DISTANCE CORRELATION FOR PHOTONS

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BROWN AND TWISS[1] have recently provided a most remarkable demonstration of a *distance-correlation* between photons in a coherent beam of light; and they have also shown why this fundamental effect could not be observed by Brannen and Ferguson[2] and others[3] under the conditions of their experiment. At first sight this observed correlation appears indeed surprising, and in fact, Brown and Ferguson have expressed the view that the existence of such a correlation would require a revision of the foundations of quantum theory. However, according to Brown and Twiss and as Purcell[4] has explained, it is a direct consequence of the theory for a Bose-Einstein gas. The purpose of this communication is to show that distance correlation for photons follows naturally from the earlier work of Uhlenbeck and Gropper,[5] and particularly of London.[6] (In a Fermi-Dirac gas this correlation is there, but negative.) London considered the case of a completely *non-relativistic* degenerate Bose-Einstein gas. Its extension to a completely *relativistic* degenerate gas—photon gas—is immediate. In Bose-Einstein degenerate gas, because of the symmetry property of the wave-function describing the assembly (which is equivalent to the tendency of Bose particles to occupy the same phase cell in the phase space), the particle density $D(r)$ at a distance r from a given particle tends to the value $2n$ for r tending to zero, where n is the average particle density for the assembly (total number of particles in the assembly divided by its volume).

As is readily shown (London, *loc. cit.*) the density of photons at a point r from any given photon is given by:

$$D(r) = n + D_1(r) \tag{1}$$

where we have

$$D_1(r) = \frac{1}{V^2 n} \sum_\rho \sum_\sigma n_\rho n_\sigma \exp.\{2i\pi(k_\rho - k_\sigma)\cdot r\} - \frac{1}{V^2 n} \sum_\rho n_\rho^2 \tag{2}$$

Here V is the volume of the assembly, and n_ρ and n_σ denote the mean number of photons in the momentum states hk_ρ and hk_σ respectively. We see from equation (2) that for r equal to zero, $D(r)$ equals $2n$. We may look upon this as a tendency for photons to form, as it were,—crudely speaking—photon pairs. As the integral of $D(r)$ over the volume of the assembly must equal nV, the integral of $D_1(r)$ must vanish. This is evident from equation (2). However, the important thing to note in equation (2) is that whereas the first term on the right-hand side of the equation makes an effective contribution to the integral of $D(r)$ over space only in the region in the immediate vicinity of a given photon (that is, small values of r, a few wavelengths at most) every volume element of the assembly makes an equal contribution so far as the second term is concerned. There is an increase in the average density $D(r)$ for r tending to zero—we call it *local* (in the immediate vicinity of a photon) increase of density—, and this is compensated by a small *general* decrease in the density throughout the volume of the assembly. Further, for a photon of momentum hk_ρ the correlation is with photons of momentum $hk_\sigma \rightarrow hk_\rho$. Thus the average number (s) of photons associated, on account of Bose-Einstein correlation, with a given photon is obtained by integrating the first term in equation (2), that is:

$$s = \frac{1}{Vn} \sum_\rho n_\rho^2 \tag{3}$$

If $n(k)\,d_3k\,(d_3k \equiv dk_x\,dk_y\,dk_z)$ be the number of photons for unit volume and with momentum lying in the domain d_3k, we have

$$s = \frac{2}{n} \int n^2(k)\,d_3k \tag{4}$$

Assuming Planck's law of black-body radiation and integrating (4) we have

$$s = \frac{\zeta(2)}{\zeta(3)} - 1 \sim 0{\cdot}3685 \tag{5}$$

Let us now confine ourselves to the case of photons lying in the momentum range k, $k + dk$. We have for this case

$$s = n(k) = \frac{1}{e^{hck/RT} - 1} \tag{6}$$

which for $hck \gg RT$ gives

$$s \sim \exp.\{-hck/RT\} \tag{7}$$

The number of photon-pairs per unit volume is, therefore, given by

$$sn = n^2(k)\,d_3k \tag{8}$$

It is these photon-pairs which are responsible for the observed distance correlation between photons. The application of the foregoing treatment to the actual experimental arrangement of Brown and Twiss will be treated elsewhere.

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