those who were expected, according to the preface of the book, to look askance at certain methods used and to say that the thing proposed could not be done.

But the plain fact is that something needs to be done. Not a few specialists in economics and in the biological sciences feel the necessity of stopping to prepare courses in elementary statistics, etc., for the benefit of their colleagues in order to deal more effectively with the problems of their own researches. These courses, applicable in subjects ranging from textiles to physiology, have a very great deal in common. Again, inept use, or the avoidance, of elementary mathematics in the physical sciences could be abundantly illustrated. These are, I think, but indications of a misdirection in the general outlook on mathematics; and certainly the ordinary courses in mathematics meet such needs not even in an indirect way. We cannot long continue to ignore this defect in our educational practice, and a first task must be to discuss the lines along which we should move away from the present mathematical courses. In *Descriptive Mathematics* is a definite proposal to this end for first year students only, not for the second (Intermediate) year; but your reviewer can see nothing new in it, and merely judges it from a conservative standpoint. (To take one simple instance: I should like to know if there be anywhere else an examination of the principles of slide rules comparable with that on page 97.) When we have achieved a reasonable measure of agreement as to what aspects of elementary mathematics should be taught, there will be no lack of endeavours to write books suited to examination purposes. But that is not the criterion to apply at this stage.

JOHN MACLEAN.

Wilson College, Bombay,
February 2, 1936.

I HAVE perused Prof. Maclean's comment. Prof. Maclean is a distinguished educationist of Bombay and we have high respect for his services to the cause of education. But so far as his present book is concerned, I cannot help expressing my frank and honest opinion without any reservations. I shall not worry myself about this charge of conservatism on my part, but I shall dwell with emphasis on one point only:

The average man has a general dislike for or difficulty to follow the theory of mathematics. It must be the endeavour of every mathematics teacher of the elementary stage to present the subject with as much simplicity as possible, confining in the earlier stages only to the *intrinsic beauty* of the subject, omitting all details and complications to a later stage. To most people, even to many mathematicians themselves, numerical work and heavy calculations are disgusting. From the boy at school who works on vulgar fractions and decimals, to the average public man, heavy arithmetic is never taken as matter of love. This is a general human weakness, and not all the slide rules in the world can remedy this to any remarkable extent. If then heavy numerical work is taken as a necessary adjunct to the elementary principles and methods of mathematics, and the result of this fusion is called *Descriptive Mathematics*, it is my frank opinion that most people would bid good-bye to this kind of mathematics. Experimental Scientists, and research workers in Social Sciences require and will automatically cultivate the required speed and accuracy in numerical work, when they settle down at their work, but to inflict this kind of work on a poor First Year Intermediate student is horrible!

I remember our learned Editor of the *Current Science*, in the course of a speech somewhere saying to the following effect: To the Physicist, everything in this world will appear as Physics and Chemistry, to the Biologist, everything will appear as Biology, etc. Likewise, shall I say that a Statistician cannot *describe* simple elementary principles of mathematics without asking his boys

(1) to use the slide rule and verify

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2}\,\frac{\sqrt{(2+\sqrt{2})}}{2}\,\frac{\sqrt{\{2+\sqrt{(2+\sqrt{2})}\}}}{2}$$

(2) to draw the curve $y = 1320x^{-0.0234}$

(3) to solve $10000\,(\sin 3x + 2\frac{1}{2}\cos 2x) = x^2 - 300x + 9000$?

These are a few specimens from the book.

I have no personal dislike for statistics and I have some little pretensions for the subject myself; I can also boast myself to be a moderately good computer. But there is a difference between having a knack for this work or cultivating it as one's needs arise, and gulping this down on a poor Intermediate student.

One can see in Prof. Maclean's book in many places the hand of an experienced and able exponent of the subject, but in my opinion this is marred by an over-filling of numerical work.

Here are two specimens (not typical, though) of *Descriptive Mathematics* :—

(*a*) "On March 21 the number of seconds taken by the sun to cross the horizon in latitude l is $32/15 \sec l$; find this for $l = 19°$ and $l = 60°$" (p. 58).

Is this illustration going to create a new interest in the student for the secant function, or is it going to be the nucleus of his future astronomical studies, or does it just show off the pedantry of the author ?

(*b*) "Note sets of words like "due, duty, dutifully" which are useful in teaching time in music. The periods of the syllables of these words are as 4 : 2 : 1. Consider the possibility that rhythm in speech and in prose may be partly and automatically determined by the essential periods and intensities of syllables. (In poetic rhythm there is of course deliberate selection of combinations of syllables)". (p. 60.)

This example follows problems like sketching the curves corresponding to $\cos^2 x$, $\cos^3 x$, etc. I have no music in me, but frankly, this piece of mathematics is beyond me.

C. N. S.

Research Notes.

On Ternions in Geometry.

HANS BECK (*Math. Zeit.*, **40**, 4, pp. 509-520) has investigated the occurrence of the linear transformation group of the system of non-commutative ternions, under various forms in several places in geometry. Let A, B, C, D be four ternions, then the linear transformation is $X' = (Cx + D)^{-1} (Ax + B)$. Now it is known that apart from the non-commutative system of the ternions, there exist two other commutative systems of ternions. In the latter cases, the linear transformation-group reduces itself to one of nine parameters. This is not of so much importance as the group in the non-commutative case, of eleven proper parameters. Beck has shown that this group occurs in the following places in geometry : (1) A special collineation group of a linear-complex ; (2) A cremona group in affine space ; (3) The group of Laguerre transformations of directed planes ; and (4) The group of rotations (in the same sense) in the four dimensional Euclidean space, etc.

A ternion of the system is represented as $A = A_0E_0 + A_1E_1 + A_2E_2$ and the multiplication table is

$$\left| \begin{matrix} E_0 & E_1 & E_2 \\ E_1 & E_0 & E_2 \\ E_2 & -E_2 & O_0 \end{matrix} \right|$$

E_0 can be taken to be the scalar unit. The norm $N(A) = A_0^2 - A_1^2$ (Hence reducible). If $\xi_0, \xi_1, \xi_2, \xi_3$ are the co-ordinates of a point in a projective R_3 and the Plucker's co-ordinates of a line are $P_{ik} = \xi_i \eta_k - \eta_i \xi_k$, then the ternions A, can be made to correspond to the lines of R_3 with co-ordinates $P_{01} : P_{02} : P_{03} : P_{23} : P_{31} : P_{12} = 0 : 1 : A_0 - A_1 : A_2 : -(A_0^2 - A_1^2) : (A_0 + A_1)$. (The transformation is not one-one.) By means of this transformation he has shown that the group is identical with the collineation which transforms $\xi_0 = 0$, $\xi_1 = 0$ into itself. Here is a nice geometrical representation of ternions.

He has also shown that the group is holomorphic with the group of the minimal complex—the straight lines having proper intersection with the conic-absolute in an Euclidean R_3.

The first representation of ternions is such that an ∞^2 st. lines of the projective space R_3 did not correspond to ternions at all. Then by considering a geometrical entity as corresponding to a ratio of two ternions, he obtains a representation in which the geometrical entities are points in an affine space ; the exceptional points for which ratio of ternions do not correspond belong to a plane which is naturally considered as the special-plane of the affine space (*Uneigentliche-Ebene*). The work is a very striking illustration of the unity in geometry stressed by Klein in his epoch-making Erlangen-Programme.

K. V. I.

Cauchy-Riemann Conditions.

MENCHOFF (*Fund. Math.* **25**, pp. 59-97) has extended Looman's classic result (*Gott. Nach.*, 1923) about the sufficient conditions for the analyticity of $f(z) = P + iQ$ in a given simply connected region. Looman had shown that if the Cauchy-Riemann partial differential equations were valid for almost all points in the region then $f(z)$ was analytic. This amounted to assuming that the derivates in two perpendicular directions (directions same

(*Continued on page 605*)