QUANTITATIVE RELATION BETWEEN YARN STRENGTH AND FIBRE PROPERTIES

K. R. SEN, D.Sc.

THE writer in 1938 gave[1] an equation for calculating the yarn strength from fibre properties as under:

$$S_L = A \frac{l\,p}{f.c}(1 - E) \qquad (I)$$

Here, S_L represents the single thread strength of a cotton yarn of count, c, obtained by using a test length, L. l denotes the mean fibre length, p, the mean fibre strength and f, the mean fibre mass per unit length. The quantity A is a numerical constant which depends on the definition of the count with respect to the units of measurement considered. E in this equation embodies the various effects brought into being by the conditions of the yarn structure. Experience shows that the resultant of these effects, which E expresses, whenever significant, is detrimental to the realisation of the full strength of the fibre aggregate in the yarn. The effect of the test length on the yarn strength is also covered mostly by the *omnibus*-term, E.

The Spencer-Smith equation for flax derived from statistical considerations, in 1947, takes the form—

$$Q_L = Q_0\left[1 - \frac{\omega_n . F(n) . V_0}{100}\right] \qquad (II)$$

Here, Q_L is the average quality number of a long specimen of yarn of length, L, Q_0 is the mean quality number of the "fracture zone" or the length within which the actual fracture is confined (Turner[5]). ω_n denotes the mean difference between the average and the minimum value of n terms selected at random from the appropriate normalised frequency distribution (Tippett[6]), $F(n)$ is a function depending upon the serial correlogram of the strength of the fracture zone; and V_0 is the percentage of variation of the strengths of the fracture zones.

The form of the Spencer-Smith equation being similar to the author's, it was considered worth while to publish the method by which the equation (I) was derived and also to scrutinise and compare the two equations.

(*A*) *Derivation of Equation* (I)

Equation (1) is built up from the following considerations:—

(i) *The strength of all fibres within the smallest specified or standard test length.* This standard length for cotton is considered to be the same as the average fibre length. We may represent the strength of the yarn of a length equal to the average fibre length by S_l.

(ii) *The increase in strength caused by* (a) *the binding effect of twist on fibres constituting the yarn;* and (b) *the frictional resistance offered by the mutual surfaces of contract opposing any slippage under tension.*

These positive effects of strength are physically known facts and may be together denoted by T.

(iii) *The decrease in strength, due to* (a) *possible non-clamping of some fibre-ends in the main region of fracture,* a fact which is likely to permit easy displacement of such fibres under the applied tension; (b) *the pre-existing tensile strain in some of the component fibres,* the magnitude of which varies according to their distribution between the surface and the core of the yarn (these differences in magnitude occur as a result of the differences in the twist angle); as well as (c) *the statistical effect of the longer actual test-length due to the presence of the thick and the thin places in the yarn.* These irregularities of yarn-structure are caused by the unequal relative displacement of the fibres along the length of the yarn during drafting, and constitute the most important source of reduction in yarn strength. The resultant negative effect on the strength of the yarn may be denoted by $-F$.

In addition to these principal factors there may yet be others which are either positive or negative in their effect on the yarn strength. Such minor factors are often local, arising out of the machine conditions and/or some peculiarities in the structure of the yarn or the constituent fibres. These are yet undefined, and their total effect, as far as experience goes, may be neglected for all practical purposes.

Now, to derive equation (I) in the simplest manner, let G represent the *maximum*† strength of the yarn from fibre fracture only, when a test length equal to the average fibre length is used. It is common experience, and also a matter which can be theoretically understood, that the fracture of the fibres, which occurs, is *not confined to one particular cross-section,* but is distributed over various cross-sections within a length $\doteqdot l$ or just $< l$. It cannot be

† It is the total aggregate strength such as would obtain on the breaking load of all the fibres concerned being realised.

greater than l*. The harder the twist, the smaller is this length. It happens thus. A fibre must break at the weakest point; this point may, however, be situated anywhere within its length. So, the points of break of the fibres, broken by the application of the tensile stress, will be situated within a zone $\not> l$ at different distances *from any particular cross-section*, for the individual fibres concerned. Thus the breakage of the yarn will spread over a region of its length, instead of being confined to a point. But the spread of this region of break along the length of the yarn, will, of course, naturally depend on the frictional and other effects brought into play by the degree of applied twist. We can not, therefore, expect to arrive at the true aggregate strength of the fibres in a test length equal to the average fibre length by confining ourselves to a particular cross-section only, to the exclusion of all others involved in a yarn break. We must take into consideration for practical reasons, all the fibres, whole or part, such as lie within the length, l.

Now, from previous considerations, $S_l = G + T - F$; also F is very much bigger than T; and $G > (F - T)$. This latter inequality is clearly proved by the fact that the resultant strength observed for a yarn is real and positive Thus we may write,

$$S_l = G - (F - T).$$

If then we express $(F - T)$ in terms of G such that $F - T = e.G$, where e is a proper fraction depending on the structural characters of the yarn, we ultimately get,

$$S_l = G(1 - e). \qquad \text{(III)}$$

In order to evaluate G, we must find a way to estimate the number of fibres which, on an average, lie within the length l of the yarn. To do this let L represent the actual test-length of the yarn in a yarn-break test. L is many times larger than l. Representing the total number of fibres which lie within the length, L, by N_L, the average number per unit length, may be taken as N_L/L. Let this quantity be represented by N. In a test length equal to the average fibre length, l, the number of fibres, on an average, will then be practically equal to Nl. (As L is kept constant for all yarns, $N \propto N_L$.) On these considerations we may put,

$$G = Nlp$$

The mean value of N can, however, be obtained in this manner. The average mass per unit length of the yarn $= A \div c$, A and c being as defined earlier. The mean fibre mass per unit length being f (see *ante*), the average number of fibres per unit length of the yarn, or N $(= N_L/L)$, should be given by $A \div cf$, Therefore, $Nl = Al/cf$. So,

$$G = Nlp = A\frac{lp}{fc}. \qquad \text{(IV)}$$

Thus we get the value of G in terms of the properties of the yarn and the component fibres. So, from equations (III) and (IV), we have for the yarn, having a test length equal to the average fibre length,

$$S_l = A\frac{lp}{f.c}(1 - e).$$

Now, calling $c.S_l$ as Q_l or the "quality number" of the yarn, when the test length $= l$ we may write,

$$Q_l = A\frac{lp}{f}(1 - e)$$

There is one point which requires to be considered here. For such a small test length as the average fibre length of cotton, the effect of the strength gradient should be extremely small for statistical reasons. It is in fact found to be practically insignificant.[9] Also, as most of the fibres are likely to be clamped (*cf.* Sen & Nodder[7]) at both ends at this test length, the possibility of slip under *tension significantly* affecting the yarn strength is negligible. Further, the diameter of the normally twisted cotton yarn being small, and the number of twists per unit length fairly large, any difference in twist on fibres in the yarn must be small. The negative effect being thus greatly reduced, since however the effect of T must be practically unaffected, the value of $F - T$ must be regarded as negligible compared with G. In other words, $e = 0$. On these considerations,

$$Q_l = A\frac{lp}{f}$$

However, for an actual test length, L, which is many times longer than the average fibre length, the specific conditions reducing 'e' to zero, do not exist. It is in fact found from experience that with the usual test length, L, the loss of strength due to the effect of yarn structure,

* As soon as one or more fibres at any point in the test length has broken, the remaining fibres about the point gets the share of the load which, therefore, increases rapidly per fibre as fibres break, precluding yarn-break anywhere outside the zone (*cf.* Turner). Use of the average fibre length, l, as the minimum test length for calculation of yarn strength therefore corresponds to the *limiting "fracture zone"*. A significantly shorter length than l cannot also fulfil this condition.

which brings down the value of G, is considerable. So, for a test length $= L$, we may represent¶ $F - T$ by E.G and get—

$$S_L = A \frac{lp}{fc} (1 - E),$$

which is the same as the equation (I). Representing $S_L \cdot c$ by the corresponding "quality number", Q_L, we may write—

$$Q_L = Q_l (1 - E). \quad (V)$$

This reduces equation (I) to a form identical with that given by Spencer-Smith.

Value of E

There is a great deal of difficulty in evaluating E physically in terms of the fibre properties. It is so because of the general structural uncertainties involved in even such a yarn as may be regarded as the nearest approach to the ideal from practical considerations. This led the writer to fall back upon the then existing results of measurement of the properties of the fibre and the yarn and, with their help, to develop an arbitrary scheme, of E with reference to the gradations of count of yarn and the intrinsic strength of the fibre. The intrinsic strength, p/f, termed by some as the fibre-breaking length and by others as tenacity, was shown by Balls[3] to be a most important character governing spinning quality. It was found (Sen[8]) that the agreement between the observed yarn strength of cotton and the yarn strength calculated from equation (I) using the given arbitrary scale of E was reasonably close.

From a large number of data[8] it was concluded that generally the observed and the calculated values based on the arbitrary scale of E differed by less than 5% of the former. A few cases with larger difference ($\sim 10\%$) were also noticed. The largest differences ($\sim 20\%$) were however found in the case of the binary mixtures of different cottons and may have been caused by lack of perfect homogeneity of the mixed product affecting the observed yarn strength.

¶ It is to be noted that while the symbol, *e*, has been used for use in case of a test length, *l*, equal to the average fibre length, for *any actual* test length, L, the symbol used is E. The length gradient of strength, for instance, is an important contributor to E, but has rather an insignificant effect on *e*. Considering the dimensions of the quantities which take part in equation (I), it appears that the factor E has the inverse dimension of length.

(B) Comparison of Equations for Flax and Cotton

Equation V derived above and Spencer-Smith's equation for flax (equation II) seem to possess the following common features—

(i) Both the equations represent the relation between the count-strength product (quality number) of a yarn at a long test length and that of the same yarn at the shortest standard or assessable test length.

(ii) Both the equations contain a reduction term for strength at the shortest standard test length. This expresses itself in the equation as deviation from unity of the ratio :

$$\frac{\text{Count-strength product at the observed test length}}{\text{Count-strength product at the specified shortest test length}}$$

Thus in both cases, the count-strength product obtained at the shortest specified test length requires to be multiplied by a suitable proper fraction to obtain the count-strength product at a desired test length.

The principal point of difference between the two equations is that the basic test length is the average fibre length in the case of cotton, and the length of the fracture zone in the case of flax. This point may be discussed further.

The mean length of cotton fibre which can be easily spun varies from 0·5 inch to slightly over 2 inches. The maximum in the case of the Indian cottons, however, hardly exceeds a little over an inch. On the other hand, the mean filament length of flax is several times longer than the maximum length of cotton. Unlike therefore the case of cotton which possesses very short filaments, there is distinct possibility of occurrence of significant effect of a fibre strength gradient in the case of the long filaments of flax. This possibility precludes the use of the average length of flax as the shortest standard or assessable length. The same reasoning also applies to any other long textile fibre such as wool, jute, nettle, ramie, etc. Now, Spencer-Smith (*loc. cit.*) has pointed out that the mean length of the fracture zone of normally twisted flax yarn is about 0·5 inch. With this length used as the test length of flax yarn, the effect of the strength gradient of the yarn is undoubtedly obviated, the clamping of the fibres at both ends may also be assumed generally, and no complication due to the strength gradient of the fibre may be expected. Thus the principal advantages of using the average fibre length of cotton as the shortest standard test length, are obtained in the case of flax if the length of the fracture zone is adopted.

There is another point calling for attention, *viz.*, the "difference" term which is E in equation (V), and

$$\left[\frac{\omega_n \cdot F(n) \cdot V_0}{100}\right]$$

in equation (II). In using the length of the fracture zone as the shortest test length, Spencer-Smith has pointed out that the relationship between the adjacent fracture zones arises from the fact that one flax fibre may spread over several fracture zones (apart from the question of any random variation imposed by the machinery). But when we use the average fibre length as in the case of cotton, *i.e.*, when short fibre materials are under consideration, the effect of this factor should be insignificant. It is therefore regarded that $F(n)$ which is based on the above consideration in the case of flax (as well as of other long fibres) *cannot persist as an important factor in the evaluation of the difference term for cotton and similar other short fibres*. In other words, the strength-reducing quantity E for cotton should not be significantly influenced by the correlation of the adjacent small lengths as indicated by Spencer-Smith to be the case with flax.

It may be worth-while to explore both the physical and the statistical possibilities which govern the values of the slippage factor E.

(C) *The Applicability of Equation (I) to Jute*

With respect to equation (V), it may be pointed out that the author[10] observed in the case of jute yarns a *linear* trend between the adopted measures corresponding to Q_L and Q_l respectively. This indicates that for jute which is a long fibre, the effect of the "slippage factor", E, is not very important.

However, the possibility of wide variations in the measurements, particularly of p, in the case of jute filaments, and the fact that "fibre-length" of jute is, so to say, created by the processes involved in spinning, make the applicability of the equation to this fibre only of academic interest.

1. Sen, K. R., *Science and Culture*, 1938, **4**, 5. 2. Balls, W. L., "*Studies of Quality in Cotton*," London, 1928. 3. —, *ibid.*, p. 252. 4. Spencer-Smith, J. L., *J. Text Inst.*, 1947, **38**. 5. Turner, A. J., *ibid.*, 1928, **19**. 6. Tippett, L. H. C., *Biometrica*, 1925, **17**. 7. Sen, K. R., and Nodder, C. R., *Tech. Res. Mem., I. C. J. C.*, No. 9. 8. Sen, K. R., *ibid.* 9. Balls, W. L., *ibid.*, 206. 10. Sen, K. R., and Nodder, C. R., *Tech. Res. Mem., I. C. J. C.*, No. 7.

FUEL RESEARCH INSTITUTE

THIRD in India's chain of eleven National Laboratories, the Fuel Research Institute at Digwadih (Dhanbad) in Bihar is due to be opened on April 22, 1950, by the President, Dr. Rajendra Prasad.

The Institute will conduct research on major problems concerning fuel — solid, liquid and gaseous—and will operate a physical and chemical survey of Indian coals, to provide a reliable assessment of the quality and quantity of the various coal resources of the country.

In addition to problems of fundamental and applied research, sampling and analysis of coal will be undertaken and pilot-plants are to be developed for various processes.

The Institution's work will be distributed among the following main divisions: Coal Survey and General Analysis; Carbonisation and by-products; Liquid fuel (including hydrogenation, synthetic fuels, petroleum and substitutes); Physics (including X-ray and Spectroscopy); Gaseous Fuels (including gasification); Engineering (including preparation of coal for the market, coal-washing, boiler plant and combustion engineering).

The Director of the Institute is **Dr. J. W. Whittaker**, who will be working in consultation with Dr. S. S. Bhatnagar.

There will be six Regional Coal Survey Stations working under the Institute for the physical and chemical survey of coals and will be located at: the Raniganj coalfield, with a laboratory near Disherghar; the Jharia field with its laboratory at the Central Institute at Digwadih, the Bokaro-Ramgarh-Karanpura fields with a laboratory at Ranchi, the Eastern States coalfields (Vindhya Pradesh) with a laboratory at Umaria, Sagra Estate; the Madhya Pradesh (C.P.) coalfields, with a proposed laboratory at Kamptee near Nagpur, and the Assam coalfield with a proposed laboratory at Dibrugarh.