NEW FORMULA FOR VARIATION OF COMPRESSIVE STRENGTH WITH GRAIN ANGLE IN TIMBER

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THERE are various formulæ for the variation of timber strength with grain angle (*i.e.*, with the inclination of grain to the direction of applied force in timber). F. Kollmann[3] suggested

$$\sigma_\theta = \frac{\sigma_{\parallel}\sigma_{\perp}}{\sigma_{\perp}\cos^n\theta + \sigma_{\parallel}\sin^n\theta} \qquad \text{(A)}$$

where $\sigma_{\parallel}$ = the strength value in the direction of grain, $\sigma_{\perp}$ = strength value in the direction of perpendicular to the grain, θ = the angle between the grain and the direction of the applied force, σ_θ = the strength required (*i.e.*, in the direction making an angle θ with the grain), and n = an exponential term depending on species and varying between 2·5 and 3 for compressive stresses and between 1·5 and 2 for tensile stresses.

This above formula is referred to as Formula A in the succeeding paragraphs.

In Madison, U.S.A.[1,2] the Hankinson Formula, *i.e.*, Formula A with $n = 2$ is used for all types of stresses and species. This is referred to as Formula B in the succeeding paragraphs. Stussi[4] claiming a more convenient formula for designing purposes proposed

$$\sigma_\theta = \sigma_{\parallel}\frac{\cos^2\theta}{\sqrt{1 + C_1\sin^2\theta}} + \sigma_{\perp}\frac{\sin^2\theta}{\sqrt{1 + C_2\cos^2\theta}} \qquad \text{(B)}$$

where C_1 and C_2 are two parameters and the rest are the same as above.

Having studied the applicability of the above formulæ for compressive stresses in *Calophyllum tomentosum* (poon), the following formula is suggested as simpler than any of the above, and may be tried for other species and stresses also.

$$\sigma_{\parallel}/\sigma_\theta = 1 + P\sin^2\theta \qquad \text{(C)}$$

where P is a parameter depending on the species and the stresses, and the rest are the same as above. This formula is denoted in succeeding paragraphs as Formula C.

Experimental.—From a plank 16′ × 2½′ of *Calophyllum tomentosum* (poon), standard sized specimens, each of 2″ × 2″ × 8″, were cut at different angles to grain varying from 0° to 90° at regular intervals of 10°. In each group three pieces were taken. All specimens were conditioned to about to 12 per cent. moisture content. Each specimen was measured and weighed before test. A Lamb's extensometer was so adjusted to the specimen that the gauge length was 6″ leaving 1″ at each end. Specimens were loaded for compression parallel to the length and the deflection of the specimen was directly plotted on a graph sheet with the increasing load as in the standard practice, until the elastic limit was passed. The extensometer was then remvoed and the load at which failure took place was noted The speed of the machine, during test, remained uniformly 0·024″ per minute as required under standard practice. The load-deflection curves were drawn connecting the plotted points on the graph sheet for each specimen and the elastic limit was noted. The maximum crushing stress, and the crushing stress at E.L. for each specimen were calculated. Their average value for each of the above properties under different grain angles are shown in Figs. 1 and 2

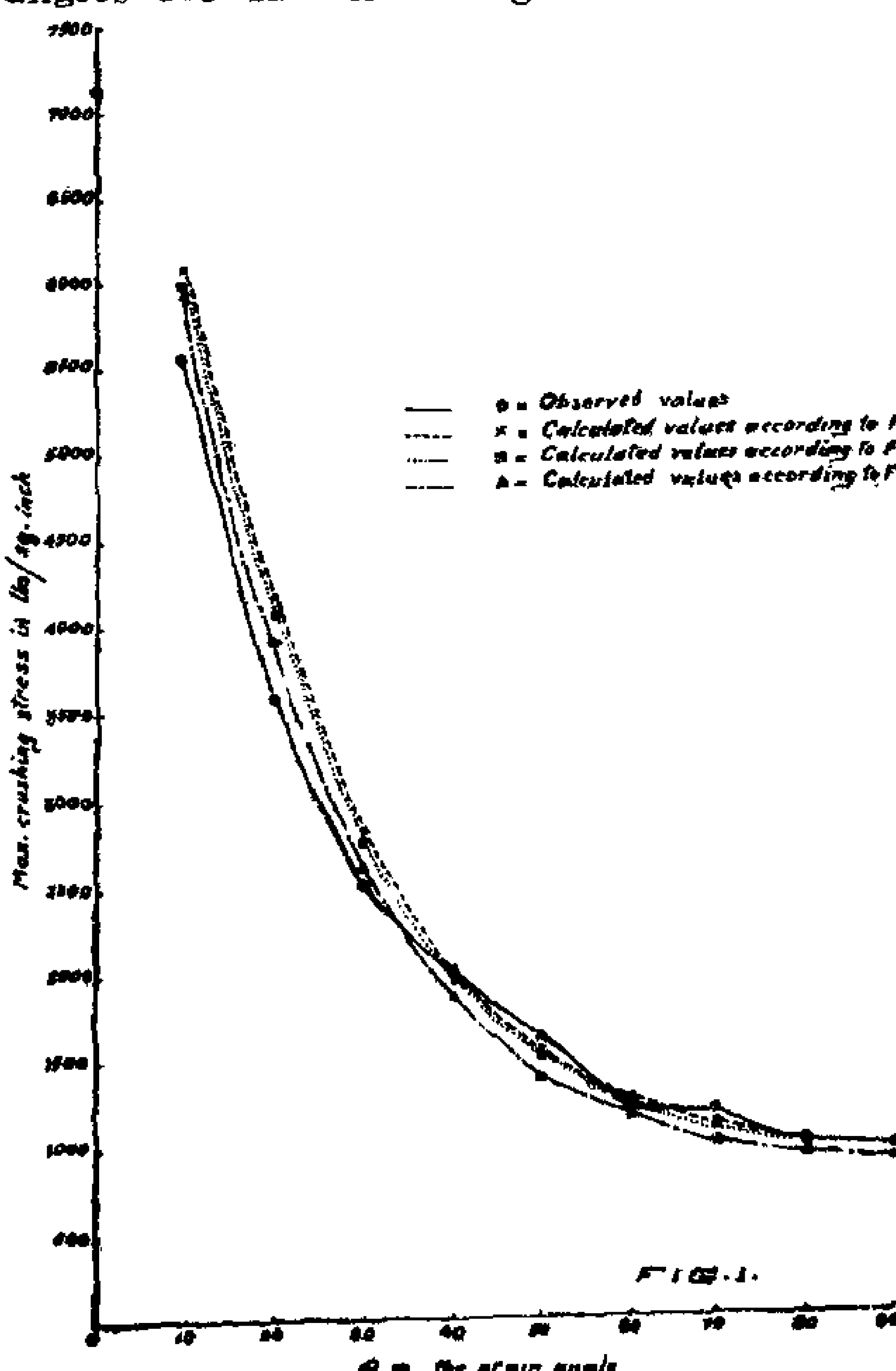

Fig. 1.

Theoretical.—In Figs. 1 and 2, the observed values and the calculated values according to the different formulæ are plotted against various angles of grain to the direction of force for maximum crushing stress and crushing stress at E.L. In trying to evaluate the required exponential n of the formula A, $\sin^n\theta$ and $\cos^n\theta$ were expanded. Approximation was made correct to the first two terms only involving $\sin^2\theta$ and $\cos^2\theta$. The value of n was thus determined for all angles taking the observed values of $\sigma_{\parallel}$ and $\sigma_{\perp}$. The average value of n was then determined and substituted in

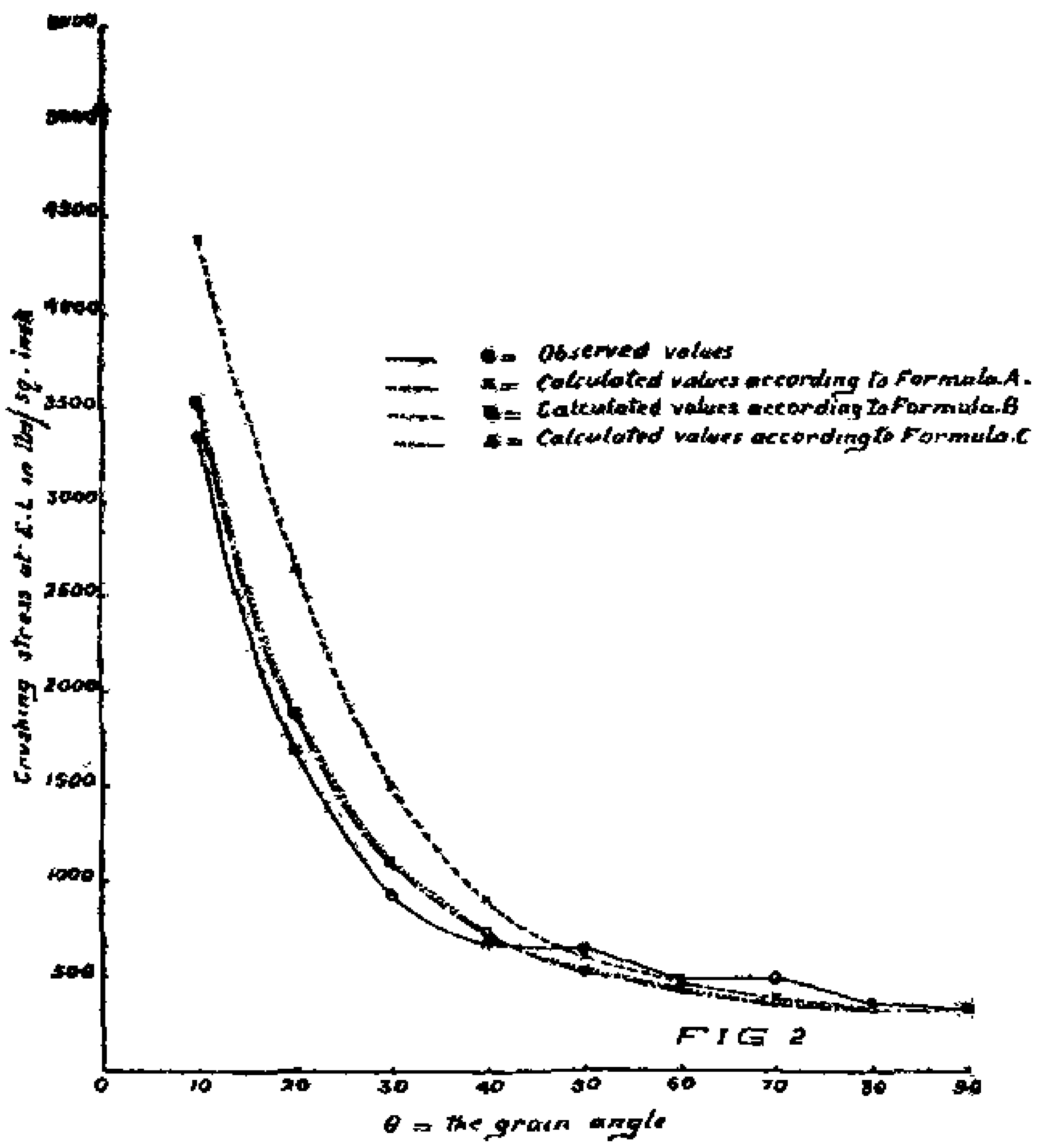

FIG 2

the Formula A for getting the calculated values of σ_{θ}. From Formula B, σ_{θ} was calculated taking the observed values for $\sigma_{\parallel}$ and $\sigma_{\perp}$

Stussi's formula was also tried for evaluation of the two parameters C_1 and C_2. In view of the above formula involving two parameters as compared with Formula A, in which there is only one parameter, it was considered that in this case the calculated values would perhaps be closer to the observed values than in any other case However, the values of C_1 and C_2 could not be easily evaluated as two identical equations involving C_1 and C_2 were obtained in evaluating the parameters either by the method of least squares or by taking any two observed values Hence this formula was discarded.

With Formula C, the parameter P was determined for all angles taking the observed values of $\sigma_{\parallel}$. The average value of P was then determined and substituted in Formula C for getting the calculated values of σ_{θ}.

Discussion

It may be noted that in solving the parameter n in Formula A, not only one approximation has to be made at one stage, but also two values are required to be observed (*i.e.*, $\sigma_{\parallel}$ and $\sigma_{\perp}$). In evaluating n for maximum crushing stress the various values of n at the different angles varied from $-63 \cdot 4$ per cent. to $+26 \cdot 8$ per cent. of its average value which worked out to be $2 \cdot 05$. In the case of crushing stress at E.L. the variation was from -28 per cent. to $+75$ per cent. of its average value which worked out to be $2 \cdot 499$. Also after obtaining the average value of n by the above method, the smooth curve of the calculated values does not necessarily pass through the fixed points and the value of n becomes ineffective for these points. Formula B appears to be more universal irrespective of species or condition of timber at test or the nature of stress applied. Formula C is the simplest of the three and required only one observed value for evaluation of its parameter P. Variation of P was from $-18 \cdot 1$ per cent. to $+38 \cdot 5$ per cent. of its average value (*i.e.*, $7 \cdot 02$) in the case of maximum crushing stress and from $-24 \cdot 8$ per cent to $+23 \cdot 5$ per cent. of its average value (*i.e.*, $14 \cdot 54$) in the case of crushing stress at E.L. Also it was found from the graphs between observed and calculated values for crushing stress at E.L. and for maximum crushing stress (not reproduced here), the scatter as well as the slope of lines fitted by the method of least squares were the best in the case of Formula C. The sum of the squares of the difference between the observed values and calculated values is least in the case of Formula C than in the other two cases. It is to be hoped that Formula C being simplest, may find applicability with sufficient or even greater accuracy than the other two. However, further experiments are required to be done on other species and different types of stresses for a more conclusive evidence.

1 Brown, H. P., Panshin, A. J. and Forsaith, G. G., *Text-book of Wood Technology*, 1952, **2**, 237, McGraw Hill.

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Stussi, *Engineers' Digest*, **7** (3), 59.