COHERENCE PROPERTIES OF ELECTROMAGNETIC RADIATION*

PART I

S. PANCHARATNAM

1. INTRODUCTION

THOMAS YOUNG in his classic experiment of 1807 showed that the light diverging from two adjacent slits, illuminated suitably from behind, gives rise to dark and bright fringes on a screen placed in front of the slits. Given that light is a wave-phenomenon, this consequence follows naturally. The apparent mystery is that light beams do not always exhibit the phenomenon of interference. Interference fringes are not formed when the two slits are illuminated by two *independent* laboratory sources of monochromatic light—the light disturbances at the two slits being then said to be mutually incoherent. On the other hand, if the two slits are illuminated by a single point source, interference effects of maximum visibility are produced—the disturbances at the slits being then described as completely coherent with one another.

Suppose a second point source be kept adjacent to the first at such a distance that the double slit interference pattern due to this illuminating source alone is shifted by half a fringe-width relative to that due to the first source alone. As a net result no interference fringes would be visible on the screen, so that the disturbances at the two slits—regarded as secondary sources—must again be described as mutually incoherent. For a smaller separation of the two point sources the fringes reappear though with diminished visibility—the minima not being absolutely dark. The disturbances at the two slits could then only be described as *partially* coherent with one another. It would be natural to take the visibility of the fringes (as defined by Michelson) as a measure of the mutual degree of coherence γ, the displacement of the fringe system from its standard position determining the effective phase difference δ. It turns out that two partially coherent disturbances could also be pictured in the following manner: *An independent fraction γ^2 of the intensity of one disturbance could be regarded as completely coherent with the second and having a phase advance δ over it—the remaining fraction being incoherent with the second disturbance.*

The phenomena depending on the interference of light (using sensibly monochromatic light and usual conditions of path retardation) merely show that for a duration long compared with the period of the light wave, the vibration cannot depart sensibly from an ideal periodic vibration having a specific amplitude and absolute phase. However, because of the extremely short period of the light wave, we may yet suppose that the temporary intensity and absolute phase fluctuate millions of times a second—the optical characteristics of a beam as observed in usual experiments depending only on certain average quantities. The fluctuations of the temporary intensity and phase occurring in two *coherent* disturbances would be absolutely correlated with one another—such disturbances usually originating from the same point source or atom. In a monochromatic source of light we could crudely picture each atom as radiating a succession of wave-trains. If the phases of the successive wave-trains are assumed to change in a random manner, the radiation reaching a point from *two different atoms will sometimes interfere* constructively, and at other times destructively—the net result being no overall interference, the average intensity being merely the sum of the average intensities of the disturbances due to each source separately. The average length of each wave-train and its duration of emission may be called the 'coherence length' and the 'coherence time' respectively. It is to be expected that if the radiation from a point source is split into two beams, one of which is allowed to suffer a very large path retardation relative to the other—larger than the coherence length—then the beams would become effectively incoherent, as displayed by the lowering of the visibility of interference fringes. Such an effect is indeed observed and we shall return to this point later. However, under normal conditions of the path retardation, two disturbances originating from the same monochromatic point source may be regarded as completely coherent.

More generally, by introducing the concepts used in the mathematical analysis of noise—such as the correlation function between two statistically fluctuating quantities—the mutual degree of coherence between two disturbances can be defined without any detailed assumption regarding the nature of the light disturbances

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* This was the title of a Conference held at Rochester, N.Y., from the 27–29 June 1960. The present article introduces some of the topics presented there, but is not meant to be a report of the proceedings.

emitted by individual atoms. Such an analysis has been developed in detail by E. Wolf[1] who, appropriately, reviewed the field at the Conference. If we assume that the light disturbance at a time t can be expressed uniquely as the real part of a complex variable $V(t)$, then Wolf introduces the mutual coherence function $\Gamma_{12}(\tau) = \langle V(t) V^*(t+\tau) \rangle$. The sharp brackets denote time average and the mutual coherence function $\Gamma_{12}(\tau)$ expresses the correlation between two disturbances 1 and 2, the first disturbance being considered at a time τ later than the second. For sensibly monochromatic radiation and for usual experiments where the path retardations involved are small compared with the coherence length, the mutual coherence function relating two disturbances may be considered a constant independent of τ.

2. Coherence and Monochromaticity

There is another point of view from which the phenomenon of partial coherence may be analysed. We have already mentioned that when two beams—obtained by the splitting of a single collimated beam—are allowed to interfere, the visibility of the interference effects goes down when the relative path retardation introduced is made very large, *i.e.*, comparable with the coherence length for the monochromatic radiation used. As is well known, this experiment was performed by Michelson who however used the variation in the visibility of fringes to determine the shape of the spectral 'line' emitted by the source. It thus becomes clear that the phenomenon of incoherence and partial coherence stands in the most intimate connection with the lack of strict monochromaticity.

The finite spectral width of all radiation that can be used or detected must be recognised as inevitable and intrinsic in the nature of things, so that only properties of radiation averaged over a small spectral range can be regarded as physically measurable quantities. A *strictly* monochromatic wave-train would be one whose amplitude and phase are constant in time and hence would extend from *plus* infinity to *minus* infinity. If the wave-train from atoms were of this nature, the radiations from different atoms could interfere and the phenomenon of incoherence would not exist. A disturbance consisting, for example, of a succession of wave-trains whose amplitudes and phase factors vary in time is therefore *not* strictly monochromatic but quasi-monochromatic. By Fourier's theorem, such a disturbance could be regarded as the sum of a number of strictly monochromatic vibrations spread over a small but finite spectral range of frequencies, the amplitude and phase factor of each monochromatic component being naturally constant quantities and not fluctuating in time. The average intensity of the quasi-monochromatic beam which alone is measurable is the sum of the 'intensities' of its monochromatic constituents.

Considering now the case of two interfering beams which are quasi-monochromatic, the strictly monochromatic component of a particular frequency in one of the beams will necessarily be completely coherent with the corresponding component of the same frequency in the second beam. In the case of two *coherent* beams the phase difference δ_m between a corresponding pair of monochromatic constituents of the same frequency ν_m in the two beams will be the same as the phase difference δ_n between the interfering pair of frequency ν_n. At the other extreme for *incoherent* beams, the phase differences δ_m, δ_n etc., between corresponding pairs of monochromatic constituents will be distributed from zero to 2π—so that the average intensity of the resultant quasi-monochromatic beam obtained by their superposition is merely the sum of the average intensities of the original beams. For intermediate cases, the degree of coherence and effective phase difference between two quasi-monochromatic beams or disturbances could be defined in a manner closely analogous to the conventional method—except that in the present view-point an averaging over frequency rather than time is involved in the definitions. This analysis was included in the paper presented by Pancharatnam,[2] which dealt with two beam interference taking into account the fact that the beams may be polarised, completely or partially—a factor which we have not till now referred to.

3. Correlation of Photons in Coherent Beams

The basic picture of interference given by the quantum theory is often discussed in theoretical text-books with reference to an imaginary two-beam interference experiment with weak light. A sufficiently accurate experiment of this nature was, however, only recently performed by Janossy[3] and co-workers in Hungary. They used a Michelson interferometer of very large dimensions in which, as is well known, a semi-silvered mirror is used to split an incident collimated beam into two coherent beams which travel along the arms of the interferometer and are then allowed to interfere. Light of such low intensity was used that on the average there would be only one

photon, at any instant somewhere in the arms of the interferometer. First an experiment was performed which, it should be noted, automatically prevents the beams from interfering: two photo-tubes were placed respectively in the paths of the two beams and connected to a coincidence counter. The absence of significant coincidences verifies that a single photon on striking the semi-silvered plate does not of course split, but is *either* reflected or transmitted with equal probability. However, according to quantum theory when the interference experiment is performed, the state function for the photon, governing the probability of its appearance somewhere in the field of interference, is now a coherent superposition of the state functions involving both beams. In accordance with this it was found on taking a very large number of counts in the field of interference that no photons fell in certain regions—'dark fringes'—and the maximum number fell in adjacent 'bright fringes'.

Considerable discussion arose in the columns of *Nature* when Twiss and Brown showed definitely that the coherence or otherwise of two beams could be detected even without allowing them to interfere, just by seeing whether the intensity fluctuations in the two beams were correlated. These discussions having already cleared the air, there was not much additional discussion of a basic nature at the Conference when they presented their work. In the first type of experiment performed by them,[4] a semi-silvered mirror was used to split the radiation from a source into two beams which were received on two separate photo-tubes with small apertures. The fluctuations occurring in the output of the two photo-multipliers were found to be correlated when the disturbances received at the tubes were expected to be coherent, and uncorrelated under conditions when they were expected to be incoherent. Classically it is immediately obvious that if there is a fluctuation of intensity above average in a wave-train falling on a semi-silvered mirror, the fluctuations will continue in the two wave-trains into which it is split. It must be noted that the intensity fluctuation mentioned is intrinsic in the nature **of things and not due to macroscopic fluctuations in the conditions of operation of the source; indeed, Brown and Twiss proved that this was not the factor causing the correlation. In the wave-picture the fluctuations arise from the fact that Fourier components of different frequencies (contained within the finite spectral width) interfere with one another giving rise to beats or fluctuations of intensity about its** average value. Clearly the intensity fluctuations in radiation from independent sources could not be expected to have any correlation.

In a second experiment which more closely illustrated the particle aspect of light,[5] two coherent monochromatic beams of light (from a mercury isotope lamp) were as before received on two photo-tubes; these were connected to a coincidence counter to record the occasions when the times of arrival of two light quanta at the two respective photo-tubes lay within the resolving time of the coincidence counter. Brown and Twiss demonstrated that when the beams were coherent the number of 'coincidences' were in excess of the random value. Considering the picture of a collimated light beam as a hail of quanta, the average intensity will be given by the number of photons received per second, but even with the steadiest source obtainable there are bound to be fluctuations from this average rate, which may be determined by statistics; in fact, since photons obey Bose-statistics and not classical statistics, there is a tendency for photons to 'clump', *i.e.*, the fluctuation in the rate will be slightly greater than for a random sequence of independent events occurring at the same average rate. This additional fluctuation in a single beam may in turn be considered as giving rise to the Brown-Twiss effect mentioned, *viz.*, that the 'coincidences' between photons received in two coherent beams exceed the random value. If the photons had obeyed classical statistics there would be no correlation between photons in two coherent beams. It was shown by Purcell,[6] as also by Brown and Twiss that these observations did not really conflict with those of Janossy *et al.*, since the latter's arrangement would be far too insensitive to detect this correlation.

4. Light Beams from Incoherent Sources

According to classical ideas, two waves of different frequency can interfere with one another giving rise to *beats* or a periodic fluctuation of intensity at a frequency equal to the difference in the frequency of the two superposed disturbances. Forrester reported on an experiment in which the beats had been detected by mixing the Zeeman components of a visible spectral line at a photo-surface. The periodicity in the emission current was detected by the excitation of a 3 cm. microwave cavity tuned to the beat frequency—a special photo-mixer tube being designed for this purpose. Since the beat is produced by the mixing of mutually incoherent radiation, the phase of the beat current could be expected to fluctuate in a period of the order of the coherence time for each Zeeman component; but the power at the beat frequency depends on the square of the current and this does not vanish on averaging—though the effect is very feeble indeed. A basic assumption made by Forrester *et al.*, in the explanation of the experiment, is that the probability of emission of an electron at the photo-surface is proportional to the square of the electric field strength of the incident radiation—rather than the sum of the intensities of the two spectral lines separately. From the comments on this paper it appeared that the state function of a photon could cover two frequencies; however, when an experiment to determine the frequency of the photon is performed it would be found to be in one or the other frequency, and beat phenomena could not simultaneously be detected. On the other hand, in an experiment where the beat phenomena are detected it would be impossible to say whether the individual photons are of one or the other frequency.

1. Wolf, E., *Proceedings of a Symposium on Astronomical Optics*, North Holland Publishing Co., Amsterdam, 1956, pp. 177.
2. Pancharatnam, S., Unpublished.
3. Adam, A., Janossy, L. and Varga, P., *Acta Hungarica*, 1955, **4**, 301.
4. Hanbury Brown, R. and Twiss, R. Q., *Proc. Roy. Soc.*, 1957, **243 A**, 291.
5. —, *Nature*, 1957, **180**, 324.
6. Purcell, E. M., *Ibid.*, 1956, **178**, 1449.
7. Forrester, Gudmundsen and Johnson, *Phys. Rev.*, 1955, **99**, 1691.

THE XII GENERAL ASSEMBLY OF THE INTERNATIONAL GEODETIC AND GEOPHYSICAL UNION

THE International Geodetic and Geophysical Union is the largest among the international scientific bodies and includes scientists from 65 countries. The XII General Assembly held recently in Helsinki was singular in that it was the first major get-together of geophysicists after the I.G.Y. It was attended by about 2,000 scientists from 45 countries.

The bulk of the Assembly's programme was devoted to discussions held in all the seven associations of the Union, *viz.*, geodesy, meteorology, seismology and terrestrial physics, geomagnetism, physical oceanography, pure hydrology and volcanology.

The geodesists discussed the results of observations of artificial Earth satellites which have added much to our knowledge of the Earth's shape. They also examined techniques of gravimetric surveys from a flying plane.

The meteorologists exchanged new data on the general circulation of air in the atmosphere, and suggested for the first time charts of circulation covering the atmosphere to an altitude of 100 kilometres.

The Geomagnetism Association was highlighted by a discussion of the geophysical phenomena observed in July 1959. July had been chosen for a comprehensive correlation of the various phenomena studied under the I.G.Y. programme. Among other things, variations in the terrestrial magnetic field were viewed against changes in the intensity of cosmic radiation, ionospheric processes, and solar activity. July 1959 was of particular interest in that a sharp ten-day outburst took place on the Sun that month. As was found out in the discussion, the streams of tiny particles coming to the Earth from the Sun cause, though in a negligible measure, the Earth's speed of rotation to slow down.

The seismologists took up problems relating to the structure of the Earth's lower crust and the layers that extend many hundred kilometres into the Earth's interior. A new finding was that the continents differ from each other not only in the structure of the crust, but also in the deeper envelope (mantle) of the Earth to a depth of at least 600 or 700 km. This discovery convincingly refutes the hypothesis of floating continents, for they are firmly anchored to the very deep zones of the globe. Intriguing results were obtained through seismographs placed for the first time on the bottom of the ocean at a considerable depth. While on mainland seismographs show what are known as microseisms, or continuous minute tremor of soil, caused, it appears, by winds, changing air pressure, and waves striking at the shores of mainland, complete quiet reigns supreme at the ocean's bottom. Thus underwater seismographs may be employed to detect very weak earthquakes which are usually obliterated by microseisms when monitored on mainland.

The oceanographers examined in detail and elaborated their joint programme involving studies in the Indian Ocean.

The Association of Hydrology summed up the results of the I.G.Y. programme. A com-