. Further, India has a varied topography. Therefore, it is suggested to have some test regions in different types of landforms (plain, undulating, mountainous and coastal) and compare different methods to identify the similarities and dissimilarities between the geoid modelling methods. It is crucial to decide on a suitable methodology to develop a consistently precise national geoid model because, with the limited dataset, it has been shown that different methods have varying precision in different regions of the country[56]. Further, if the geoid model is being developed to be adopted as the national vertical datum, it is suggested to re-commence astro-geodetic observations for geoid validation as it gives a check which is independent of levelling errors.

Since a geoid model is required for achieving some milestones of the NGP of the years 2030 and 2035, it is strongly suggested that a Working Group on Indian geoid modelling may be formed, which will also work towards establishing/redefining gravity datum, gravity data standardization and its densification (including airborne gravity). It will be important to avoid future complications arising from non-standardized data collection and archiving procedures being practised for decades.

Concluding remarks

This article discusses the vertical datums for India defined in 1909 and 2018, and the height systems associated with them. It would have been useful and clearer if clause 8(iv) in the geospatial guidelines, i.e. the threshold for data sharing of vertical or elevation is 3 m, had been provided with some details on the height system and datum. Both the vertical datums were defined with the then-best available data and methods, thus requiring re-definition to meet the present-day accuracy requirements by accounting for errors introduced in the datums due to limited data and methods. If there will be any future adjustment of the levelling network, it is suggested to constrain only one TGBM to avoid tilts and biases in the datum.

The published gravimetric geoid modelling studies over India have also been discussed in detail, showing that all studies have certain limitations. Despite numerous efforts, no consistently precise official geoid model for any part of India is available in the public domain. The less-precise Indian gravimetric geoid models available in the public domain, though better than GGMs, must be validated in the region of interest before being used for surveying applications. After analysing the current status of geoid modelling in India, a roadmap has been suggested for achieving an immediate goal of the NGP, i.e. developing a geoid model by 2025, for which collaboration between SoI and academic institutions is inevitable.

Meanwhile, the surveyors can develop local geoid models using GNSS-levelling-based geometric geoid undulations for their respective geodetic and engineering survey requirements to avoid costly and laborious differential levelling. The rationale is that such a geoid model would be consistent with IVD2009 and more precise than EGM2008. Further, if the competent authority decides to keep levelling and gravimetric geoid both in practice, it is suggested that hybrid geoid models be developed to reduce the tilts and biases in the two datums. It will also maintain consistency in the large infrastructural projects in regard to the existing fundamental national elevation dataset.

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