TOWARDS A UNIVERSAL MASS FORMULA FOR ELEMENTARY PARTICLES

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ABSTRACT

A formula for calculating the masses of various elementary particles is developed on the basis of a dynamical theory. The calculated masses are in good agreement with the observed values.

1. INTRODUCTION

RECENTLY a dynamical model for elementary particles was developed[1]. In this model it was envisaged that a primordial object in its classical equilibrium state is stressless and massless having a finite dimension $\sim 10^{-13}$ cm. This object executes harmonic oscillations around the equilibrium position and the various excited states of such a system are identified with the known elementary particles. It will be shown in a separate note that elastic restoring forces of a general relativistic origin are indeed set up at this equilibrium position and that at this position the primordial object is indeed massless and stressless, thus justifying the basis for the above model.

However, the entire treatment of the above model was confined to a non-relativistic oscillator. This is found to be valid for an electron-like system where the frequency is relatively low. For more massive particles such as mesons and baryons it turns out that a relativistic treatment is called for. These points will be amplified later in a separate note.

2. THE MODEL HAMILTONIAN AND DERIVATION OF MASS FORMULAE

The expression for the energy of a one-dimensional relativistic oscillator is given by[2]:

$$E = m_0 c^2 + V, \tag{1}$$

where m_0 is the rest mass, c = velocity of light and V is the potential energy which has the form:

$$V = \frac{1}{2} m_0 \omega^2 x^2 \left(1 + \frac{3}{8}\frac{\omega^2 a^2}{c^2} + \frac{9}{256}\frac{\omega^4 a^4}{c^4} + \cdots\right). \tag{2}$$

Here a is the maximum displacement corresponding to zero velocity and ω is the angular frequency of oscillation. For low frequencies the expression (2) reduces to the classical form for the potential energy of the harmonic oscillator.

Now the corresponding quantum-mechanical energy equation can be obtained by using the time-independent Schrodinger equation:

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0. \tag{3}$$

The energy eigen values are then obtained as:

$$E_n = \hbar\omega\left[(n + \tfrac{1}{2}) + \tfrac{3}{4} b\,(n^2 + n + \tfrac{1}{2}) + \cdots\right], \tag{4}$$

where $b = \hbar\omega/m_0 c^2$, and is related to the force constant.

It is found from general relativistic considerations as well as empirical arguments regarding the nature of weak, electromagnetic and strong interactions[3], that the oscillation frequencies of the three families of particles (*i.e.*, leptons, mesons and baryons) can be described by the relations:

$$\omega_L = \frac{1}{(2\alpha)^{n_L}} \cdot \omega_e \text{ (Leptons)}$$
$$\omega_M = \frac{1}{2\alpha} \cdot \omega_e \text{ (Mesons)}$$
$$\omega_B = \frac{1}{\alpha^{3/2}} \cdot \omega_e \text{ (Baryons)}, \tag{5}$$

where

$$\omega_e = \sqrt{8\pi}\,(m_e/M_P)\,\omega_0$$

and

$$\omega_0 = \left[\frac{G}{r_0^3}\left|\frac{6e^2}{c^2 r_0} - 4M_P\right|\right]^{\frac{1}{2}}. \tag{6}$$

Here $\alpha = e^2/\hbar c = 1/137 \cdot 04$ is the electromagnetic fine structure constant; e is the elementary charge; G is the Newtonian gravitational constant; $M_P = (\hbar c/G)^{\frac{1}{2}}$, is the Planck mass; m_e is the electron rest mass and r_0 is the equilibrium radius of the object whose bare mass is M_P.

It can be shown[3] from simple arguments using the uncertainty principle that the oscillations of frequency ω_0 of a primordial object whose bare mass is the Planck mass, generates the frequency ω_e corresponding to the observable electron rest mass at the electron Compton length. This shows that at this frequency $3b/4 \sim 1$.

Thus for the three families of particles the energy eigen value equation (4) can be written as:

Leptons:

$$E_n\,(\text{Leptons}) = \hbar\omega_e\left[\frac{2n_L + 1}{(2\alpha)^{n_L}}\right]. \tag{7}$$

As remarked earlier, for leptons we do not require the relativistic corrections for the energy of the oscillations.

Mesons :

$$\mathrm{E}_n\,(\text{mesons}) = \hbar\omega_e \left[\frac{n_M^2 + 2n_M + 1}{2a}\right] \tag{8}$$

Baryons :

$$\mathrm{E}_n\,(\text{baryon}) = \hbar\omega_e \left[\frac{n_M^2 + 2n_M + 1}{2a} + \frac{n_B}{a^{3/2}} \pm \frac{(2n_L + 1)}{(2a)^{n_L}}\right]. \tag{9}$$

For leptons the quantum numbers $n_L = 0,\ 1$; higher numbers are not needed. For mesons, $n_M = 0,\ 1,\ 2, \ldots\ldots$ etc. n_B turns out to be 1 for all baryons and zero for leptons and mesons. Thus n_B should be identified with the baryon number.

The various values of the masses are calculated from the relations (7), (8) and (9) and using $m_n = \mathrm{E}_n/c^2$.

Mass Formula as obtained by the Quantization of the Relativistic Harmonic Oscillator

TABLE I
Leptons

$$\mathrm{M} = m_e \frac{(2n_L + 1)}{(2a)^{n_L}}$$

n_L	M MeV	Observed MeV	Identification
0	0·51	0·51	Electron $e\pm$
1	105·4	105·6	Muon $\mu\pm$

TABLE II
Mesons

$$\mathrm{M} = m_e\left[\frac{n_M^2 + 2n_M + 1}{2a}\right] = \frac{m_e}{2a}(n_M + 1)^2$$

n_M	M MeV	Observed MeV	Identification
0	35·5	..	..
1	140·2	139·7	Pion $\pi\pm$
2	314·2	..	..
3	559·5	549	Eta η
4	869	865	K*
5	1260	1220	C*

TABLE III
Messons with lepton component

$$\mathrm{M} = m_e\left[\frac{n_M^2 + 2n_M + 1}{2a} \pm \left\{\frac{(2n_L + 1)}{(2a)^{n_L}}\right\}\right]$$

n_M	n_L	M MeV	Observed MeV	Identification
3	1	454·5	495	K±, K⁰
4	1	974·1	950	η^1
5	1	13[illegible]3·7	1310	K*
5	1	1253·7	1247·5	K K°
6	1	1786·9	1785·6	K π π
6	1	1105·5	161[illegible]·6	f^1

The values are tabulated along with the observed values in the following tables.

TABLE IV
Baryons

$$\mathrm{M} = m_e\left[\frac{n_M^2 + 2n_M + 1}{2a} + \frac{1}{a^{3/2}}\right]$$

n_M	M MeV	Observed MeV	Identification
1	957·8	931	P
2	1131	1115·4	Λ
3	1375	1320	Ξ
4	1669·5	1672	Ω

TABLE V
Baryons with Lepton Component

$$\mathrm{M} = m_e\left[\frac{n_M^2 + 2n_M + 1}{2a} + \frac{1}{a^{3/2}} \pm \frac{2n_L + 1}{(2a)^{n_L}}\right]$$

n_M	n_L	M MeV	Observed MeV	Identification
2	1	1237	1195	Σ
3	1	1272	124[illegible]	$N^*_{3/2}$
4	1	1581	15[illegible]0	Ξ
4	1	1792	1752	Y_1^*
5	1	2[illegible]81	2110	Y_0^*
5	1	197[illegible]	2035	Y_1^*
6	1	2634·8	2[illegible]01	$N_{1/2}^*$
6	1	2124	2418	$N_{3/2}^*$

3. DISCUSSION

It is seen that there is good agreement between the calculated and observed values. On examining the relations (7), (8) and (9) it emerges that the baryon 'core' mass is related to the electron rest mass as :

$$m_B = \frac{m_e}{a^{3/2}}. \tag{10}$$

This is in conformity with the current picture of baryons (say the proton) as consisting of a hard core ('parton') surrounded by various meson clouds. Accordingly, as seen from the tables, the proton mass (*i.e.*, the lowest baryon mass) is comprised of this core mass *plus* the pion cloud (*i.e.*, lowest meson mass). Also from Table IV for the baryon masses, it is interesting to note that the number $(n_M - 1)$ appears to be related to the corresponding strangeness quantum number S for the baryons. This is indicated in Table VI.

TABLE VI

$(n_M - 1)$	S	Particle
0	0	P
1	1	Λ, Σ - hyperon
2	2	Ξ — hyperon
3	3	Ω — hyperon

Also the fact that the oscillation frequencies of mesons and baryons are scaled off by $1/\alpha$ and $1/\alpha^{3/2}$ respectively, suggests that the strength of the binding interactions for these particles is of this order. In this context, it may be noted that the strength of the interaction between two Dirac[4,5] monopoles is roughly of this order, *i.e.*, $1/\alpha \sim 137$.

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A STUDY OF METASTABLE EQUILIBRIUM IN ALUMINIUM-COPPER ALLOYS

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ABSTRACT

Hardness and electrical resistivity measurements are carried out on Al–0·75 and 1·50 at.% Cu alloys to study the reversion phenomenon. Critical reversion temperatures for these alloys are found to be in good agreement with the findings of earlier work. The metastable solvus curves for both G.P. zones and θ' phase are thus confirmed with minor modifications.

INTRODUCTION

In recent times the search for stronger metallic materials has aroused more interest in metastable equilibrium in alloys. By resorting to suitable thermal treatments it is today possible to convert the so-called equilibrium alloys into different non-equilibrium states associated with superior mechanical properties. Apart from its practical aspects, metastability in alloys constitutes a fascinating area for theoretical studies. Among alloys systems explored from the point of view of metastable equilibrium, those based on aluminium occupy a very special place because of their great commercial importance, as also their pronounced age-hardening potential.

Ever since the discovery of the age-hardening phenomenon by Wilm[1] in aluminium-copper (Al-Cu) alloys, this binary alloy system has attracted considerable attention in the metallurgical world. The sub-microscopic structural changes on ageing dilute alloys of this system have been investigated by many workers[2]. The pioneering work of Guinier[3] and Preston[4] led to the identification of the complex sequence of precipitation on ageing in this system as:

Guinier-Preston (G.P.) Zones $\rightarrow \theta'' \rightarrow \theta' \rightarrow \theta$ ($CuAl_2$).

Two peaks are generally observed in the ageing curves of these alloys. It has been shown[5] that the G.P. Zones are responsible for the initial rise in hardness while θ'' and θ' metastable phases give rise to the second hardness peak. Thus the metastable equilibrium on ageing is quite complex in this system.

The metastable solvus for Al-Cu alloys was studied for the first time by Beton and Rollason[6] through a study of the changes in hardness on reversion of the aged alloys. Later, Borelius and Larsson[7] established the metastable phase boundary for G.P. Zones through calorimetry in support of their earlier study[8]. Around this period Gerold and his coworkers[9-11] established and demonstrated the usefulness of the metastable miscibility gaps in aluminium-silver and aluminium-zinc systems. There have, however, been no attempts so far to confirm the earlier work on metastability in Al-Cu alloys although many techniques like tensile testing, calorimetry, resistometry, X-ray small angle scattering and others are today available for the study of the metastable solvus in alloy systems.

The present investigation was undertaken primarily to verify earlier results and incidentally to arrive at a better understanding of metastable equilibrium in the Al-Cu system. Two compositions were selected, one in the lower and the other in the higher concentration range for age-hardenable alloys. Hardness measurement was first used in an analogous way to that of Beton and Rollason[6] to verify earlier results. Later, electrical resistivity measurements were made use of for the first time

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