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ISOTOPE SHIFTS AND INTERNAL CONVERSION OF γ-RAYS*

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INVESTIGATION of isotope shifts in energy of atomic spectral lines provides a tool for studying nuclear charge distribution. Isotope shifts due to finite mass of the nucleus are important in light elements but become negligible for atoms of heavy elements. For elements in medium and heavy mass region, $A > 80$, isotope shifts due to non-zero volume of the nucleus, called volume effect, become important. It is the study of this volume effect in isotope shifts which can be used for investigating properties like nuclear charge distribution, deformation of nuclear shape and compressibility of nuclear matter. In these areas, therefore, measurements on volume effect in isotope shifts will provide valuable information, supplementary to the results obtained from nuclear spectroscopic studies of nuclear energy levels. Detailed interpretation of volume effect in isotope shifts in the mass region, where nuclear shape deformation is pronounced, will also involve consideration of specific nuclear models, which is of great interest to nuclear physicists.

For a nucleus treated as a point charge, the potential at a distance r is $-Ze^2/r$. For a finite nucleus with a certain charge distribution of radius r_0, the potential at $r < r_0$ will be different from this value. The energy of an electron in an orbit which penetrates the nuclear volume, *i.e.*, s-states and to a slight degree $p_{\frac{1}{2}}$-state, will be different from that calculated for a point charge nucleus. The addition of neutrons to a given nucleus alters its radius and charge distribution and the effect of penetration on the position of energy level of the s-electron is different in different isotopes. This gives rise to volume effect in isotope shifts. On the basis of such considerations, the change in energy ΔE, of an s-state electron is obtained as follows :

$$\Delta E = \int P(r) \left[V(r) + \frac{Ze^2}{r} \right] d\tau$$

where $d\tau$ is volume element and $P(r)$, which is s-electron density in the neighbourhood of a point charge in Dirac theory is

$$P(r) = \frac{2(2\rho + 1)}{[\Gamma(2\rho + 1]^2} \cdot \psi^2(0) \left(\frac{2Zr}{a_H} \right)^{2\rho - 2}$$

where $\rho = (1 - Z^2\alpha^2)^{\frac{1}{2}}$, α being the fine-structure constant.

a_H = first Bohr radius and $\psi(0)$ is the non-relativistic Schrodinger wave-function at $r = 0$.

$$\therefore \quad \Delta E = F(Z)\, \psi^2(0) \cdot R_1^{2\rho}$$

where

$$F(Z) = \frac{12\pi Z e^2 (\rho + 1)}{[\Gamma(2\rho+1)]^2 \cdot \rho(2\rho+1)(2\rho+3)} \left(\frac{2Z}{a_H} \right)^{2\rho-2}$$

$$R_1 \equiv \left[\left(1 + \frac{2\rho}{3} \right) \langle r^{2\rho} \rangle \right]^{1/2\rho}$$

$$\langle r^{2\rho} \rangle = \int f(r)\, r^{2\rho} \cdot \frac{d\tau}{Z}$$

and $f(r)$ is normalized so that $f(r)\, d\tau = Z$. The perturbation energy shift between two isotopes differing by δR_1 in equivalent radius, *i.e.*, the isotope shift due to volume effect is then given as follows :

$$\delta \Delta E = 2\rho F(Z)\, \psi^2(0)\, R_1^{2\rho} \frac{\delta R_1}{R_1}.$$

If nuclei are spherical and the nuclear radius varies as $A^{1/3}$, one gets certain results about isotope shifts, which are, however, different from what is actually observed. For even isotopes of a given element, for instance, the addition of successive two neutrons should give equal shifts. Actually this is not the case. There are fluctuations in the value of the ratio of observed to theoretically predicted (on spherical model) shifts, $\delta \Delta E_{obs.}/\delta \Delta E_{th.}$ in isotopes

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* Based on a Review talk given at Nuclear Physics Symposium, Madras, in February 1962.

of a given element with neutron number and this variation is strongly reminiscent of the change in quadrupole moment with neutron number. These fluctuations have been explained in terms of nuclear deformation and shell structure. (Fig. 1).

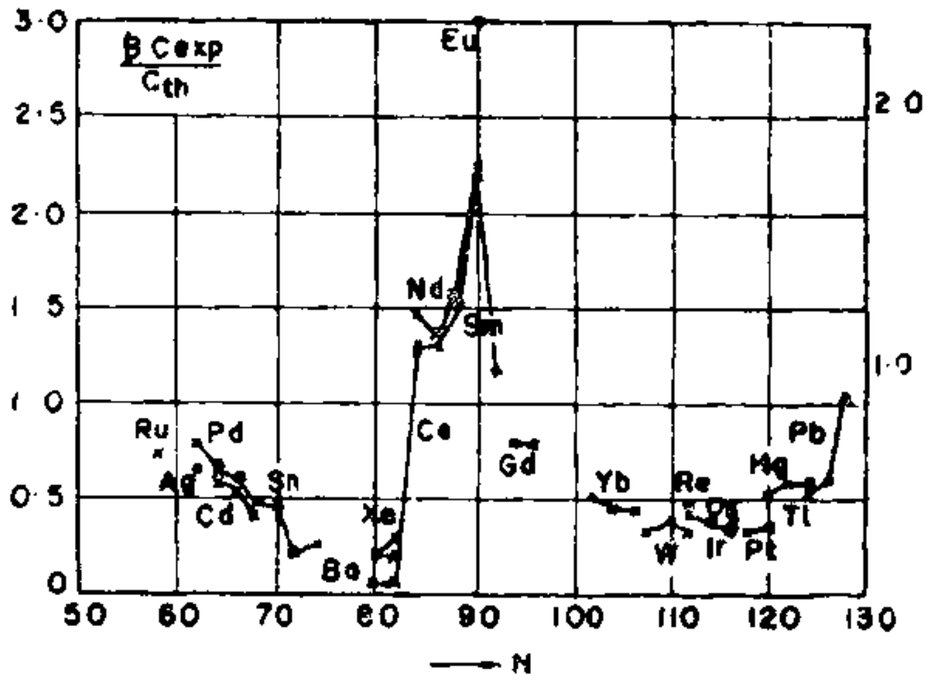

FIG. 1

Nuclei which are non-spherical will appear more extended radially than spherical nuclei of the same volume, after averaging over all angles. Considering a uniformly charged spheroidal nucleus, an expression relating the isotope shift to the change in square of deformation parameter with neutron number, $\delta\beta^2/dN$, has been derived (Willets, Hilland Ford)

$$\frac{\delta\beta^2}{dN} = \frac{10}{3\,(2\rho + 3)}\,\frac{A}{}\,\frac{(\delta\triangle E)_\beta}{(\delta\triangle E)_v}\,.$$

If one plots the ratio of observed shift to predicted volume effect on spherical model against neutron number, there should be a large value where the rate of change of deformation is large. When one uses values for deformation parameter obtained from nuclear spectroscopic data on Coulomb excitation and E2 transition probabilities and the data on isotope shifts, the above relation is seen to hold good fairly well. The data on isotope shifts, however, are not as plentiful as one would like. The case of the six even isotopes of Gadolinium A = 152 to 160 is instructive in this regard. Going from A = 152 to 154, there is a large change from near spherical to deformed shape, and this is reflected in a large value for the ratio $q = (\delta\triangle E)_\beta/(\delta\triangle E)_v$. In the heavier isotopes of mass number 156, 158, 160, though the deformation itself is large, the rate of change of deformation with addition of two successive neutrons is small. This may be seen from Table I which gives the values of the ratio q, deformation parameter β and $\delta\beta^2/dN$ for these isotopes.

A similar trend is observed for Samarium isotopes A = 148 to 152, though the data are not adequate. Recently Kuhn and Turner have reported their results on isotope shifts in even

TABLE I

Gadolinium isotopes, Z = 64

A	152	154	156	158	160
N	88	90	92	94	96
β	·057	0·3	0·41	0·46	0·47
q	1·6	0·9	0·05	·01	
$\delta\beta^2$	·09	·08	·04	·01	

isotopes of Tellurium A = 120 to 130. Using interferometric method they determined the relative shifts in fringe systems of the 4049 Å line due to a mixture of isotopically enriched samples. Their values corrected for normal mass-shift are reproduced in Table II.

TABLE II

Teliurium isotopes, Z = 52

A	120	122	124	126	128	130
N	68	70	72	74	76	78
$\delta\triangle E$ 10^{-3}cm.$^{-1}$	20·9	19·7	15·6	15·0	12·9	

The general decrease in the value of the shift from neutron number 68 to 78 follows the same trend as the variation in deformation parameter, β^2, for these isotopes obtained from the results on Coulomb excitation measurements of Stelson and McGowan. $\delta\beta^2/dN$ is found to decrease regularly in this range of neutron numbers. In Fig. 2 are plotted the isotope shifts against neutron number for even isotopes of Tin, Z = 50 and Tellurium Z = 52. It is seen that the shift decreases more steeply for Tin than for Tellurium, which is presumably due to the effect of the completed magic shell of 50 protons in Tin.

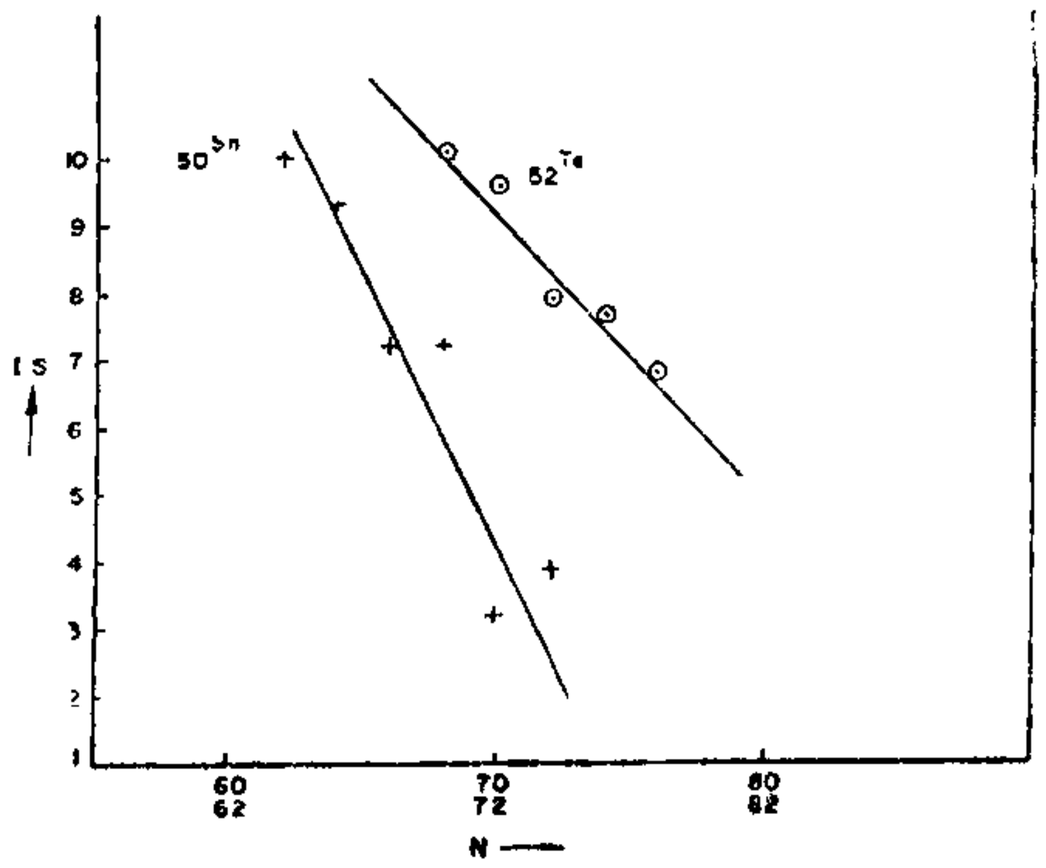

FIG. 2

The trends in variation of volume effect in isotope shifts are thus reasonably well explained in terms of the nuclear deformation. To interpret the actual values of isotope shifts, however, the effects due to nuclear compressibility and

surface effects have to be taken into account. The shifts, on the average, are smaller than the *predicted values.*

If nuclear radius $R = r_0 A^{\frac{1}{3}}$ where r_0 is a constant, $\delta R/R = 1/3\,A$. However, R is a function of N and Z. Taking Coulomb forces into account.

$$\frac{1}{R}\cdot\frac{dR}{dN} = \frac{1}{3A}\left[1 - \frac{4Ec}{E_0'' + Ec}\right]$$

where $Ec = 3\,Z^2e^2/5\,r_0\,A^{\frac{1}{3}}$ and $E_0'' = K.A$, K being compressibility. The isotope shift depending on dR/R will be reduced by a factor $1 - 4\,Ec/E_0'' + Ec$.

Another observation in isotope shift measurements which has not yet received proper explanation is that called "Even-odd staggering". It is found that the centroid of an *hfs* pattern due to an odd isotope lies closer to the line of its lighter even-even neighbouring isotope than to the line due to the heavier neighbour.

Internal Conversion Process

In making a transition from one level to another, the nucleus can give up energy K, angular momentum L and parity π, in the form of a γ-ray or these can be transferred to one of the orbital electrons. The ejection of the electron occurs through interaction of the electron and nuclear currents and charges through the electromagnetic field. Outside the region of nuclear transition currents and charges, the form of the field depends only on K, L, π and not on details of transition currents and charges. The rate of electron ejection depends on the electron wave-function and the form and strength of the E.M. field while the strength of the field is measured by γ-ray emission probability. The internal conversion coefficient, α, which is the ratio of the rates of ejection of electron and γ-ray will depend, for a non-penetrating electron, on the electron wave-function and K, L, π. The effect of finite size of nuclear charge distribution (as against a point charge) on the Dirac wave-functions of the electron has been computed (Rose, Sliv) and their effects on I.C.C, particularly for M 1 type transitions, have been experimentally verified.

The electron, however, does penetrate the nuclear region a small fraction of time. This leads to the so-called dynamical penetration effects in internal conversion, as shown by Church and Weneser. The whole electric monopole, E0, mode of de-excitation occurs through such penetration effects, since there is no parallel γ-ray mode. E0 transition is thus the dynamical analogue of the static penetration effects as shown in volume effect in isotope shifts. It would, therefore, be of some interest to correlate data on isotope shifts and E0 transition probabilities in nuclei, particularly in the 'deformed' region. While E0 type of de-excitation alone is possible for $O^+ \rightarrow O^+$ transitions, for transitions of type $I \rightarrow I$, with no change in parity, E0 transition probability competes strongly with *E2* and *M1* types of transitions (Fig. 3) in heavy elements.

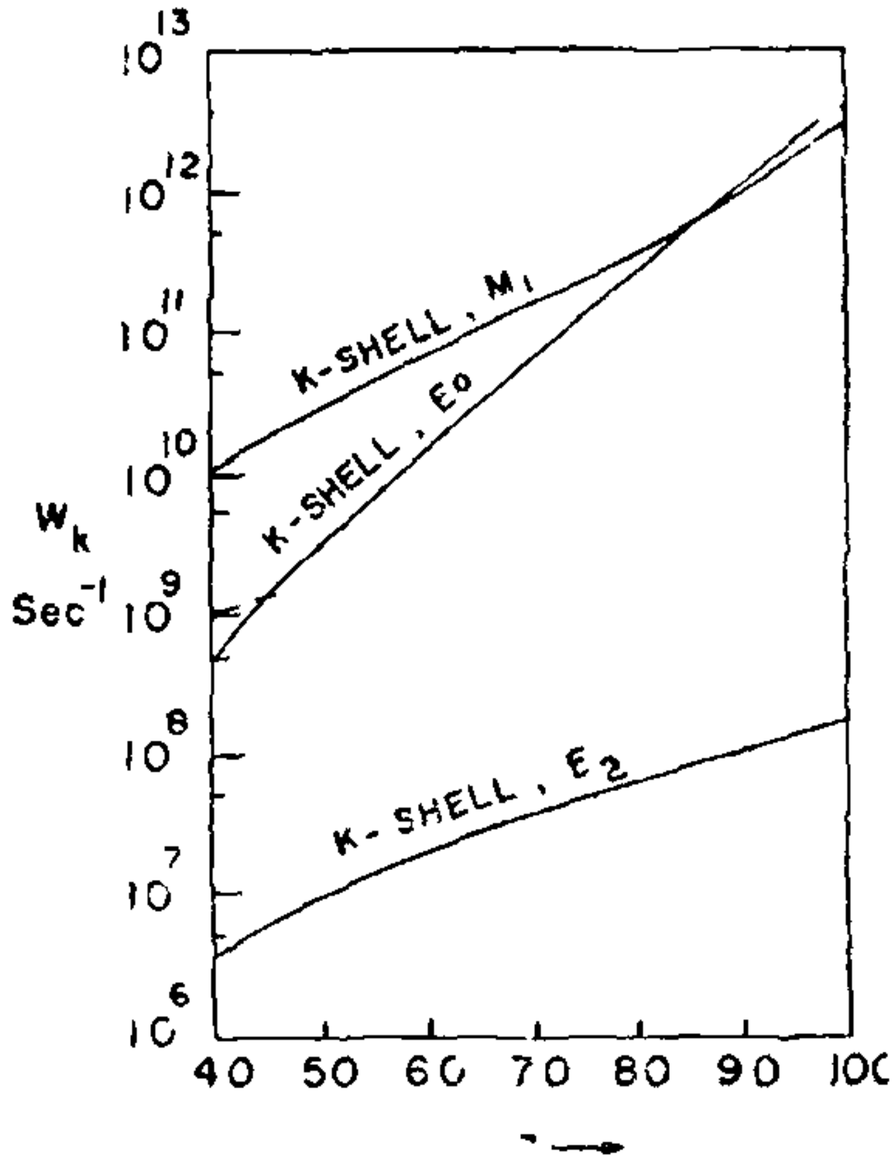

FIG. 3

It is, therefore, of great interest to search for second excited O^+ levels in even isotopes of deformed nuclei and to look for variation of branching ratio E0/E2 with the deformation parameter. In analogy with isotope shifts, this ratio should vary as $\delta\beta^2/dN$. There is some slight evidence for such a trend of variation in the values of E0/E2 for the even isotopes of Gadolinium—Gd 152, 154 and 156. Adequate experimental data on this point are at present not available.

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