

in the parallel setting both the halo and the direct beam appear but the two are of complementary colours.

The rate at which the azimuth changes depends upon

$$\psi = \left( \frac{R_{\parallel}^2}{\sigma_{\parallel}^2} - \frac{R_{\perp}^2}{\sigma_{\perp}^2} \right) \approx \frac{R^2}{(\delta n)^2 \bar{\tau}} \quad (4)$$

where  $(\delta n)^2$  and  $\bar{\tau}$  are the average crystallite birefringence and crystallite size. When  $\bar{\tau}$  is small  $\psi$  will be large making the polarization effects conspicuous, a fact supported by experiments. Values of  $(\delta n)^2$  for magnesium fluoride, barium sulphate, calcium sulphate and  $\alpha$ -quartz are respectively 0.02, 0.009, 0.0005, 0.0006. Hence  $\psi$  is a minimum for magnesium fluoride while it is a maximum for calcium sulphate and  $\alpha$ -quartz. Thus there is hardly any change in polarization in the case of magnesium fluoride which therefore gives a halo that can be extinguished between crossed polaroids. On the other hand, the change in polarization is large for  $\alpha$ -quartz or calcium sulphate rendering it impossible to extinguish the halo between crossed

polaroids. These are the very experimental observations of Raman and Bhat (1955).

In the case of oriented polycrystals like fibres of wool and cotton, we find maximum effects of birefringence (which itself is very small) for an incident light wave polarized at  $45^\circ$  to the fibre axis. Thus we expect polarization and chromatic effects to be conspicuous for this state of polarization. This has also been observed by Raman and Bhat (1954). Thus most of the observed facts can be explained by the theory presented here.

1. Sethi, N. K., *Proc. Ind. Assoc. Cult. Sci.*, 1920, 6, 121.
2. Raman, C. V. and Nath, N., *Proc. Ind. Acad. Sci.*, 1935, 2 A, 406.
3. Mueller, H., *Z. Kristallogr.*, 1938, 99 A, 122.
4. Ramachandran, G. N., *Proc. Ind. Acad. Sci.*, 1942.
5. Raman, C. V., *Ibid.*, 1949, 29 A, 381.
6. — and Ramaseshan, S., *Ibid.*, 1949, 30 A, 211.
7. — and Jayaraman, A., *Ibid.*, 1950, 32 A, 123; *Ibid.*, 1953, 37 A, 1.
8. — and Bhat, M. R., *Curr. Sci.*, 1953, 22, 31.
9. — and —, *Proc. Ind. Acad. Sci.*, 1954, 39 A, 109.
10. — and Viswanathan, K. S., *Ibid.*, 1955, 41 A, 37; *Ibid.*, 1955, 41 A, 41.
11. — and Bhat, M. R., *Ibid.*, 1955, 41 A, 61.
12. Ramaseshan, S., *Curr. Sci.*, 1972.
13. Ranganath, G. S. and Ramaseshan, S., 1972 (unpublished).

## MODULATION OF ELECTRON WAVES BY LASER BEAM\*

K. P. SINHA

Department of Physics, Indian Institute of Science, Bangalore-12, India

OTHER speakers have already given you an account of the growth of ideas emanating from Prof. C. V. Raman. Perhaps the richest among these has been his interest in the interaction of radiation with matter in various forms. I wish to talk about one of the latest developments in this field. This involves the interaction of a coherent laser light with free electron waves in crystalline

medium. The experimental results are rather puzzling and have aroused considerable theoretical interest<sup>1</sup>.

Before giving a quantum mechanical interpretation of the results with which we have been concerned, we will briefly discuss the experimental results.

The original experiment of Schwarz and Hora<sup>2</sup> is a transmission electron diffraction of a 50 KeV electron beam from a single crystal film of aluminium

\* This paper was published in *Current Science*, February 20, 1972 and is reprinted here.

oxide or silicon dioxide (thickness  $2d \sim 1000 \text{ \AA}$ ). They first observed the diffraction spots on a luminescent screen. As expected, on replacing this by a non-luminescent screen the diffraction pattern was no longer visible. However, when they irradiated the single crystal film with a monochromatic coherent polarised light beam from a 10-Watt argon-ion laser (wavelength  $\lambda = 4880 \text{ \AA}$ ) with its electric vector parallel to the electron beam, the diffraction spots reappear emitting the colour of the laser light. The effect weakens on changing the polarisation of the light and the spots shift on the application of a magnetic field. Also, the effect disappears in the absence of the single crystal film.

Several theorists all over the world have been seized with the problem of interpreting this unusual result<sup>1</sup>. Quite often similar ideas have been advanced by different groups without being aware of the results of the other. Two kinds of attempts have been made. Some authors believe that the effect is similar to the classical bunching of electrons as in a Klystron. However the electron density is rather small. Also, we shall show that certain parameters relevant to the Schwarz and Hora experiment lie in the quantum domain. In what follows, we shall discuss a quantum treatment of the effect developed by us recently<sup>3</sup>. Other authors elsewhere have also given a quantum treatment but they do not bring out the essential role of the single crystal film in explicit mathematical form. We follow the path-integral approach which gives the desired results in a few steps and in a more natural and transparent way.

First we make note of the fact that photons do not interact with free electrons in the first order for reasons of energy and momentum conservation in

vacuum. Second order processes are exceedingly weak. Thus the observed modulation can be achieved only after incorporating the role played by the crystalline film.

We are required to find the amplitude of the electron waves at the screen ( $x_2$ ) at time ( $t_2$ ) after the initial wave has traversed the irradiated film. This is given by

$$\psi(x_2, t_2) = \int K_v(x_2, t_2; x_1, t_1) \phi(x_1, t_1) dx_1, \quad (1)$$

where  $\phi(x_1, t_1)$  is the initial amplitude (wave) function and it will have the usual form for a free electron, i.e., a plane wave (see ref. 3),  $K_v(x_2, t_2; x_1, t_1)$  is the particle Kernel propagating from  $x_1, t_1$  to  $x_2, t_2$ . The propagator is related to the classical action evaluated along all possible paths. Its explicit form is given by

$$K_v(x_2, t_2; x_1, t_1) = \int_{x_1}^{x_2} \exp\left(\frac{i}{\hbar} \int_{t_1}^{t_2} L dt\right) Dx(t), \quad (2)$$

where  $Dx(t)$  is the element of volume in the path space.  $L$  is the Lagrangian and must contain the interaction between the electron and laser field in the presence of the crystal. For a one-dimensional situation, which is adequate for the present problem, we have

$$L = \frac{m\dot{x}^2}{2} - eV(x) \sin \Omega t, \quad (3)$$

where  $m$  is the electron mass,  $x$ -axis is along electron beam path and  $\Omega$  is the circular frequency of the laser light. The electric field vector is taken parallel to the electron beam and  $V(x)$  represents the scalar potential produced by the laser field. The interaction term [(cf, second term of equation (3))] must contain the crucial information, i.e., the discontinuities to the normal component of the electric field associated with light beam at the crystal boundaries. For this purpose we introduce  $\delta$ -functions at the two

boundaries. Accordingly the second derivation of the potential is expressed as

$$\frac{d^2\psi}{dx^2} = E_0 f(\epsilon) [\delta(x-d) - \delta(x+d)] + \text{a smooth function } F(x), \quad (4)$$

where  $E_0$  is the constant electric displacement,

$$f(\epsilon) \equiv \frac{\epsilon - 1}{\epsilon},$$

$\epsilon$  being the optical dielectric constant of the crystalline film. The smooth function represents the diffuse character of the light beam and does not change over lengths of the order of de Broglie wavelength of electrons. The precise formulation of the discontinuity given in Eq. (4) is of central role here. This affords a distinct mathematical advantage over the treatments given by others.

We shall now quote the final result. For details reference may be made to the original paper<sup>3</sup>. Once we get  $\psi(x_2, t_2)$  by evaluating Eq. (1), the current density at the screen is easily obtained. We get

$$\begin{aligned} j(x_2, t_2) &\approx ev \left[ 1 + \frac{2eE_0}{mv\Omega} \sin\left(\frac{\Omega d}{v}\right) \right. \\ &\quad \times \sin\left(\Omega t_2 - \frac{x_2\Omega}{v}\right) + \frac{eE_0 f(\epsilon)v}{\hbar\Omega^2} \\ &\quad \times \sin\left(\frac{\Omega d}{v}\right) \cos\left(\Omega t_2 - \frac{x_2\Omega}{v}\right) \\ &\quad \left. \times \sin\left(\frac{x_2\hbar\Omega^2}{mv^3}\right) \right], \quad (5) \end{aligned}$$

where  $v$  is the electron velocity. This equation gives current due to each individual electron. The first term is the usual d.c. part. The other terms give spatially modulated and (an unmodulated) oscillating parts. The current oscillates in time for a given distance of the screen with the same frequency as the laser beam. The current intensity also varies sinusoidally with the screen distance. These features explain the observed results. To ascertain whether the experimental results are in the classi-

cal or quantum domain, let us consider the modulating function  $\sin(x_2 \hbar \Omega^2 / mv^3)$ . In the classical limit the argument should be very much smaller than unity. However, for the Schwarz-Hora result it has the value  $\sim 10^2$ . Thus we are in the quantum domain. The question of the mechanism by which such optically modulated electrons emit the same radiation is not clearly answered. It has been surmised that the modulated electrons impinging on the inert screen re-radiate through Bremsstrahlung processes. It is difficult to establish that it must re-radiate the light of the laser frequency. Some have suggested that the screen has argon-ions imbedded in it. Accordingly, there will be a resonance radiation when the electrons hit the screen. One could also throw some exotic suggestions. For example, the modulated electron wave may be considered a composite unit of an electron and a photon. On striking the target a Raman-like scattering takes place wherein the electron is absorbed and a photon emitted.

It is possible to speculate further. But this can await until some precise formulation is given.

Finally, there exists the intriguing possibility of observing a temporal counter-part of the Aharonov-Bohm effect<sup>3</sup>. This can arise from the following. In regions well beyond the effective range of the radiation field there will persist a time-dependent oscillator potential. This will produce a time-dependent phase factor in the wave function in the field free region. In order to see a physically observable effect the experiment must deviate from the accepted concept of time ( $t$ ) and consider the situation relevant to the proper time interval  $\tau_{12}$ .

$$\begin{aligned} \tau_{12}^2 &= (t_2 - t_1)^2 - \frac{1}{c^2} [(x_2 - x_1)^2 + (y_2 - y_1)^2 \\ &\quad + (z_2 - z_1)^2], \quad (6) \end{aligned}$$

A few words about the future developments of the Schwarz-Hora effect will be in order. The effect will find applications in the colour television. Also, when other experimentalists succeed in observing this effect under different conditions, it

may open some new avenues of research.

1. *Physics Today*, 1971, 24, 17.
2. Schwarz, H. and Hora, H., *Appl. Phys. Letters*, 1969, 15, 349.
3. Kumar, N. and Sinha, K. P., *Proc. Ind. Acad. Sci.* 1971, A, 74, 91.

## INFORMATION TO CONTRIBUTORS

CURRENT SCIENCE is the Premier Science Fortnightly of India published by the Current Science Association, Bangalore and issued on the 5th and 20th of each month. All material intended for publication in CURRENT SCIENCE and books for review should be addressed to the Editor, *Current Science*, Raman Research Institute, Bangalore-6.

The following types of scientific communications will be considered for publication in *Current Science*: (1) Review Articles on topics of major current interest, not exceeding 4,000 words or 12 pages of foolscap; (2) Original Research Papers not exceeding 2,000 words; (3) Letters not exceeding 700 words; (4) Short scientific notes not exceeding 200 words. Articles and papers should include a short abstract at the beginning.

Manuscripts should be typewritten in double space on one side of the paper, carefully revised by the authors and in final form for printing. Illustrations should be minimum in number, drawn in black and white with Indian ink on bristol board, and preferably enlarged to about twice the size they are intended to appear in print which will be mostly in column size.

Business correspondence, remittances, subscriptions, advertisements, reprints, exchange journals, etc., should be addressed to the Manager, Current Science Association, Raman Research Institute, Bangalore-6.

### Subscription Rates :

*Institutions* : India Rs. 48.00 ; Foreign £ 5.00 ; \$ 12.00

*Individuals* : India Rs. 24.00 ; Foreign £ 3.50 ; \$ 8.00