SIMILAR SOLUTIONS OF COMPRESSIBLE LAMINAR BOUNDARY LAYER EQUATIONS FOR AN AXISYMMETRIC BODY

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ABSTRACT

Similar solutions of the steady laminar viscous compressible fluid at the stagnation point of an axisymmetric blunt body have been studied taking into account the variation of the product of density and viscosity across the boundary layers. The effect of this variation on skin friction and heat transfer is appreciable. The skin friction coefficient increases but the Nusselt number decreases as the total enthalpy at the wall increases.

1. INTRODUCTION

THE solutions of the steady laminar viscous compressible boundary layer equations in the stagnation region of the two-dimensional and axisymmetric bodies have been obtained by Cohen and Reshotko,[1] Levy,[2] and Li and Nagamatsu,[3] taking the product of density and viscosity as constant across the boundary layer. Kemp *et al.*[4] have considered the above problem in the presence of dissociation when the wall is highly cold.

In the present analysis, the solution of the same problem is obtained without imposing any restriction on the variation of the product of density and viscosity, the total enthalpy at the wall, and the Prandtl number. The velocity and the total enthalpy are obtained by numerically solving the two coupled non-linear ordinary differential equations. Further, the skin friction coefficient, the Nusselt number, and the various characteristics of the boundary layer are also obtained.

2. FORMULATION AND SOLUTION OF THE PROBLEM

We shall consider the supersonic viscous flow in the axisymmetric stagnation region of a blunt body. It is assumed that the flow is steady, laminar and axially symmetric. In addition, the fluid is perfect and the wall of the boundary is not very cold. With these assumptions, using the notations of Ref.,[5] the governing equations due to similarity requirements can be expressed as:

$$(Cf'')' + ff'' + \beta\left(\frac{\rho_e}{\rho} - f'^2\right) = 0 \qquad (1)$$

$$\left(\frac{C}{P}\,g'\right)' + fg' = \frac{u_e^2}{I_e}\left[\left(\frac{1}{P} - 1\right)Cf'f''\right] \qquad (2)$$

with the boundary conditions:

$$f(0) = f'(0) = 0;\quad f'(\infty) = 1$$
$$g(0) = g_w;\qquad g(\infty) = 1 \qquad (3)$$

where

$$\beta = \frac{2S}{u_e}\frac{du_e}{dS},\quad C = \frac{\rho\mu}{\rho_e\mu_e},$$

and prime denotes differentiation with respect to η.

At the stagnation point,

$$\beta = \tfrac{1}{2} \text{ and } \frac{u_e^2}{I_e} = 0.$$

The density and viscosity are given by:

$$\frac{\rho_e}{\rho} = \frac{T}{T_e} = g;\quad \frac{\mu}{\mu_e} = \left(\frac{T}{T_e}\right)^{\lambda};\quad C = g^{\lambda-1} \qquad (4)$$

where λ is constant and T is the absolute temperature.

The numerical solution of the coupled equations (1) and (2) with boundary conditions given by equation (3) was carried out on 803, Elliot Computer using Runge-Kutta-Gill method for $\beta = 0{\cdot}5$, $u_e^2/I_e = 0$, $P = 0{\cdot}72$, $\lambda = 0{\cdot}5$, $g_w = 0{\cdot}2$ and $0{\cdot}6$. The velocity and the total enthalpy profiles are given in Fig. 1 and C in Fig. 2. $f'(\eta)$ and $g(\eta)$ increase, but C decreases as g_w increases. If the present results are compared with the corresponding results of Ref.[1], it can be seen that f_w'' and g_w which determine the skin friction and heat transfer respectively are reduced due to the variation of $\rho\mu$ in the boundary layer.

3. SKIN FRICTION, HEAT TRANSFER AND OTHER BOUNDARY LAYER CHARACTERISTICS

The skin friction coefficient, C_f, at the stagnation point can be expressed as:

$$C_f\,(R_{ex})^{\frac{1}{2}} = \frac{2^{3/2} f_w''}{(g_w)^{\frac{1}{2}}} \qquad (5)$$

where

$$C_f = \frac{\left(\mu\frac{\partial u}{\partial y}\right)_w}{\frac{1}{2}\rho_e u_e^2},\quad R_{ex} = \frac{\left(\frac{du_e}{dx}\right)x^2}{\nu_e}$$

is the Reynolds number, du_e/dx is the velocity gradient at the stagnation point in the inviscid

TABLE I

Skin friction coefficient, nusselt number, and other boundary layer characteristics

$\beta = 0{\cdot}5,\ u_e^2/I_e = 0,\ P = 0{\cdot}72,\ \lambda = 0{\cdot}5$

g_w	f_w''	g_w'	$C_f\,(R_{ex})^{\frac{1}{2}}$	$N_u/(R_{ex})^{\frac{1}{2}}$	D	M	E	$\bar{E}$
0·2	0·3464	0·1853	2·1905	1·6377	0·0265	0·5071	0·7906	0·5707
0·6	0·6615	0·1519	2·4155	0·8951	0·4555	0·4173	0·6661	0·2716

flow, and ν_e is the kinematic viscosity at the edge of the boundary layer.

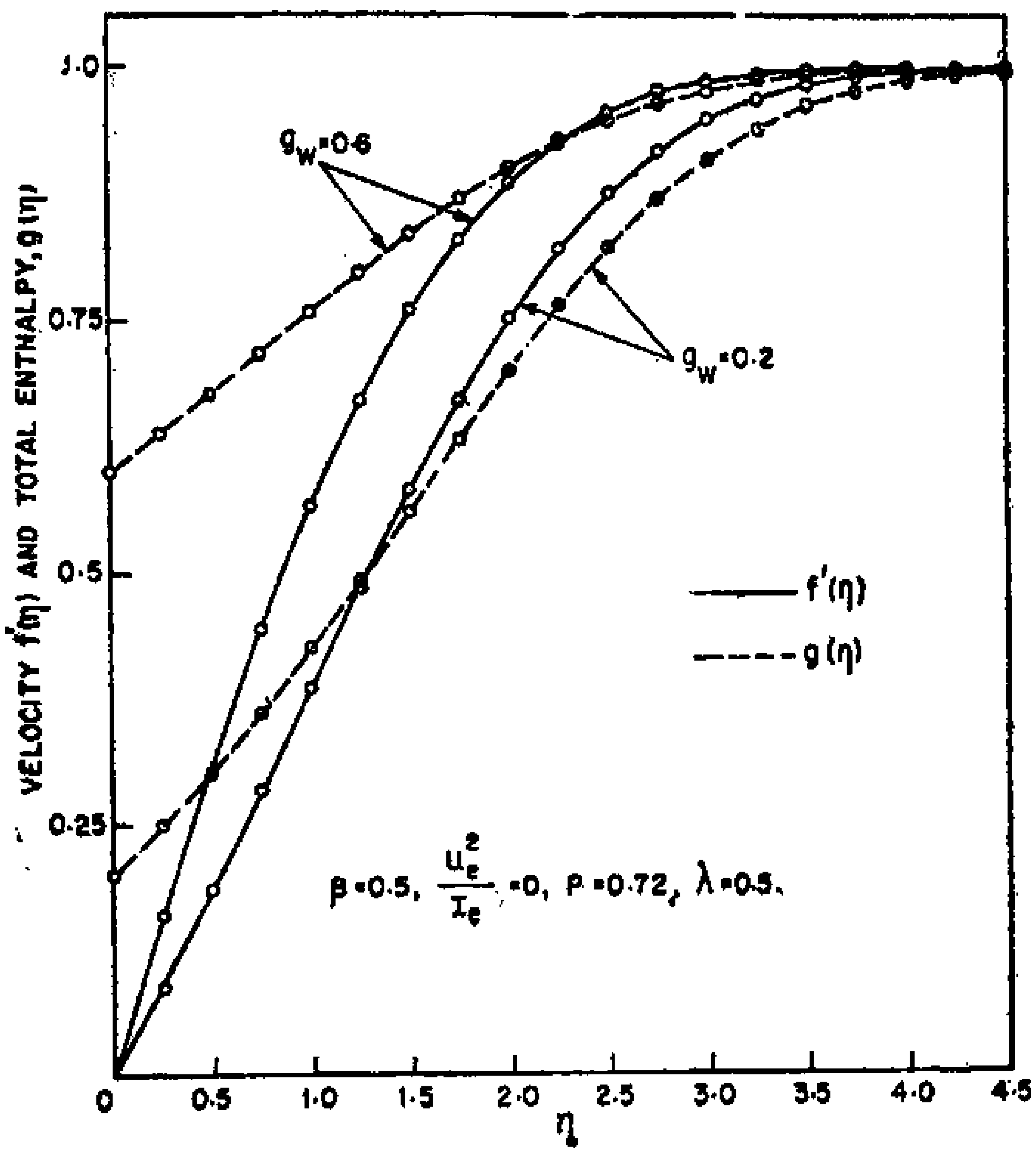

FIG. 1. Velocity and total enthalpy distributions.

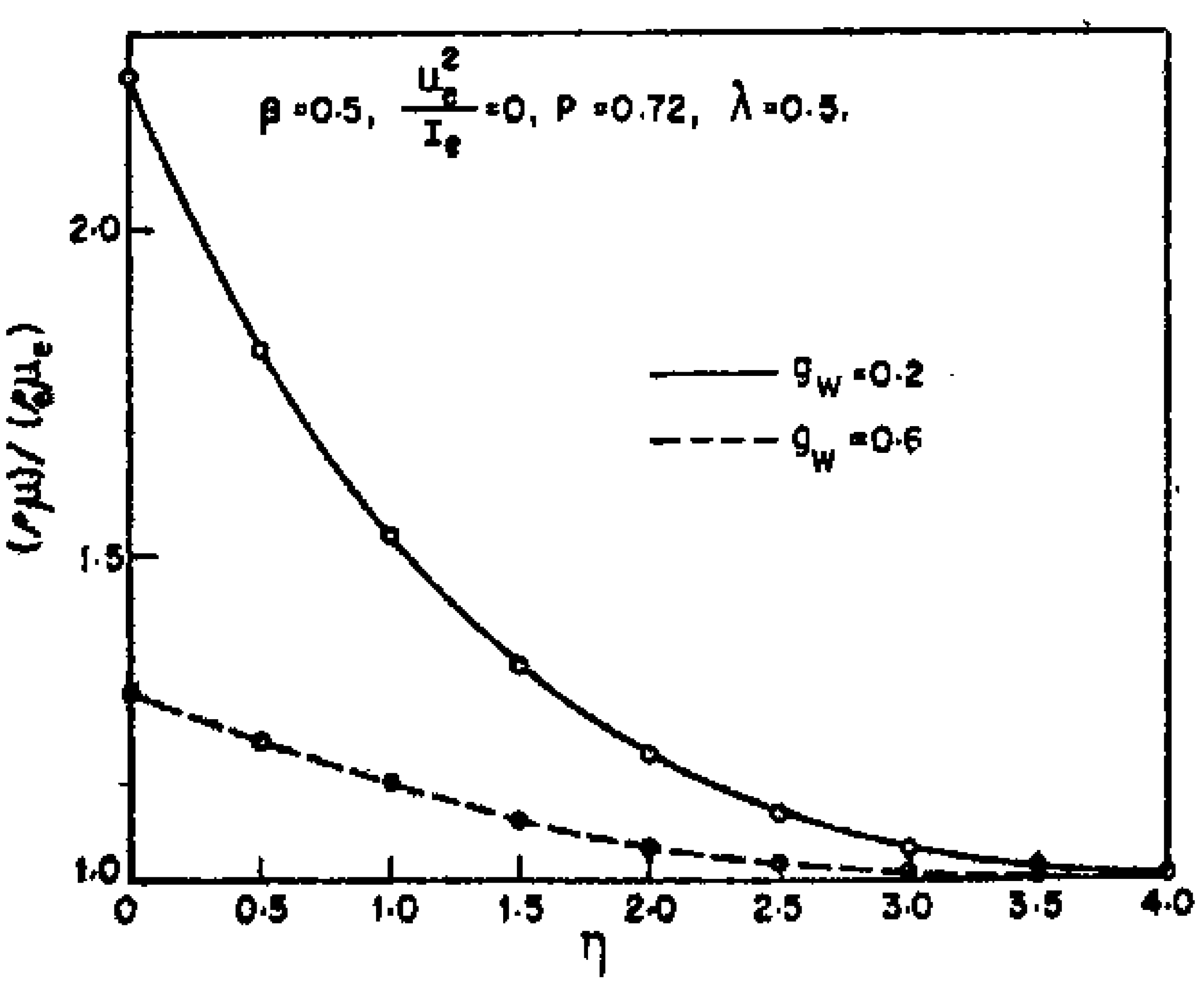

FIG. 2. Variation of $(\rho\mu)/(\rho_e\mu_e)$ with η.

Similarly, the Nusselt number, N_u, can be written as:

$$\frac{N_u}{(R_{ex})^{\frac{1}{2}}} = 2^{\frac{1}{2}}\,\frac{g_w'}{g_w\,(1 - g_w)} \tag{6}$$

where

$$N_u = \frac{\left(K\frac{\partial T}{\partial y}\right)_w C_p x}{K\,(I_e - I_w)}$$

K and C_p are conductivity and specific heat respectively and they are taken as constant across the boundary layer.

The dimensionless displacement, momentum, enthalpy-defect and energy thicknesses are expressed respectively as:

$$D = \int_0^\infty (g - f')\,d\eta\,;\quad M = \int_0^\infty f'\,(1 - f')\,d\eta \tag{7}$$

$$E = \int_0^\infty f'\,(1 - g)\,d\eta\,;\quad \bar{E} = \int_0^\infty f'\,(1 - f'^2)\,d\eta$$

The skin friction coefficient, the Nusselt number, the displacement, momentum, enthalpy-defect and energy thicknesses are given in Table I. The skin friction coefficient and the displacement thickness increase, but the Nusselt number, momentum enthalpy-defect and energy thicknesses decrease as the total enthalpy at the wall increases.

4. CONCLUSIONS

1. The Nusselt number, the displacement, and energy thicknesses are much more sensitive to the change in the total enthalpy at the wall as compared to skin friction coefficient, momentum and enthalpy-defect thicknesses.

2. The effect of the variation of the product of density and viscosity on the skin friction coefficient and the Nusselt number is quite appreciable.

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5. Dorrance, W. H., *Viscous Hypersonic Flow*, McGraw-Hill Book Company, Inc., 1962, p. 32.