

of the Dipterous family Chironomidæ. It has recently been found by Hungerford¹⁵ in the Notonectid, *Buenoa*.

The most important pigment of protein origin found in insects is melanin; it is commonly found in many groups. Of the pigments with purine bases, uric acid and

its derivatives have been shown by Hopkins²² to be the cause of white and yellow colour of wings of butterflies of the family Pieridæ.

²² Hopkins, Sir F. G., *Phil. Trans. Roy. Soc.*, 1896, B 186, (2), 661.

The Mathematical Theory of a New Relativity.*

BY SIR SHAH MUHAMMAD SULAIMAN—A CRITICAL REVIEW.

§ 1. In the two papers published in the *Proceedings of the U. P. Academy of Sciences*, the author claims to have given a modification of Newtonian kinematics and Newtonian dynamics which not only yields all the results deducible from relativity but disproves the assumptions of relativity by deriving results more in accord with observation. He further derives some equations which, superficially, at any rate, look like generalisations of relativistic equations and then deduces Newton's forms as first approximations and Einstein's as higher ones. The first article consisting of Chapters 1 and 2 is devoted mainly to the theory of gravitation and the second article consisting of Chapters 3, 4 and 5 deals with Cosmology and questions of special relativity.

The first of these articles was included by Shapley¹ as "one of the high lights of Astronomy during 1934" in his remarks at the annual dinner of the American Association of Variable Star Observers on October 20, 1934.

It is not clear from Shapley's speech whether such a reference was based on a critical study of the article in question or on a tacit assumption, at its face value, of the claims put forward by the author. Quite recently this article has been critically reviewed by D. R. Hamilton,² who, confining himself to Sulaiman's explanation of the advance of perihelion, comes to conclusions which suggest that Sulaiman's work is absurdly erroneous. On the mathematical side not much notice has been taken of the work, the *Zentralblatt für Math.*,³ satisfying itself with a bare mention of the article.

§ 2. Before undertaking a detailed review,

a few general observations might be made. In the first place, it must be remarked that for the author to call his theory a new relativity is to give a completely false impression of his own work. If anything at all, the main thesis of the work is purely anti-relativistic and is vehemently opposed to a principle of relativity in any form whatsoever. Further one is struck by the large preponderance of books on popular expositions of relativity in the references to literature given at the end of the articles and this perhaps gives a clue to the great aversion to relativity which is manifest in the author's work. For, as is well known, the champions of the Theory of Relativity too often delight to bring forward those results of the theory which appear to them to be specially fitted to shock the common sense of people who take statements too literally and relativity is not the only example of a physical theory which appears absurd when its logical consequences are pushed to their very limit. In the list of references placed at the end of the second article it is curious to find the book "*Mysterious Universe*" ascribed to Eddington.

There are some mis-statements of facts in the author's references to relativity the most serious of which are in connection with the observational verifications of the general Theory of Relativity. The author says, (p. 4, Ch. 1), "It is now established that the supposed verifications are not exact," but the references to literature in support of this statement do not refer to the best observational data which are universally accepted. For the advance in the longitude of perihelion of Mercury the observational value is given as 40".00 per century (the reference being to Eddington's *Mathematical Theory of Relativity*) whereas the best determinations are due to Chazy⁴ and give 43".5 as against the

* The Mathematical Theory of a New Relativity by Sir Shah Muhammad Sulaiman, *Proceedings of the U. P. Academy of Sciences*, 1934-35, Vol. IV. Part 2, pp. 1-36 and Vol. IV, Part 4, pp. 217-261.

¹ *Science*, 1934, 80, 439.

² *Science*, 1935, 81, 271-272.

³ *Zentralblatt für Math.*, 1935, 10, 88.

⁴ *Comptes Rendus*, 1926, 182, 1134.

theoretically predicted $42''.9$. In the case of the gravitational deflection of light the author refers on p. 25, Ch. 1, to values obtained at several eclipse expeditions but significantly omits to mention the most satisfactory data available at present, *viz.*, those of Campbell and Trumpler,⁵ who obtained the results $1''.72 \pm 0''.11$ and $1''.82 \pm 0''.15$ with two different sizes of cameras in the 1922 expedition of the Lick Observatory. As regards the gravitational shift of spectral lines a reference is made to the older work of St. John as quoted in Eddington's *Math. Theory* whereas in the cases of both the Sun and the dense companion to Sirius the agreement between the Theory of Relativity and observation is quite satisfactory as a result of the later work of St. John⁶ and of Adams.⁷ It appears therefore that the claims of Sulaiman's theory that it gives results more in accordance with observation than relativity are to be taken with some reservation. Other mis-statements of a minor nature are that relativistic invariance holds in vacuum only (p. 3, Ch. 1), that Einstein arbitrarily assumes $c+v=c$ and $c-v=c$ (p. 32, Ch. 2), that Milne's theory ignores gravitation and evades collisions (p. 224, Ch. 3) and that, in relativity, time is *wholly imaginary* and space *illusory* (p. 253, Ch. 5).

The mathematical part of the work is quite elementary and does not go beyond the solution of an ordinary differential equation of the second order. Looking from an æsthetic-mathematical point of view, one searches here in vain for such concepts like groups, tensors and generalised spaces characteristic of relativity or functional equations, sets of points, and Finsler spaces relevant to Milne's new relativity. On the other hand we have a set of drab differential equations as a series of approximations ninety per cent. of which is not relevant even to the author's own work. In dealing with the relativistic equation of a planetary orbit the exact solution of which can, as is well known, be expressed in terms of elliptic functions, the author claims to have devised a method superior to the methods of Forsyth, Morley and Pierpoint (p. 14, Ch. 1). A little scrutiny however shows that this superiority of method is achieved at the cost

of a little wrong mathematics (see Section 9, p. 14, Ch. 1).

To obtain a solution of

$$\frac{d^2u}{d\theta^2} + u - \frac{3\mu}{D^2}u^2 = \frac{\mu}{h^2}(1-2k\theta) \quad \dots (2.1)$$

the author considers the solution of

$$\frac{d^2u}{d\theta^2} + u - \frac{3\mu}{D^2}u^2 = 0 \quad \dots \dots (2.2)$$

which is correctly obtained as

$$u = \frac{D^2}{6\mu} + \wp \left\{ \frac{\sqrt{\mu}}{D\sqrt{2}}(\beta - \theta) \right\}.$$

It is then stated that the solution of (2.1) is given by

$$u = \frac{\mu}{h^2}(1-2k\theta) + \frac{D^2}{6\mu} + \wp \left\{ \frac{\sqrt{\mu}}{D\sqrt{2}}(\beta - \theta) \right\} \quad \dots (2.3)$$

presumably on the strength of the theorem that the general solution is the sum of the complementary function and a particular integral. It is, however, absurd to use this theorem here since it cannot apply to non-linear differential equations like (2.1) and, moreover, $\mu(1-2k\theta)/h^2$ is not a particular integral.

As examples of the author's attitude towards scientific investigation we might mention his views (1) that Nature's limits are not fixed by our capacity to observe them (p. 230, Ch. 4), (2) that relative velocity cannot mean relative velocity as actually observed and we cannot go by measurements only (p. 242, Ch. 5), and (3) that a certain concept in relativity is unacceptable because the concept is philosophically an impossible one (p. 226, Ch. 3).

Finally on a point relating to a question of priority, it is highly amusing to see the author refer to a paper by P. Jordan mentioning gravitational quanta and claim priority by pointing out that his own theory was published in 1933 and again in 1934. It might be pointed out that, if it be a question of the "gravitons" of the Sulaiman type subject to the impulsive pulls and pushes of Newtonian dynamics, a whole literature⁸ about them already exists. These "gravitons" have in fact a very close family resemblance to the "corpuscules ultra-mondains" of Le Sage,⁹ the "radiating

⁵ *Lick Observatory Bull.*, 1923, 11, 41 and 1928, 13, 130.

⁶ *Astrophysical Journ.*, 1928, 67, 195.

⁷ *Proc. Nat. Acad.*, 1925, 11, 382.

⁸ J. Zenneck, Article on "Gravitation" in the *Ency. Math. Wiss.*, Bd. V2, §§30-33, 57-63.

⁹ *Berlin Mém.*, 1782.

atomules" of O. Keller,¹⁰ and the "residual attraction" of Crehore.¹¹ If, on the other hand, it be a question of gravitational quanta the possibility of whose existence is a consequence of the complementarity of the wave and corpuscular aspects of modern quantum mechanics, it is needless to say that such a concept is now quite well known for a number of years and finds a place even in elementary books on wave mechanics.¹²

§ 3. GENERAL RELATIVITY. (a) *Advance of Perihelion*.—The two main ideas which the author uses for dealing with gravitational phenomena are the finiteness of the velocity of propagation of gravitation and the introduction of a correction to Newton's law for the case of moving bodies. Both these ideas have no novelty in them going back in fact to the work of Laplace¹³ and a series of later investigators.¹⁴ Laplace himself did not assume any variations of Newton's law for moving bodies but only the finiteness of the velocity of propagation. He was thus led to apply a sort of an *aberration principle*, but his results were completely against all observed values in planetary perturbations. Assuming D the velocity of propagation of gravitation equal to c the velocity of light, his theory did not correctly give the advance of Mercury's perihelion and, in addition, gave a secular variation of the mean longitude contrary to observation. An attempt to bring down this secular variation to the observed value necessitated the assumption that $D=500c$. It was later shown by Lehmann-Filhès¹⁵ that such an attempt in the case of the perturbation of the moon's longitude required the assumption for D a value nearly a million times c .

Coming now to theories which assume both the finiteness of D and a modification of Newton's law we have the theories of Weber, Riemann, Gauss, Neumann, Clausius, Anding and Gerber. On the unsatisfactory nature of the first five theories reference may be made to the sources¹⁶ mentioned above and we might confine our attention to

the last two, specially to Gerber's theory which bears a great resemblance to Sulaiman's work. Anding¹⁷ substituted for the Keplerian equations of motion the following equations

$$\frac{d^2x}{dt^2} + \frac{\mu x}{r^3} = \frac{\mu}{c} \cdot \frac{x}{r^3} \cdot \frac{dr}{dt};$$

$$\frac{d^2y}{dt^2} + \frac{\mu y}{r^3} = \frac{\mu}{c} \cdot \frac{y}{r^3} \cdot \frac{dr}{dt}$$

in order to explain the perihelion advance of Mercury, but these give rise, in addition, to a large perturbation in the eccentricity which is quite contrary to observation. Gerber¹⁸ started with an expression for the potential in the form

$$P = k^2 m_1 m_2 : r \left[1 - \frac{1}{D} \frac{dr}{dt} \right]^a$$

and determined the constant a in order that the perihelion advance thus given may equal the observational value which he took $41''.25$ per century (with $D=c$). He thus obtained two possible values of a , viz., $a_1=2$ and $a_2=-3$ and assumed the former value for his correction to Newton's law. It is remarkable that the other value $a_2=-3$ gives Sulaiman's law if we observe that it is derivable from the above potential, remembering that Sulaiman's correction factor does not depend on r . The criticisms levelled against Gerber's theory therefore apply to Sulaiman's theory equally well and reference in this connection might be made to the remarks of Seeliger,¹⁹ Lane²⁰ and Oppenheim.²¹ Any one who has worked in the perturbation theory of celestial mechanics knows quite well that modifications of the Newtonian law introduced to explain a certain anomaly give rise to unforeseen perturbations in other elements of the planetary orbit. This is exactly what happens with Gerber's theory which, like the theory of Anding, gives unwanted perturbations in the eccentricity or alternatively an assumption that D is nearly 10^4c . We should therefore expect similar absurdities to arise in Sulaiman's theory and this has been confirmed by Hamilton who has shown that this theory gives an yearly increase of eccentricity equal to 0.0026

¹⁰ *Comptes Rendus*, 1908, 147, 853-56.

¹¹ *Electrical World*, 1912, 59, 307-11.

¹² See for e.g., J. Frenkel, "Introduction to Wave Mechanics," who uses the word 'gravons'.

¹³ *Méc. cél.*, 4, Livre. X, Chap. 7.

¹⁴ See (8) above, §§ 20 and 21-24. Also S. Oppenheim, Article on "Kritik des Newtonschen Gravitationsgesetzes," *Ency. Math. Wiss.*, VI 2, 22, § 31, 152-58; also F. Tisserand, *Méc. cél.*, 4, Chapter 28.

¹⁵ *München, Ber.*, 1895, 25, 371.

¹⁶ See references (8) and (14) above.

¹⁷ *Astr. Nachr.*, 1924, 220, 353-60.

¹⁸ *Ann. d. Phys.*, 1917, 52, 415.

¹⁹ *Ibid.*, 53, 31 and 54, 38.

²⁰ *Ibid.*, 53, 214. Also Article on "Relativitätstheorie" by W. Pauli in *Ency. Math. Wiss.*, V, 19, § 58, 732.

²¹ *Ann. d. Phys.*, 53, 163.

which means that Mercury goes off in a parabolic orbit within about three centuries! Alternatively an attempt to bring down this perturbation in eccentricity to Newcomb's value of $(-4.3 \pm 2.5 \times 10^{-8})$ per year requires the assumption $D = 6 \times 10^4 c$!

It would not be out of place to mention here other theories relating to the advance of Mercury's perihelion. We have Asaph Hall's²² alteration of the law of Newtonian attraction from r^{-2} to $r^{-(2+\delta)}$ which gives the required perihelion advance if δ be put equal to 0.00000016 but, apart from its arbitrary nature, it gives a movement of 135" in the apse of the Moon which is negatived by observation. Again the assumption of the oblateness of the Sun²³ explains the perihelion advance but gives very large perturbations of the inclination of Mercury's orbit. Finally the zodiacal theory of light²⁴ also gives rise to unwanted perturbations. One is therefore led, almost by a process of exhaustion, to Einstein's theory which by its very nature does not give any perturbations.

(b) *Gravitational Deflection of Light*.—The author's derivation for the deflection of light of a value equal to 4/3 times the Einstein value can only be described as truly amazing! He states (Ch. 1, p. 4) that Gerber's equation does not yield the value for the deflection which is certainly true and the same should also be true of Sulaiman's equations if properly handled. His method consists in taking the equation

$$\frac{d^2 u}{d\theta^2} + u = \frac{\mu}{h^2} + \frac{3\mu}{D^2} u^2 \quad (D = c) \quad \dots (3.1)$$

as the differential equation of the path of a light particle in a gravitational field. This equation is the relativity equation for the path of a material particle and in Sulaiman's theory it is only an approximate equation (*viz.*, the third approximation) there being approximations of four higher orders. The general equations of Sulaiman's theory are obtained by treating v (velocity of the particle) as small compared with D and consequently neglecting higher powers of $\left(\frac{1}{D} \frac{rd\theta}{dt}\right)$ than the first. It is therefore obvious that when one is dealing with the

motion of a particle whose velocity is D itself (*i.e.*, a light particle, since D is taken equal to c), it is wrong to start with an approximate equation. The author himself sees the need of this when he is dealing with the motion of an electron in connection with his explanation of the fine structure of spectral lines (Ch. 5, p. 258). The straightforward thing to do in this case is to write down the equations of motion *ab initio* using the relation $v = D$ and when this is done with the equations in Sec. 5, Ch. 1, p. 9, we easily obtain the equation to the path of the light particle

$$\frac{d^2 u}{d\theta^2} + u = 0 \quad \dots \quad \dots (3.2)$$

that is, a straight line showing that there is no deflection for a light particle in a gravitational field! This could also be *qualitatively* verified from Sulaiman's law of attraction, *viz.*,

$$-\frac{\mu}{r^2} (1 - v/D)^3 \text{ by putting } v = D$$

and is in consonance with what Gerber's equations can give for the deflection. Thus while on the old pure Newtonian theory we could deduce a deflection at least equal to half the Einstein value, this generalisation by Sulaiman yields no deflection at all!

Assuming for a moment that (3.1) correctly gives the path of a light particle, the author still fails to justify his final result, for when he states (Ch. 2, p. 25) that $r \frac{d\theta}{dt}$ can never exceed the *tangential velocity* c , he assumes unconsciously that the tangential velocity is constant, but this cannot certainly be true. Even in the derivation of the deflection on Newtonian mechanics c is assumed to be the velocity at infinity of the light particle. The assumption of a constant tangential velocity is equivalent to taking the central orbit as circular and it becomes meaningless to talk of the deflection as the angle between the asymptotes of the orbit. Such circular orbits²⁵ are also possible for light particles according to general relativity but they are excluded²⁶ for purposes of obtaining the deflection.

There is a third mistake in this derivation of the deflection. By showing that the least value of the expression on the right hand side of (3.1) is $\frac{4\mu}{D^2} u^2$ ($D = c$) the conclusion

²² *Astr. Journal*, 14, 45.

²³ See A. C. D. Crommelin, *Nature*, 1920-21, 106, 788.

²⁴ See H. Jeffrey's *M.N.R.A.S.*, 80, 138.

²⁵ D. Hilbert, *Gott. Nachr.*, 1917, 73-75.

²⁶ Laue, "Relativitätstheorie," Bd. 2, §24, 224-27.

is drawn that the deflection is exactly $4/3$ times the Einstein value, but the correct conclusion to draw is that it is *at least* $4/3$ that value. In a case like this where observational verification is essential one would naturally enquire what would be the *maximum* deflection possible, but the work is silent on this point!

(c) *Shift of Spectral Lines*.—No remarks appear to be necessary in this case for, according to the author's own showing, the "corrections provided by the 'New Relativity' are not appreciably large and the value of the ratio is the same as Einstein's" for spectral lines in the solar spectrum. For planetary spectra he remarks, "Unfortunately the ratios for the planets are too small as compared to that of the Sun and the more accurate formulæ cannot give any better results at present."

Having examined the achievements of this 'New Relativity' in the three crucial tests, we can well conclude by saying that no one would seriously think of adopting it as an alternative to the general theory of relativity for the explanation of gravitational phenomena.

§ 4. SPECIAL RELATIVITY. (a) *Relative Velocity*.—The author enunciates his 'first universal principle' as follows (Ch.5, p. 247):

The relative velocity v between two bodies moving with velocities u and v' , measured by employing a messenger travelling with a velocity D in a to-and-fro journey, is given by the formula

$$\frac{v}{v'-u} = \frac{D(D+v'-u)}{(D+v')(D-u)} \quad \dots (4.1)$$

and claims that this formula is more general than the corresponding formula of special relativity

$$v = \frac{v'-u}{1 - \frac{v'u}{c^2}} \quad \dots \quad \dots (4.2)$$

by showing that (4.2) is an approximation obtained from (4.1) by neglecting terms like $(v'^2u - v'u^2)/D^3$, etc.

It is difficult to see how a correspondence could be established between (4.1) and (4.2) if it be noted that in the derivation of (4.1) the notions of absolute space and absolute time are retained while these are foreign to relativity. (4.1) applies even to the case where u and v' are velocities relative to an observer who is at rest in his own system, while in such a case both classical and relativistic kinematics give $v'-u$ for

the relative velocity. As an example of confused thinking it is hard to find anywhere in relativistic literature a parallel to the author's derivation of equation (4.1). An absolute distance between two moving points is assumed as r independent of all measurement and on this are made to depend a real and an apparent distance. This leads on to the notions of absolutely real relative velocities, apparently real relative velocities and really apparent relative velocities! Let us however assume that (4.1) actually corresponds to the relativistic equation (4.2). We can then deduce some absurd consequences.

(i) Putting $v'=D=c$ in (4.1) we deduce $v=c-u/2$ while classical kinematics gives $v=c-u$ and relativity gives $v=c$. We can therefore describe the Sulaiman kinematics as a sort of a hybrid form or as a sort of a semi-emission theory similar to the emission theory of Ritz.²⁷ Sulaiman's kinematics founders therefore on the rock of de Sitter's binary star test²⁸ as all other emission theories do.

(ii) Formula (4.1) looks superficially like a generalisation of Einstein's formula for addition of velocities and the author applies it to derive Fresnel's formula for the dragging coefficient and claims to have obtained a better approximation than the usual expression

$$c'_1 = c_1 - u \left(1 - \frac{1}{\mu^2} \right) \quad \dots \quad \dots (4.3)$$

where c'_1 is the velocity of light in moving water, $c_1=c/\mu$ the velocity in stationary water and u the velocity of the water. The corresponding expression deduced from (4.1) reads

$$c'_1 = \frac{(c_1 - u) \left(1 + \frac{c_1}{c} - \frac{u}{c} \right)}{\left(1 + \frac{c_1}{c} \right) \left(1 - \frac{u}{c} \right)} \quad \dots (4.4)$$

The actual reduction of (4.4) to an equation of the same form as (4.3) (which the author has not carried out) gives after a slight simplification

$$c'_1 \approx c_1 - u \left[1 - \frac{1}{\mu(\mu+1)} \right] \quad \dots (4.5)$$

In the case of water (4.3) gives for the second term on its right hand side the value of $0.44 u$ which has been well confirmed by the

²⁷ *Ann. de. Chim. et Phys.*, 1908, 13, 145.

²⁸ *Proc. Amsterdam Acad.*, 1913, 15, 1297 and 1913, 16, 395.

experiments of Fizeau²⁹ and the later very accurate researches of Zeeman.³⁰ On the other hand formula (4.5) gives in the same case the absurdly high value of $0.68u$ which is contrary to all observational results.

We can therefore safely dismiss as idle speculation all the results derived on the basis of this 'universal principle' and in particular the ridiculous analogues to Lorentz transformations on pp. 247-48, Ch. 5, between two moving systems which have a common time $t=t'$!

(b) *The Principle of Aberration* (Ch. 5, p. 251).—This principle which follows as a consequence of the finiteness of the velocity of propagation of a force has been mentioned already in connection with Laplace's theory of gravitation and is made extensive use of by the author who takes it as his second universal principle. According to him it is merely the necessary result of the compounding of two dynamical velocities, but it is difficult to see any justification for the reduction in the intensity of force along its apparent direction. It really makes no sense to say that when the velocity of flow is D , the effective component of force observed along the apparent direction is $D \cos \alpha$. There is an utter confusion here between velocity and force. This confusion is also responsible for the meaningless phrase "*the velocity of light on a body moving with velocity v* ". The claim of universality of application of this principle is belied by assuming that in the case of light the velocity is reduced while in other cases the intensity of force is changed (for example H in the explanation of Bucherer's experiment). There is yet another inconsistency in the application of this aberration principle to the case of "gravitons". The universality claimed would certainly require the modification in Newton's law of attraction to be $-\frac{\mu}{r^2} : \left(1 + \frac{v^2}{D^2}\right)^{\frac{1}{2}}$ leading on to Gerber's equations, but the author uses, instead, the factor $\left(1 - \frac{v}{D}\right)^3$ deduced from special consideration of 'graviton' pulls.

In his explanation of Minkowski's equation and of the possibility of velocities exceeding that of light the equations made use of are

$$\left. \begin{aligned} c_1 &= c \cos \alpha \\ \tan \alpha &= v/c \end{aligned} \right\} \dots \dots (4.6)$$

where c_1 the apparent velocity of light has

its direction perpendicular to that of v and α is the angle of aberration. If, as the author states, the principle of aberration is merely the result of compounding dynamical velocities it is impossible to see how both the equations in (4.6) could be simultaneously true. It is on the basis of such 'flawless' mathematics that the possibility of velocities up to ∞ is deduced and one might well suggest to the author the derivation of his first universal principle when one of the bodies is moving with such a velocity, for example a velocity greater than that of the messenger employed.

(c) *Michelson and Morley Experiment*.—The explanation offered is briefly as follows:—
Time of longitudinal journey

$$\begin{aligned} &= \frac{l}{c+v} + \frac{l}{c-v} \\ &= \frac{2lc}{c^2-v^2} \dots \dots \dots (4.7) \end{aligned}$$

Time of transverse journey

$$= \frac{2l}{\sqrt{c_1^2-v^2}} \dots \dots \dots (4.8)$$

where c_1 is the same quantity as in (4.6).

Hence the difference in times

$$\begin{aligned} &= \frac{2lc}{c^2-v^2} - \frac{2l}{\sqrt{c_1^2-v^2}} \\ &= \frac{2lc}{c^2-v^2} - \frac{2l}{\sqrt{c^2-2v^2}} \text{ using (4.6)} \\ &= -\frac{l}{c} \left(\frac{v}{c}\right)^4 \text{ nearly,} \end{aligned}$$

which cannot be detected by experiment.

By using the author's own 'universal principles' it can easily be shown that this explanation is untenable. For, according to the first universal principle of relative velocities, (4.7) should be replaced by

$$\frac{l}{c-v/2} + \frac{l}{c+v/2} = \frac{2lc}{c^2-v^2/4} \dots (4.9)$$

Again (4.8) is obtained by a wrong application of the second universal principle according to which the effect of a finite velocity of flow is the same as if the body were stationary and the direction of flow were shifted forward by an angle α and the velocity changed from c to c_1 . It is therefore wrong to again compound c_1 with v and hence (4.8) should be replaced by

$$\frac{2l}{c_1} = \frac{2l}{\sqrt{c^2-v^2}} \dots \dots (4.10)$$

Hence the difference in times

$$= \frac{2lc}{c^2-v^2/4} - \frac{2l}{\sqrt{c^2-v^2}} = -\frac{l}{c} \cdot \frac{v^2}{2c^2}$$

²⁹ *Comptes Rendus*, 1851, 33, 349.

³⁰ *Amsterdam Proceedings*, 1914, 23, 245; 1915, 24, 18.

which can certainly be measured but is contradicted by the null result of the Michelson-Morley experiment.

(d) *Fine Structure of Spectral Lines*.—The author has not derived the formula for fine structure on the basis of his own theory but only talked about in a certain hazy way which tends to suggest that he is unaware of the methods of even the old Bohr-quantum theory. One would naturally start with the proper expression for the Hamiltonian, then set up the Hamilton-Jacobi equation and introduce the angle and action variables, the quantum conditions being derived by equating the non-degenerate action variables to integral multiples of Planck's constant. Nothing of the sort is done here and it is suggested that the same equations of Newtonian form as used for planetary orbits should be employed with the retention of the term $\left(\frac{1}{D} \frac{rd\theta}{dt}\right)^2$ and the reader is left to proceed as best as he can with the help of the third universal principle which deals with the force acting on a spinning spherical shell. It is really a complete mystery what this universal principle has got to do with the motion of an electron in a central field of force and where this spinning spherical shell comes into the picture. In the absence of any quantitative results, it is impossible to attach any weight to the author's explanations.

§ 5. COSMOLOGY. We may well spare the author the joy of his profound cosmological speculations and trenchant criticism of other cosmological theories and proceed to examine those positive results of his theory which are expressed in a mathematical form. The only such result is the derivation of Hubble's famous velocity-distance law on the basis of the author's emission theory of matter and the conclusion therefrom that not only velocities of recession of nebulae but also velocities of approach are possible. The fundamental equation is

$$\frac{d^2R}{dt^2} = \gamma \frac{dR}{dt} \dots \dots \dots (5.1)$$

where R may be measured in any direction and from any origin and $\gamma = \frac{n\mu}{3}$, n being

the number of gravitons emitted from unit mass per unit time and μ the mass of each graviton. From (5.1) we obtain by integration

$$\frac{dR}{dt} = \gamma R + A_0 \dots \dots \dots (5.2)$$

which is the expression for the velocity-

distance law. From (5.1) also follows a cosmological principle that the relation of acceleration and velocity presents the same picture to all observers. It might be observed, in passing, that this cosmological principle can be considered as a particular form of Milne's principle of equivalent observers. On the basis of (5.2) it is claimed that velocities of recession and approach are both possible.

It might be remarked in the first place that the deep-lying velocity-distance proportionality could be very simply deduced³¹ from pure classical kinematic considerations only on the basis of a cosmological principle of equivalent observers of the type derived by the author himself. Even in relativistic cosmology it is a simple deduction³² from the form of the metric assumed in non-static models of the Universe. It appears therefore that it is quite redundant for the purposes of deriving Hubble's law to invoke the aid of an emission theory of matter which calls to aid supernatural agencies for the production of gravitons by the explosion of a sub-atomic shell.

It is again wrong to say that (5.2) explains both recession and approach. For, since R can be measured in any direction and from any origin, a simple change of origin reduces (5.2) to the equivalent form

$$\frac{dR}{dt} = \gamma R \dots \dots \dots (5.3)$$

and γ , by its very definition, is a *positive* quantity unless one were to indulge in Schuster's³³ "holiday dreams" of negative masses. Thus (5.3) shows that $\frac{dR}{dt}$

has always the same sign as R , i.e., the velocity is one of recession. All the enchanting speculation about approaching and receding nebulae and a stable Universe are therefore seen to be without a foundation. Finally the author's criticism of relativistic cosmology loses much of its force if it be observed that he confines himself to the de Sitter static model whereas the trend of modern work³⁴ is in the direction of considering non-static models as better suited to explain observed facts.

B. S. M.

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³¹ E. A. Milne, *Relativity, Gravitation and World Structure* 1935, §§ 71-72, 73-74.

³² R. C. Tolman, *Relativity, Thermodynamics and Cosmology*, 456.

³³ *Nature*, 1898, 58, 367 and 618.

³⁴ Tolman, *ibid.*, Chap. X, Part IV, 443.