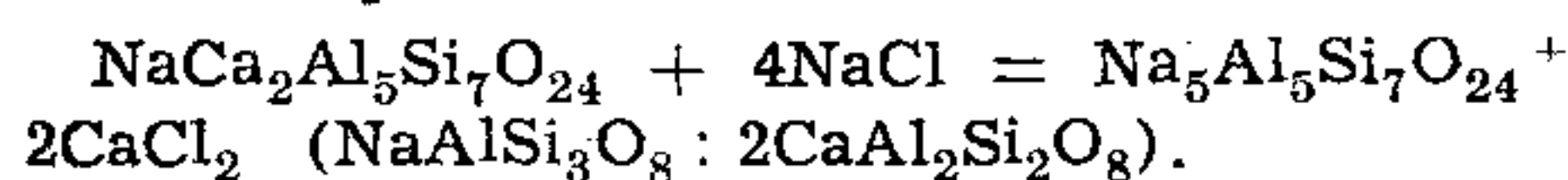


of Ca^{+2} or Na^+ occurs along with the addition or removal of Al^{+3} or Si^{+4} from the silicate framework. From the present set of experiments, it is evident that during the exchange of cations of unequal charges, the rearrangement of the silicate structure occurs in preference to the substitution or depletion of Al^{+3} or Si^{+4} . This is much more important in the case of 67% anorthite for which the reaction can be represented as:



This results in a nepheline with Si:Al ratio of 1.4 with a lower sodium content. It is known that the natural samples of nepheline contain excess of Si over the theoretical amount and the variation in Si:Al ratio varies up to 1.4 (B. Mason, 1966). Therefore, such cation exchange reactions may be occurring in nature as well, resulting in nepheline of higher silica content.

The resemblance of the infrared spectrum of the natural sample of nepheline and that from the solid-state reaction is very striking. This clearly indicates the formation of nepheline by such solid-state reactions in nature. The natural samples of nepheline contain more of K^+ and Ca^{+2} as impurity. The difference in the spectrum II and III ($650\text{--}750\text{ cm}^{-1}$ and

$400\text{--}450\text{ cm}^{-1}$ region) is due to the presence of these ions. On the other hand, nepheline obtained from the molten sodium chloride is completely devoid of K^+ and Ca^{+2} ions. Therefore, the I.R. absorption bands are clearly split. The same situation exists in the I.R. spectrum of albite and microcline (Lyons, 1967).

It may be expected that by heating plagioclase in molten potassium chloride gives rise to kaliophillite. But, it is observed that cation exchange takes place only to a certain extent (2.0% K_2O is taken by 67-anorthite, in molten KCl). This can be attributed to the difference in size of Ca^{+2} and K^+ . In the case of sodium chloride melt, initially complete substitution of Ca^{+2} by Na^+ in plagioclase takes place readily. Consequently, the silicate framework rearranges to accommodate another ion of Na^+ . The initial substitution of K^+ can be expected to be very slow. On the other hand, kaliophillite may be formed by heating celsian in molten KCl. Due to the non-availability of celsian, this has not been attempted.

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A NOTE ON AN INVARIANT OF SPHERICALLY SYMMETRIC SPACE-TIMES

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IN our recent investigations of spherically symmetric metrics, we have found that for the most general spherically symmetric metric

$$ds^2 = -A dr^2 - B d\Omega^2 + C dt^2 + 2D dr dt, \quad (1)$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2,$$

where A, B, C, D are functions of r and t alone, the quantity

$$I = \frac{f_1 f_4 - f_5^2}{f_2 f_3} \quad (2)$$

is an invariant under arbitrary non-singular transformations $(r, t) \rightarrow (\bar{r}, \bar{t})$ preserving spherical symmetry. Here f_1, f_2, f_3, f_4 and f_5 are the independent non-vanishing components

of the curvature tensor R_{hijk} in the notations of Takeno¹ as given by

$$f_1 = R_{1212} = \frac{R_{1313}}{\sin^2\theta}, \quad f_2 = R_{1414}, \quad f_3 = \frac{R_{2323}}{\sin^2\theta},$$

$$f_4 = R_{3434} = \frac{R_{3434}}{\sin^2\theta}, \quad f_5 = R_{1224} = \frac{R_{1334}}{\sin^2\theta}. \quad (3)$$

Since the variation of the gravitational field from point to point is mainly brought out by the curvature tensor, this invariant I is of interest. As we have found, this also seems to have a good geometrical interest.

Firstly, we have found that for all the solutions of the field equations

$$R_{ij} = 0 \quad (4)$$

with spherical symmetry, this invariant I has the value $\frac{1}{4}$. This covers the case of Schwarzschild's external metric. Further, we find that for all the spherically symmetric metrics of constant curvature which satisfy the relation

$$R_{hijk} = K_0 (g_{hj}g_{ik} - g_{ij}g_{hk}) \quad (5)$$

where K_0 is a constant, the invariant I has the value unity.

A necessary and sufficient condition for the space-time

$$ds^2 = -Adr^2 - r^2 d\Omega^2 + Cdt^2 \quad (6)$$

to be of class one, as obtained by Takeno,² can be expressed as

$$f_3 \neq 0, \quad f_2 f_3 = f_1 f_4 - f_5^2 \quad (7)$$

which when $f_2 \neq 0$ means $I = 1$. This is in conformity with the well-known result that every space of constant positive curvature is of class one.

In order to understand the significance of I , we have evaluated it for some well-known general relativistic metrics.

1. The Nordström metric :

$$ds^2 = - \left(1 - \frac{2m}{r} + \frac{4\pi e^2}{r^2} \right)^{-1} dr^2 - r^2 d\Omega^2 + \left(1 - \frac{2m}{r} + \frac{4\pi e^2}{r^2} \right) dt^2,$$

$$I = \frac{\frac{1}{4} \left(m - \frac{4\pi e^2}{r} \right)^2}{\left(m - \frac{2\pi e^2}{r} \right) \left(m - \frac{6\pi e^2}{r} \right)} \quad (8)$$

2. The Schwarzschild-deSitter metric :

$$ds^2 = - \left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3} \right)^{-1} dr^2 - r^2 d\Omega^2 + \left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3} \right) dt^2,$$

$$I = \frac{\left(\frac{\Lambda}{3} - \frac{m}{r^2} \right)^2}{\left(\frac{\Lambda}{3} + \frac{2m}{r^3} \right)^2} \quad (9)$$

3. Vaidya's metric³ for a radiating star :

$$ds^2 = - \left(1 - \frac{2m}{r} \right)^{-1} dr^2 - r^2 d\Omega^2 + \frac{\dot{m}^2}{f^2} \left(1 - \frac{2m}{r} \right) dt^2,$$

where

$$m = m(r, t), \quad f = m' \left(1 - \frac{2m}{r} \right), \quad I = \frac{1}{4}. \quad (10)$$

Although here $R_{ij} \neq 0$, the conditions prescribed on T_{ij} are such as to lead to $I = \frac{1}{4}$, even for

$$ds^2 = -Adr^2 - Bd\Omega^2 + Cdt^2.$$

4. The Schwarzschild internal metric :

$$ds^2 = - \left(1 - \frac{r^2}{R^2} \right)^{-1} dr^2 - r^2 d\Omega^2 + \left(A - B \sqrt{1 - \frac{r^2}{R^2}} \right)^2 dt^2, \quad I = 1. \quad (11)$$

5. The Robertson-Walker metric :

$$ds^2 = \frac{-R^2(t)}{\left(1 + \frac{kr^2}{4} \right)^2} (dr^2 + r^2 d\Omega^2) + dt^2, \quad I = 1. \quad (12)$$

6. The Hoyle-Narlikar⁴ metric :

$$ds^2 = - \left(1 - \frac{p}{r} \right)^{-2} dr^2 - r^2 d\Omega^2 + \left(1 - \frac{p}{r} \right)^2 dt^2$$

where $\mu = m_0 p / (r - p)$, $2\pi m_0 p - 1$, m_0 and p being constants,

$$I = \frac{m_0^2}{(4m_0^2 - \mu^2)}. \quad (13)$$

Thus the value of the invariant is finite for almost all the well-known spherically symmetric metrics. However, we may note here that it is possible at least in principle to construct metrics with one or more of f_1 , f_2 , f_3 , f_4 and f_5 being zero. In such cases the invariant is either zero, infinite or indeterminate.

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