Research Notes.

Theory of Meromorphic Functions.

LARS AHLFORS ("über eriue Methode in der Theorie der Meromorphen funktionen," *Com. Phy.-Math.*, T. **8**, Nr. 10) has contributed a valuable paper on the introduction to the theory of meromorphic functions. After the fundamental contribution of Rolf Nevanlinna characterised by his first two fundamental theorems, the introduction was simplified in two directions. One was due to the Japanese workers headed by Shimuzu and the other was due to the Finnish school headed by Selberg and Frithiof Nevanlinna. Combining and crystallising all these methods the author has proved the chief properties of the characteristic function $T(r)$, and has deduced the second fundamental theorem of Nevanlinna in 14 pages. The ideas underlying the contribution are these :—First of all $m(r, a)$ is defined to be

$$\frac{1}{2\pi}\int_0^{2\pi}\log\frac{1}{\{f, a\}}\,d\theta$$

where $f(z)$ is the meromorphic function and (a, b) denotes the straight-line distance between the stereographic projections of a and b on a sphere of diameter unity touching the plane at the origin. (This is essentially the same as the earlier definition of Nevanlinna.) $N(r, a)$ is defined to be the same. Next by a simple differentiation and applying the principle of argument the independence of $T(r. a)$ w.r.t. a is deduced. There is no need to apply the Poisson-Jensen formula at all. Next taking $\rho(a)$ to be any positive function such that $\Phi\ \rho(a)\ d\omega(a)$ taken over the surface of the sphere $= 1$, he has shown in a single step that if

$$m_\rho(r) = \oint m(r, a)\ \rho(a) d\omega(a) \text{ and } N_\rho(r) = \oint N(r, a)\ \rho(a) d\omega(a).$$

then $T(r) = m_\rho(r) + N_\rho(r)$ also. From this he has easily deduced the convexity of $T(r)$ w.r.t. $\log r$. This is extremely simple when we remember the fact that this is proved by Nevanlinna by making use of the properties of the Green's function corresponding to a ring-region. The second fundamental theorem of Nevanlinna is obtained as a corollary of this result by taking a suitable density function $\rho(a)$; *viz.*,

$$\text{Log } \rho(f) = c + 2\sum_1^q \text{Log}\frac{1}{\{f, a_\nu\}} - \alpha \log\left[\sum_1^q \log\frac{1}{\{f, a_\nu\}}\right]$$

where c is a normalising constant and α is any number > 1. Some words are to be said with respect to the form of the theorem we obtain when we consider functions meromorphic in the unit circle. We get

$$\sum_1^q m(r, a_\nu) < 2T(r) - N_1(r) + \log T(r) + k' \log\frac{1}{1-r} + 0\,(1)$$

except in a set of intervals I_t such that $\Sigma\int_{I_t}\frac{dr}{(1-r)^{1+\epsilon}}$ is finite, k' being a constant which can be made as near to unity as we like. This is much sharper than the corresponding theorem by Nevanlinna and others and in fact as sharp as the result obtained by F. Nevanlinna by the use of suitable modular functions.

The second fundamental theorem mentioned here is not the *proper*—Nevanlinna by applying his *proper* second fundamental theorem in connection with his proof of Picard-Borel theorem. The proper theorem itself is applied by Nevanlinna to deduce further theorems, such as the theorems of unicity, etc. Probably these theorems can all be deduced directly by Ahlfors's method, but the proper theorem itself can be proved by Ahlfors's method as follows. Take

$$\rho(f) = k\frac{1}{\{f, \infty\}^2\{f, 0\}^2}\left[-\log\{f, \infty\}\{f, 0\}\right]^{-\alpha}$$

where $\alpha > 1$ and k is the normalisation constant. After some further analysis it can be proved that

$$m_r\left[\frac{f'}{f}, \infty\right] \leqslant \tfrac{1}{2}\log[\lambda(r)] + \alpha\log T(r) + 0\,(1)$$

where $\log\lambda(r) = 0[\log r + \log T(r)]$ except in the exceptional intervals. It should also be mentioned that this is the sharpest form in which the theorem is put. In fact if we work out the values of the actual constants we get a much sharper form than that obtained recently by Shimuzu and others.

K. V. I.

Uniform Distribution of Points.

KOKSMA ("Ein mengen theoretischen Satz uber die Gleich-verteilung modulo Eins.," *Comp. Math.*, **2**, Fasc. 2, pp. 250–258) has

recently contributed an interesting theorem concerning the uniform distribution of points $\{\theta^n\} = \theta^n - [\theta^n]$ (*i.e.*, the fractional part of θ^n.) He has proved that the distribution of such points is uniform in (0, 1) for almost all $\theta > 1$ (*i.e.*, excluding a set of points of measure zero utmost.) The most important theorem he proves in this connection is the following:—Let α and β be real numbers $\alpha > \beta$. Let $f(x, \theta)$ be a real continuous function of θ in $\alpha \leqslant \theta \leqslant \beta$ for every +ve integral value of x. For every $x_1 \neq x_2$, let $\phi(x_1, x_2, \theta) = f(x_1, \theta) - f(x_2, \theta)$ be a continuously differentiable function of θ (*i.e.*, its differential derivate ϕ_θ' is monotomic and $\neq 0$. Let

$$A_N = \frac{1}{N^2} \sum_{x_1=2}^{N} \sum_{x_2=1}^{x_1=1} \text{Max} \left[\frac{1}{|\phi'\theta\,(x_1, x_2, \alpha)|} \frac{1}{|\phi'\theta\,(x_1, x_2, \beta)|} \right]$$

and N_v be a sequence such that ΣA_{N_v} converges and $\frac{N_{v+1}}{N_v} \to 1$. Then $\{f(n, \theta)\}$ is uniformly distributed for almost all θ. The proof of this theorem is established by utilising Weyl's criterion, *viz.*,

$$\frac{1}{N} \sum_{1}^{N} e^{2\pi i f(n, \theta)} \to 0 \text{ with } N \to \infty.$$

From this the result follows as a particular case.

K. V. I.

Creation of the Radio-Elements belonging to a New Radioactive Family.

So far there were only three radioactive families known namely (1) the thorium series with atomic weights of the form $4n$ where n is an integer, (2) the radium series with mass numbers expressed by $W = 4n + 2$ and (3) the actinium series with $W = 4n + 3$. A series of the form $W = 4n + 1$ must consist of unstable elements since they do not occur normally in nature. Now I. Curie, H. von Alban Jr. and P. Preiswerk (*Journ. de Physique*, 1935, **6**, 361) have artificially produced some elements which they have definitely proved to belong to this family. Since Fermi has shown that slow neutrons have a large probability of being captured by a nucleus resulting in the formation of an isotope having the mass number greater by one, they bombarded Thorium with slow neutrons. The new element formed was expected to be β-active and hence to prove its existence, the Thorium had to be carefully purified to remove the radiothorium associated with it. After addition of Ba and Pb and Bi and precipitation with H_2SO_4 and H_2S respectively, La was added and the Thorium precipitated with H_2O_2. The hydroxide formed was converted into nitrate or chloride and dried before irradiation. The Thorium was placed in a bakelite case with a cellophane covering since such a case was found to be free from activity when irradiated. The β-activity was examined by means of a Geiger-Müller counter of thin aluminium 0·2 mm. thick. The activity curves were analysed and showed products of 1 min., 25 min. and 3·5 hrs. periods. These were chemically separated and then the repetition of the analysis showed two new products of periods 2·5 min. (associated with the 25 min. product) and 12 min. When the irradiated thorium was precipitated with H_2O_2, the 25 min. product came down with it showing that it is an isotope of Thorium. Addition of paraffin with consequent slowing down of the neutrons increased the yield of this product thus confirming the above view.

The 3·5 hrs. product accompanied La in the chemical separation thus showing itself to be an isotope of actinium. The 2·5 min. product was shown to arise from a Thorium isotope and to be an isotope of protactinium and hence the conclusion was drawn that the 2·5 min. product was produced from the 25 min. product. The several results could be explained on the basis of the following equations:

$$^{232}_{90}\text{Th} + {}^{1}_{0}n = {}^{233}_{90}\text{R}\ (25\ \text{min.})$$

$$^{233}_{90}\text{R}\ (25\ \text{min.}) \xrightarrow{\beta} {}^{233}_{91}\text{R}\ (2{\cdot}5\ \text{min.})$$

The production of the 1 min. product which was shown to be an isotope of radium was explained as follows:

$$^{233}_{90}\text{Th} + {}^{1}_{0}n = {}^{229}_{88}\text{R}\ (1\ \text{min.}) + {}^{4}_{2}\text{He}$$

$$^{229}_{88}\text{R}\ (1\ \text{min.}) \xrightarrow{\beta} {}^{229}_{89}\text{R}\ (3{\cdot}5\ \text{hrs})\ \text{(isotope of actinium)}$$

The 12 min. product probably results from the process:

$$^{232}_{90}\text{Th} + {}^{1}_{0}n \to {}^{232}_{89}\text{R} + {}^{1}_{1}\text{H}.$$

Thus the new radioactive family is probably developed thus:

$$^{237}_{92}\text{R} \xrightarrow{\alpha} {}^{233}_{90}\text{R}\ (25\ \text{min.}) \xrightarrow{\beta} {}^{233}_{91}\text{R}\ (2{\cdot}5\ \text{min.}) \xrightarrow{\beta} {}^{233}_{92}\text{R} \xrightarrow{\alpha} \text{etc.}$$

A search for the origin of this family $^{237}_{92}\text{R}$ in Uranium minerals was however without result.

T. S. S.

Electrets.

M. EGUCHI performed a remarkable experiment in 1925. A suitable wax was poured in the molten state into a condenser, a voltage was applied, and the wax allowed to solidify under the electric stress. The result was a permanently electrified body, an "electret," which retained its charge constant over a period of more than a year. In a recent paper (*Phil. Mag.*, 1935, **20**, 929) A. Gemant has carefully studied these highly interesting electrical analogues of permanent magnets, and finds that there are two possible kinds of charges, one having the opposite sign to the adjacent polarising electrode, of short duration, and caused by ionic space charges. The other has the same sign as the adjacent polarising electrode. It is caused by orientation of dipole molecules leading to an oriented crystallisation and accompanied under circumstances by a secondary piezo electric effect. This charge is the steady one, and is of great interest. Technical applications for such electrets can be found in the construction of electrometers and in a kind of electrostatic microphone.

M. A. G. RAU.

The Hammersten Effects.

J. W. McBAIN (*J. Am. Chem. Soc.*, 1935, **57**, 1916) has cleared up much of the confusion that has arisen by calling different Phenomenon "Hammersten Effect". Linderstörm-Lang conferred the name to the fact that the osmotic coefficient of the thymonucleates depends upon the size of the kation. Pauli and Valko and later others, included under this term the fact that in certain colloidal electrolytes the osmotic coefficient (obtained from freezing point lowering) is far less than that necessary to account for the hydrogen ion alone as measured by the electromotive force. McBain classes the former as the genuine Hammersten Effect and has put forth a new explanation of the same based on steric hindrance to close packing The second phenomenon (the "Hammersten Effect" of Pauli) appears to be more fundamental in view of the considerations of McBain. The divergence between the freezing point and the electromotive force data finds a rational explanation on the basis that the activity coefficient of the hydrogen ion is about half of that of the chloride ion in the concentrated solutions of hydrochloric acid. This marks one of the very first steps in getting at the individual activity coefficients of the ions without making any arbitrary assumptions.

K. S. G. D.

The Use of Ortho-para Hydrogen Conversion in the Detection of Free Radicals.

THE fact that paramagnetic substances can catalyse the ortho-para conversion of hydrogen has been used for the first time in the detection of free radicals by W. West (*J. Am. Chem. Soc.*, 1935, **57**, 1931). He finds definite indications of the production of paramagnetic free radicals on illumination of methyl iodide and acetone. On the other hand, propionyl iodide does not seem to give rise to free radicals under similar conditions. This is pointed out as a strong evidence in support of Norrish's hypothesis of a difference in mechanism in the photodissociation of aliphatic aldehydes and ketones.

K. S. G. D.

Deuterium as an Indicator in the Study of Intermediary Metabolism.

IN a series of four papers published in the *Journal of Biological Chemistry* (1935, **111**, 163–192), Rudolf Schœnheiner and D. Rittenberg have published the results of their work on intermediary metabolism, employing a very elegant and novel technique. The difficulty of following the course of transportation of normal physiological substances in the body and their conversion into other substances has long been realised and very often laborious experiments have yielded uncertain results. While the use of synthetic derivatives for following the metabolism of physiological substances has yielded interesting results, the method is open to the objection that such derivatives are not natural, and it is open to doubt whether they successfully imitate the physiological substances whose metabolism is under investigation. By replacing one or more of the elements in the molecule of the substance, by their respective isotopes, it is reasonable to expect that the chemical properties of the substance will not be greatly altered. Schœnheiner and Rittenberg have employed this method and by replacing the hydrogen in the organic compound by its heavy isotope, deuterium, which is capable of being detected with a precision of 0·001 per cent. they are enabled to follow the fate of such substances after administration to animals. The employment

of deuterium as an indicator opens out a new chapter in the study of metabolism.

For a successful working of the method, the deuterium should be placed in the molecule in such a position that it is not interchangeable with the hydrogen of water. It was, therefore, necessary to introduce deuterium into carbon groupings by catalytic hydrogenation of unsaturated compounds. The application of the method is, therefore, restricted to the substances containing *stable* hydrogen. "A substance like oxalic acid cannot be investigated as the two hydrogens in it are labile." The method gives information only on processes in which the hydrogen (deuterium) remains fixed at the carbon atom, but not on the fate of these carbon atoms to which deuterium is not attached. There may be a few other reactions which restrict the universal application of the method and further work will be necessary to understand such limitations.

As a result of the experiments on mice fed from 2 to 16 days on a diet comprising 20 per cent., 4 per cent. and 1 per cent. deuterium-containing fats, the authors conclude that the largest part of the dietary fat, even when present in small quantities, is deposited in the fat tissues before it is utilised

The application of the method to the investigation of the intermediary products in the general cholesterol metabolism has yielded very interesting results. Thus, cholestenone is an intermediary substance in the sterol metabolism. It is probable that it is formed from the oxidation of cholesterol as a first step, and is then reduced to coprostanone.

Further applications of the method will be awaited with interest.

B. N. S.

On the Biology of the Psyllidæ (*Homoptera*).

THE work of R. U. Mathur (*Indian Forest Records*, 1935, Entomology Series, Vol. I, No. 2) on the biology of the *Psyllidæ*, furnishes important details about the habits, life-history and economic value of a family of insects that has long remained comparatively untouched by Entomologists in India generally; not only in economic (Forest) but in taxonomic entomology as well, this work supplies a long-felt want, specially for investigators in forest Psyllid fauna all over India; indeed these valuable notes will also guide economic entomologists in India, in their study of Psyllids, as affecting cultivated crops of all kinds.

B. K. M.

Immature Stages of Indian Coleoptera (*Scarabæoidea*) and Eucnomidæ.

THE studies of immature stages of Insects in general and of Coleoptera and Lepidoptera specially, cover a very large field and offer great scope. In insect classification, the custom has mostly always been to take into consideration the taxonomically valuable characters of the imaginable stages, to the utter exclusion of the several important features of the larval or immature stages. This has not only rendered classification incomplete but has made the task of Coleopterists and Lepidopterists, in specific determinations based on characters of larval instars, doubly difficult, and well-nigh impossible.

The descriptions of the immature stages of Scarabæoidea, particularly, are of vital interest to economic entomologists who have frequently to deal with the larval stages of Scarabiid, Melolonthid and Rutellid beetles; considerable damage is caused to the roots of cultivated crops of all kinds by them and an exact knowledge of their important features largely facilitates not only correct determination but effective control. Entomologists all over India welcome the valuable work of J. C. M. Gardner, (*Indian Forest Records*, 1935, Entomology Series, Vol. I, Nos. 1 and 4) on the immature stages of Coleoptera.

B. K. M.

Asexual Reproduction in Tunicates.

N. J. BERRILL in his paper on the Asexual Reproduction in the Tunicata ["Studies in Tunicate Development," *Phil. Trans. Royal Society, London*, (B), 1935, **225**, No. 526] has described the nature of budding in twenty genera of ascidians; eight of them are described for the first time, such as, Diazona, Tylobranchion, Morchellium, Euherdmania, Endistoma, Archidistoma, Pycnoclavella and Chonorostachys. He has noted that the majority of the forms possess a regular alternation of sexual and asexual phases which coincide more or less with summer and winter, but the onset of budding seems to be dependent more on the physiological conditions of the animals than on the change of environment. The budding results in a tendency on the part of the zooid

to degenerate, the degeneration being brought about by autolysis or regression in an antero-posterior direction, which is associated with the formation of yolk-laden trophocytes. The posterior extension of migration of the trophocytes depends upon the presence or absence of the posterior-abdomen, enlarged ventral vessel and other similar factors. The medium of nutrition for the bud may be fluid where there is a physical continuity or by trophocytes in cases of physical isolation. During autolysis the tissues least specialised survive readily and the presence of at least one unspecialised cell is necessary to the development of a new individual, and buds are isolated from the parent by transverse constrictions and where a part of the old alimentary tube is present, the development is only a case of regeneration, while, where it is absent it is only reorganisation of the contained tissues. The author has contradicted the accepted view that the vascular septum of the ventral vessel has no connection whatever in embryonic development with the pericardium of the adult and hence cannot be considered as extension of the epicardium into the ventral stolon. He points out that though it is assumed that the mode of budding is primitive, there have been two major trends in specialisation, the one culminating in Distaplia, Diplosoma and probably Thaliacea where the type of budding is localised being epicardial and œsophageal, and the other through the posterior extension of zooids and buds to form Synoicids, Clavelinids and possibly ending in Perophoridæ, and the budding in the Botryllidæ and Polystylidæ is only a reacquisition of the faculty and not phyletically continuous with any other type.

Spermatogenesis of Man.

J. B. GATENBY AND H. W. BEAMS have for the first time described the behaviour of the Cytoplasmic inclusions during Spermatogenesis in man (*Quart. Journ. Micros. Sci.*, 1935. 78, Pt. 1, No. 309). While the general behaviour of the constituents probably does not differ from that found in many other mammals, there are certain important variations which are pointed out by the authors. Three types of golgi apparatus can be distinguished; that in the sertoli cells being filamentar while it is sub-spherical in the spermatocytes and semi-dispersed in the spermatogonia. During division, the golgi apparatus breaks up into a large number of very small bodies which show a tendency of grouping round the nucleus rather than round the asters. The mitochondria are granular throughout and probably do not differ from the mitochondria in other mammals. The rôle of the centriole appears interesting. The single centriole of the spermatid soon divides into two which move close to the cell-membrane. The flagellum arises from both of them and the distal one becoming larger, assumes a ring-shape. The whole apparatus moves close to the nucleus and the proximal centriole is eventually seen to attach to the nuclear membrane. The distal ring-shaped centriole moves a certain distance down the flagellum and marks the posterior end of the middle piece. The authors describe a neck granule in addition to the two centrioles, embedded in the posterior part of the nucleus close to the proximal centriole. They do not attach any relation between this body and the centrioles and tend to think it is a derivative of the nucleus. The golgi apparatus takes very little part in the formation of the acrosome, a single bead-like body developing within the archoplasmic area becoming attached to the nuclear membrane and giving rise to the acrosome. The rest of the golgi apparatus is discarded. A post-nuclear cap is described. The sperm head has a vacuole whose function is not determined with certainty. Crystals, of either rod, batonette or pointed shape are found in a number of cell-elements of the testis and are probably no more than reserve nutriment in the cells.

The Cranial Morphology of Some Examples of Pelobatidæ (Anura).

W. K. PARKER described the gross structure of the skull of various anuran examples as early as 1881 and recently, however, the subject has received greater attention and has been studied by modern methods. In a paper in *Anat. Anz.* (Bd. 81, Nr. 4/6, S. 65–96) Mr. L. S. Ramaswami has described some aspects of the cranial morphology of the two pelobatid examples Scaphiopus and Megophrys. The cranial anatomical features of these forms are compared with those of the ancestral group 'Liopelmidæ". Further, it is pointed out that Megophynæ, which according to Noble occupies a basal position, anticipates some of the characters of the next group—the pelobatinæ, of which

Scaphiopus has been studied. Moreover, Scaphiopus possesses certain exclusively individual characters. Thus the Nobelian viewpoint, that in the order of evolution, Megophrynæ occupies the lowest while Sooglossinæ the highest rungs, is questioned. The author suggests that the three sub-families Megophrynæ, Pelobatinæ and Sooglossinæ are all of equal rank and they have moved on parallel lines.

The Atomic Nucleus.

[Prof. Born first explained how he had chosen to talk about the nucleus although his main line of work was not nuclear structure: the intimate touch with the pioneers in nuclear Physics, which he had during his stay at Cambridge, was influential in his choice. Incidentally he referred the audience to his new book *The Restless Universe* with its novel illustrations which produced the same impression as a cinematographic picture when the leaves of the book were rapidly turned over. The lecturer then proceeded to give a very lively discourse on the discovery and investigation of atomic nuclei.]

THE view that an atom consists of opposite electric charges concentrated at a great distance from each other and not uniformly distributed throughout the atom was put forward by Lenard in his theory of dynamides, long before Rutherford proposed the nuclear theory of the atom. The idea behind the attempts to unravel the mystery of the nucleus was to pierce it by swift particles and thus gain a knowledge of its contents just as a closed mechanism has to be taken to pieces in order to lay bare its inner details. The discovery of Radioactivity had placed such swift projectiles in the hands of the physicist. The radioactive elements like Uranium, Polonium and Radium emit three different kinds of radiation which were named α-, β- and γ-rays. The α-rays consist of helium atoms which have lost two electrons, the β-particles are swift electrons and the γ-rays are waves like X-rays but of higher frequency. It was natural that Rutherford who had done important work in Radioactivity should study the effect of bombarding different substances by means of α-particles. It was also natural that one of the best means of studying the tracks of these particles, *viz.*, the Wilson cloud chamber should have been invented in England which is famous for its fog. When a sudden expansion is caused to take place in a chamber saturated with water vapour, the fall of temperature produces supersaturation and the vapour will condense into drops wherever some dust or charged particles are present. Since the α-particles are heavy projectiles with high energy their path is thickly studded with charged ions and droplets forming on these show the track of the α-particle as a thick straight line. Just as the projectiles from a heavy gun spread swift destruction thickly along their path while the bullets from pistols produce here and there a chance casuality and are also more easily turned from their course, the β-particles in contrast to the α-rays, produce zig-zag tracks sparsely covered with droplets. The γ-rays, on the other hand, first produce electrons along their path and the tracks of the latter then show up the passage of the γ-rays.

The experiments of Rutherford on the scattering of α-particles showed that these particles were generally deflected by small amounts, but now and then there occurs a very large deviation. Rutherford saw that such a deviation was like the passage of a comet round the sun and applying a similar calculation he was able to deduce the distance to which the α-particle had approached the positively charged part of the atom in order to suffer such large deflections as were observed, and found the distances to be sub-atomic.

The periodic system of the elements assigns to each element a number denoting its place in the table called the atomic number. This was now identified by Mosely to be identical with the positive charge on the nucleus. The simplest nucleus is that of hydrogen having unit charge and is called the proton. The other charges are multiples of these but the chemical atomic weights of the various atoms are not exact multiples of the weight of a proton. The explanation was furnished by the work of J. J. Thomson on positive-ray parabolas and the refinement which Aston introduced by designing his mass-spectrograph. This work showed that the masses of the different atoms were really integral multiples of that of the proton, thus reviving Prout's hypothesis. The chemical atomic weights were shown to be different from integers because the chemical elements are mixtures of atoms of different mass but with the same charge. Such atoms occupy the same place in the periodic table and are called isotopes. The separation of isotopes is very difficult; but in the case of the most interesting isotope, *viz.*, the heavy isotope of hydrogen discovered by Urey (for which he obtained the Nobel Prize) has been separated, with the help of its most important compound: heavy water. G. Hertz has also succeeded in separating the isotopes of neon and of hydrogen by repeated diffusion through a large number of porcelain vessels connected to diffusion pumps. Prof. Born had seen his apparatus filling a large room and witnessed its working.

Starting from the nuclear model of the atom, Bohr assumed that the electron, *e.g.*, in the hydrogen atom was revolving round the nucleus but it could do so only at definite distances from the centre so that its angular momentum changed from one position to another by integral multiples of h—*Planck's constant*—*and that the difference in the* energy when an electron jumped from one orbit to another was radiated as a single quantum $h\nu$ of frequency ν. Bohr was thus able to explain *Balmer's formula for the lines of hydrogen and* to deduce the constant occurring in it with great *accuracy*. He elaborated his theory further by the correspondence consideration that in the limit, classical theory and quantum theory should lead