SOME NEW CONCEPTS IN NON-LINEAR SYSTEMS

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Of late active interest has been displayed in the general field of non-linear systems and various techniques have been developed to analyse different aspects of the system under investigation. A technique developed by the author giving a new perspective and greater insight into the physical significance of the response of non-linear systems is presented in this communication.

Its principal features, and the major conclusions arrived at, are indicated in brief.

The behaviour of many complex physical systems is governed by higher-order non-linear differential equations. The solution of such equations is closely related to the number and nature of roots of the associated linear differential equation. The solution of the governing equation itself, is here obtained by the principle of the variation of parameters. It has been recognized that any non-linear system can be easily portrayed by a simple mathematical representation by means of a 'response-vector' and an orthogonal frame of reference. This reference-frame can have its directions along the inphase, quadrature and zero-phase components, with reference to the phase of the assumed solution. Thus, it can be easily visualized that the response-vector belongs to a space of as many dimensions as the order of the original governing equation; and, that it can be resolved into components along the above-referred directions of remarkable physical significance. These can be suitably denoted by the term 'space-phase components'.

One of the important features of the mathematical representation is that cross-harmonic terms are no longer present, yet the non-linear behaviour has been adequately accounted for by the d.c. and first harmonic terms. Thus it can be significantly referred to as the 'transient system-equivalent'. Such a representation can take care of the different types of inputs along with specified initial conditions. Thus it can reveal valuable information about peak overshoot, time of rise, time of settling, as well as possible limit-cycle behaviour. Though geometric representation is possible only for first, second and third order systems, the analysis itself is not limited by the order of the equation, and the principles can be extended to higher-order systems, with suitable engineering judgment.

It is further observed, that the zero-phase component influences only the amplitude of the response, while the inphase and quadrature components modify the amplitude, as well as the frequency of the response. Thus it is seen that these new concepts isolate important characteristics into quantities which represent a better criterion of the controlling factors. This feature is of great help in the synthesis procedures.

Another important feature of this technique is its striking analogy to the classic technique of symmetrical components. The more important of the analogues are indicated in Table I, while Table II shows the main points of difference.

The technique has been applied to evaluate the transient response of a multiplicative feedback control system.[1] The instantaneous error of the system to unit step input is governed by the equation

$$\frac{d^3e}{dt^3} + \frac{d^2e}{dt^2} + \left(1 - \frac{1}{2}e^2\right)\frac{de}{dt} + \frac{1}{2}e = 0. \quad (1)$$

The initial conditions of the error are given by

$$e(o) = 0 \cdot 5,\ e'(o) = 0 \text{ and } e''(o) = 0. \quad (2)$$

The solution of the above non-linear equation is obtained by the principle of variation of parameters; and is given in equation (3).

$$e(t) = -\frac{1}{6}e^{-2t}\sin\left(t + \frac{\pi}{2}\right) + \frac{2}{3}e^{-t/2}. \quad (3)$$

Thus, the transient response of the system for unit step input is given by

$$C(t) = 1 + \frac{1}{6}e^{-2t}\sin\left(t + \frac{\pi}{2}\right) - \frac{2}{3}e^{-t/2}. \quad (4)$$

In equation (4), C (t) is the response-vector, $1-(2/3)e^{-t/2}$ is the zero-phase component, which modifies only the amplitude of the response, while $(1/6\, e^{-2t}\sin(t+\pi/2))$ is the inphase component which modifies both the amplitude and the frequency of the response. The qudrature component is zero.

Equation (4), derived analytically, shows very favourable agreement with the analogue computer study as given in ref. 1.

It is believed that this technique will lead to further developments in the theory, and application to non-linear control systems, possibly even for random inputs.

The detailed paper will be published elsewhere.[2]

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TABLE I
Analogues

	Symmetrical components	Space-phase components
1	Number of phases of the electrical system	Order of the governing equation
2	Unbalance effects	Non-linear effects
3	Mutual coupling between phases due to dis-symmetry	Presence of cross-harmonic terms
4	Positive and negative sequence components	Inphase and quadrature components
5	Zero-sequence component	Zero-phase component
6	A simple mathematical representation of an unbalanced electrical system	A simple mathematical representation of a non-linear physical system
7	Enables stability study of power systems	Enables stability study of physical systems governed by the non-linear differential equation
8	Valid for analysis of transient as well as steady state behaviour (transient analysis by using Lyon's approach)	Valid for analysis of transient as well as steady state behaviour
9	Various time constants of the system can be determined	Time of rise, time of settling and other relevant time constants can be determined
10	Initial currents or voltages can be accounted for	Initial conditions can be accounted for
11	Suitable for study of transient disturbances and fault conditions	Any type of input in closed form can be studied, and instruments can be devised to control extraneous disturbances
12	Grounding phenomena associated with zero sequence component	Zero-phase component influences only the amplitude of the response, while the inphase and quadrature components influence both the amplitude and the frequency

TABLE II
Differences

	Symmetrical components	Space-phase components
1	Linearity is assumed in the derivation of the equivalent representation	The only assumption made is that higher harmonics in the response of the non-linear component are adequately filtered out by the linear part. This assumption is quite valid and practical with non-linear control systems
2	Sequence currents, voltages and impedances	The only variable is the system response
3	Evaluation of sub-transient parameters	None
4	Synthesis of networks to represent faults	Synthesis of components to represent non-linear effects is not possible because of the assumption made
5	Special features in the application to rotating machines	No essential difference in application to static and dynamic non-linear components
6	Sequence components are coplanar for any number of phases	Phase-space components are mutually orthogonal and located in space

1. Ku, Y. H., *Analysis and control of non linear systems*, (Book). The Ronald Press Co., New York, 1958, pp. 312-320.

2. Accepted for publication in the *Journal of the Franklin Institute*, U.S.A.

ALPHA-ACTIVITY OF DRINKING WATERS OF BRITAIN

INVESTIGATION of the nature and levels of naturally occurring radioactivity in human foods and potable waters is of vital interest. Prof. Mayneord of the Institute of Cancer Research, London, has reported the results of experiments on the α-activity of drinking waters supplied to the population of Britain. (In passing it may be noted that as early as 1902, J. J. Thomson made observations on the radioactivity of Cambridge water).

Mayneord and his collaborators have measured the radium-226 content of 71 drinking waters of Britain. The water samples were collected from several points of view, including the size of population supplied, and the nature of the associated geological formations. The samples can be divided broadly into five types, namely, (A) Spa waters which possess high contents of mineral matter; (B) Waters of Cornwall parts of which county are known to have deposits of uranium and radium; (C) Ground waters from boreholes in geological strata other than chalk; (D) Ground waters from boreholes in chalk, and (E) Surface waters from rivers, lakes and reservoirs.

In the experimental procedure, a litre of each specimen of water was evaporated to dryness and the α-activity of the residue measured