89. Rasetti, F., *Nature*, 1931, 127, 626.
Born, M., *Dynamik der Kristallgitter Tenbuer*, 1915.
90. Placzek, G and Van Hove, L., *Phys. Rev.*, 1954, 93, 1207.
91. Brockhouse, B. N. and Iyengar, P. K., *Phys. Rev.*, 1957, 108, 894.
92. Van Hove, L., *Ibid.*, 1953, 89, 1189.
93. Johnson, F. A. and Loudon, R., *Proc. Roy. Soc.*, 1964, 281, 274.
94. Birman, J. L., *Phys. Rev.*, 1963, 131, 1489.

SIR C. V. RAMAN'S WORK ON THE DYNAMICS OF VIBRATIONS

B. S. MADHAVARAO
Indian Institute of Science, Bangalore

I WISH to refer here briefly to some mathematical aspects of the earlier work of Raman related to the theory of vibrations of several types. This work was done during the years 1914 onwards at Calcutta when he was greatly interested in acoustics, specially in the subject of Indian musical instruments. It is interesting to recall in this connection a few remarks made by Dr. Ganesh Prasad, a pure mathematician of high calibre, in his Presidential Address to the Physico-Mathematics Section of the Indian Association for the Cultivation of Science (Report for 1914, p. 39), *viz.*, "Prof. Raman is well known to you all as an experimentalist of worldwide reputation. But some of you will feel surprised to learn that by his recent mathematical researches, he has firmly established his claim to be considered a sound mathematician. The two papers which he read before the Calcutta Mathematical Society during the current year are very valuable, and I trust he will continue his mathematical researches." It is indeed fortunate for science that Raman did not take Ganesh Prasad's advice to continue research in mathematics for its own sake, but, like a sound mathematician, decided to use mathematics to help him expound new physical phenomena of his own creation.

His contributions to the theory of vibrations have been numerous, but I shall just choose a few which appear to be of mathematical interest. The first one relates to the general investigations on the maintenance of vibrations in strings (*Bull. No.* 6, 1912, *Ind. Assoc. Cult. Science*), and deals with several topics, specially the difficult case of the inhomogeneous string. The nature of the mathematics employed is the one based on Rayleigh's classical book on the "Theory of Sound", which Raman had thoroughly mastered. Plenty of mathematics has been used, but is always accompanied by experimental details illustrating the mathematical results, and also the different types of maintenance are explained by beautiful figures. It is well known that the topic of vibrations of an inhomogeneous string is really the Sturm-Liouville eigenvalue problem (Hilbert-Courant, *Methoden der Math. Phys.*, 1924, p. 237), and it is interesting to note that Raman has beautifully succeeded in transferring this eigenvalue problem for the case of maintenance of vibrations also, by simple physical explanations rather than by the use of the difficult mathematical techniques involved. Such an approach is also evident in his investigations (*Bull. No.* 11, 1914) on the maintenance of combinational vibrations by two simple harmonic forces, and by other types of periodic fields.

The most interesting of his researches on the vibration of strings is contained in the comprehensive memoir (*Bull. No.* 15, 1918, pp. 1-157) on the mathematical theory of the vibration of bowed strings, and of musical instruments of the violin family with experimental verifications of the results. The necessary dynamical theory is developed using Langrange's equations of motion for simple and forced vibrations, and the notion of normal co-ordinates. A Fourier analysis is made of the types of possible motions, and the difficult question of convergence of the Fourier series is dealt with in a physically intuitive manner, using just the essential mathematical steps. In view of the complexity of the problem, emphasis is laid upon the cases which are of practical interest, and the choice of these cases exhibits clearly his ability to overcome difficult mathematical hurdles. This certainly goes to his credit if only we realise that, even with the present-day techniques of mathematical physics relating to vibration and sigenvalue problems, the complex case dealt with in this *Memoir* cannot be solved in full. One of the important results in this paper is the quantitative explanation of the effect of a mute on the quality of the violin tune. Another remarkable result of mathematical interest is his analysis of the nature of motion when n, the number of discontinuities, is a prime > 1, or is composite. A careful examination of the manner in which this analysis is accomplished shows that he has not used any sophisticated results of prime number theory, but just the definition of a prime number!

Another investigation relating to the vibrations of circular membranes is also of great interest. It is well known that percussion instruments, generally speaking, give rise to inharmonic overtones, and are thus musically imperfect. The Indian Mridanga forms a remarkable exception to this rule as found by Raman experimentally. The character of vibrations of the associated heterogeneous membrane which gives rise to these significant properties was investigated by sand figures, and definite quantitative results obtained. The problem of oscillations of heterogeneous membranes is a complicated one, and it is difficult to solve it completely with all the resources of advanced mathematical physics. It is again a general Sturm-Liouville eigenvalue problem, whose solution demands an extensive knowledge of integral equations theory (Hilbert, *Ibid.*, pp. 255, and 10, p. 273). It is very interesting to note that the result of physical interest mentioned at the end of paragraph 2, p. 279 of the above reference is inherent in Raman's experimental results.

Lastly, a result of rather academic interest is Raman's attempt at a classical derivation of the Compton effect (of which the Raman Effect is the optical analogue) by using the theory of vibrations (*Ind. J. Phys.*, 3, p. 357) based on the result that even the most arbitrary type of wave disturbance can be represented as the superposition of plane trains of waves travelling in all directions in space.

In conclusion, I might mention that Raman always evinced a keen interest in topics of mathematics, even of the purest type. I remember, when I was a student of the M.Sc. Pure Mathematics course of the Calcutta University during 1919-21, and he was the Palit Professor of Physics, his finding time to question me (a young student of about 20 years then) as to what I was learning, and listen to my talking about topics like Algebra of Quantics, Projective and Differential Geometry. He retained this interest in Pure Mathematics, which he often likened to an art like painting or music, throughout his life. Such great men of wide interests, and broad vision are rare indeed.

ELASTIC CONSTANTS AND SUB-HARMONICITY

B. R. SETH
Dibrugarh University, Dibrugarh, Assam

1. Introduction

ONE of the subjects which interested C. V. Raman was that of internal couple-stresses produced in crystals subjected to severe deformation. This required the introduction of additional elastic constants. He applied it to the determination of the elastic constants of cubic crystals at the macro-level.

Internal moments are essentially a microphenomenon and cannot be of much use in macro-measurements of elastic constants. Recently, unsuccessful attempts have been made to use this concept to explain fatigue. It is also being used in turbulence where it may bear some fruit, as any further work in this subject must now be at the macro-level.

As regards rotation effects, these can be produced by the second order terms in the strain tensor itself, which are neglected in the classical treatment. Actually, the elastic fields are sub- or super-harmonic in character, and, contain rotation or spin effects as particular cases. The classical bi-harmonic or harmonic fields are only degenerate forms, and hence do not exhibit these effects. If this is taken into consideration, the assumption of additional elastic constants for crystals, isotropic at the macro-level, need not be made. We shall now prove the sub- or super-harmonicity of the elastic fields.

2. Equations of Equilibrium

Let the undeformed and the deformed coordinates of the position of a particle be X^r and x^r, $(r = 1, 2, 3)$ respectively, so that the deformation is given by

$$u^r = x^r - X^r, \quad x^r = f^r(X^1, X^2, X^3)$$
$$(x^1, x^2, x^3) = (x, y, z). \qquad (2.1)$$