

Suppose that on \mathcal{H} ,

$$V(t) \rightarrow e^{iHt},$$

$$\mathcal{P} \rightarrow P,$$

$$\mathcal{J} \rightarrow T$$

where H is the Hamiltonian. Then

$$P e^{iHt} P^{-1} = e^{iHt},$$

$$T e^{iHt} T^{-1} = e^{iHt}$$

where it may be shown that no extra phases need be inserted in the above equations. The basic hypothesis required to decide the nature of P and T is that the eigenvalues of H are all nonnegative and that they are not all zero. This is a hypothesis not related to geometrical principles and may be questioned.

Now suppose that P is anti-unitary. Then the linear terms in t give

$$P(iHt) P^{-1} = iHt$$

or

$$PH P^{-1} = -H.$$

Therefore if $H\psi = |\lambda|\psi$, then

$$HP^{-1}\psi = -|\lambda|P^{-1}\psi.$$

Hence, P must be unitary.

Similarly, T must be anti-unitary.

The composition laws obeyed by P and T among themselves and with the operators which occur in theorem 3 have been discussed by Lurcat and Michel and by Wigner at the Istanbul Summer School (1964).

Let me conclude by observing that there seem to be several points regarding the implementation of discrete symmetries which are not entirely clear. In addition to those which were mentioned at the beginning of the talk, we may note here that if there were tachyons present, the spectrum of the Hamiltonian will no longer be nonnegative. Then the possibility arises of implementing for example \mathcal{P} by an anti-unitary operator. To the best of my knowledge there is no exhaustive analysis of such possibilities in the literature.

PROBLEMS IN ELECTROMAGNETIC MASS-DIFFERENCE CALCULATIONS*

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THE basic idea of the electromagnetic mass difference is best expressed in the first two sentences of the initial paper by Feynman and Speisman¹ in 1954: "Suppose all deviations from isotopic spin symmetry are due solely to electromagnetic effects. Then such things as the mass difference of charged and neutral π -mesons, and the neutron-proton mass difference would have to be just electrodynamic." The elegance of this hypo-

thesis is illustrated by the very small deviations (see Table I) in the masses of particles² belonging to the same iso-multiplet. In the last eighteen years several attempts³ have been made to calculate this mass difference but all these calculations are either incomplete or have other difficulties. In this talk we shall briefly discuss how far we have progressed, at least in principle, in calculating the electromagnetic mass differences of elementary particles.

TABLE I
Mass differences of neutral and charged particles with same isospin

Particle	Spin and parity	Isotopic spin (I)	Average mass MeV	$m_1^0 - m_1^{\pm 1} = \Delta m$ MeV	$\frac{100 \Delta m}{m_1^0}$
π^-, π^0	0^-	1	137.274	4.6943 ± 0.0037	3.41
K^0, K^+	0^-	1/2	495.815	3.95 ± 0.13	0.79
π^+, ρ	$1/2^+$	1/2	938.006	1.29344 ± 0.00007	0.14
Σ^0, Σ^+	$1/2^+$	1	1190.95	3.06 ± 0.16	0.26
Σ^-, Σ^0	$1/2^+$	1	1194.91	4.86 ± 0.06	0.41
Σ^-, Σ^+	$1/2^+$	1	1193.38	7.92 ± 0.13	0.66
Ξ^-, Ξ^0	$1/2^+$	1/2	1318.0	6.6 ± 0.7	0.50
K^{*0}, K^{*+}	1^-	1/2	891.7	6.1 ± 1.5	0.68
ρ^0, ρ^+	1^-	1	765	2.4 ± 2.1	0.31
Δ^0, Δ^{++}	$3/2^+$	3/2	1236	2.9 ± 0.85	0.23
Δ^-, Δ^{++}	$3/2^+$	3/2	1236	7.0 ± 0.8	0.64
Y^{*0}, Y^{*+}	$3/2^+$	1	1385	0.3 ± 2.0	0.45

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Before proceeding with the theoretical aspects, let us note some phenomenological points. Except for pion all mass differences are less than 0.8%. The percentage mass differences of the observed particles are largest for the spin zero particles. Further π^+ has a higher mass than π^0 , but K^+ has a lower mass than K^0 ! There is however no real problem with the mass difference of Σ^+ and Σ^- because, for example, their form factors are not expected to be the same. We observe a general regularity of the masses of the elementary particles: all observed spin $\frac{1}{2}$ particles obey the following inequality:

$$m_{I^0} > m_{I^{\pm 1}} \quad (1)$$

where m_{I^Q} denotes the mass of a particle with charge Q and iso-spin I . To order α one can write $\delta m_{I^Q} = a + b I_3 + c I_3^2$; our observation then tells us that b has to be negative, and puts a condition on c . If we propose Eq.(1) as a law of nature for all particles, then we have to regard π^+ (and perhaps ρ^-) as an antiparticle like K^- and \bar{K}_0 .

The problem of e.m. mass-difference calculations has been highlighted by the fact that Feynman's hope of explaining the main contribution of the mass-difference in a simple way, namely by the Born term (i.e., $N \rightarrow N\gamma \rightarrow N$) with suitable e.m. form factors, did not materialize. The nucleon pole produces a result ~ -0.8 MeV (a wrong sign!) for the n - p mass difference. Similarly this idea does not work for the $K^+ - K^0$ mass difference⁴, but it produces a reasonable value¹ for the $\pi^+ - \pi^0$ mass difference! It may be thought that the low-lying nucleon resonances (i.e., $N \rightarrow N\gamma^* \rightarrow N$) will contribute enough to reverse the wrong sign of the nucleon pole result. But only $\Delta I = \frac{1}{2}$ resonances can contribute to n - p mass difference, and it is found that they contribute very little, destroying all hopes of any simple explanation.

The electromagnetic mass difference (δm) calculations are done with the belief that even if one may not be able to predict the self-masses of the particles (which could be divergent in the theory!), their mass differences are finite and calculable. It is in this context that one works with quantum electrodynamics, which is the most successful theory we know of. The expression for δm in second order is then given by

$$\delta m = (2\pi)^{-3} \alpha i \int d^4q \frac{g^{\mu\nu} T_{\mu\nu}(\nu, q^2)}{q^2 + i\epsilon} \quad (2)$$

where $T_{\mu\nu}(\nu, q^2)$ is the difference of the forward spin-averaged virtual Compton amplitudes of the two particles whose mass difference is being calculated. q^2 is the photon momentum squared and ν is its energy in the laboratory frame. This

expression for δm involves an integration over all q^2 -both spacelike and timelike. We can obtain some information on the spacelike photon amplitude from electron-scattering experiments, but we have, at present, no way of obtaining data of the timelike photon Compton amplitude. Thus the main problem of calculating the e.m. mass difference this way is how to express Eq. (2) in terms of measurable quantities.

An important development in δm calculations was made by Cottingham⁵ in 1963 when he showed that one can perform a Wick rotation ($q^0 \rightarrow iq^0$) and express δm in terms of $T_{\mu\nu}$ with spacelike photons ($q^2 = -q_0^2 - \vec{q}^2 < 0$) but imaginary energy. With this trick the problem of calculating e.m. mass difference reduces to the problem of knowing the dispersion relations for the Compton amplitude as a function of q_0 for a fixed spacelike q^2 . Although this method has been widely used in the last nine years to evaluate δm there are two problems here. First, one has to establish that the outer circle at $|q_0| = \infty$ in the Wick rotation does not contribute anything to δm . This is related to the asymptotic behavior of the Schwinger term and no clear result has been established here⁶. Even if we forget this, the second problem is more serious and in fact limits its applicability. This is due to the fact that Cottingham's method demands the use of fixed mass dispersion relations (FMDR) and, then, Regge theory dictates a subtraction term⁷ in the virtual forward Compton amplitude (FCA). This has been further complicated by the recent indications that fixed poles are present in the Compton amplitude⁸. Thus unless we have a complete prescription of calculating the subtraction term for all q^2 , Cottingham's method cannot be used to evaluate δm .

With the discovery of scaling in the inelastic e - p and e - D scattering, the mass-difference problem received a new dimension. It has been shown^{9,10} that if the structure functions $W_1(\nu, q^2)$ and $\nu W_2(\nu, q^2)$ of the virtual forward Compton amplitude scales, we obtain divergent contributions from the large q^2 and ν part of the integral in Eq. (2). A finite δm would then require a very special cancellation of the terms involving the subtraction constant and the Bjorken scaling functions. Suppose in the Bjorken limit $W_1(\nu, q^2) \rightarrow F_1(\xi) - H_1(\xi)/\nu + \dots$ and $\nu W_2(\nu, q^2) \rightarrow F_2(\xi) + \dots$, where $\xi = -q^2/2\nu$. Then we must have

$$\int_0^1 \frac{d\xi}{\xi^2} [F_1^p(\xi) - F_1^n(\xi)] = 0 \quad (3)$$

in order that there be no quadratic divergence ($F_1 = F_2 - 2\xi F_1$),

and

$$\int_0^1 d\xi [F_2(\xi) + 2H_1(\xi)]^{p-n} = 0 \quad (4)$$

in order that there be no log divergence in δm . The inelastic $e-p$ scattering data indicate that $F_1^p(\xi)$ is perhaps zero, thereby suggesting that there is no quadratic divergence even in the self-energy of the proton. The data also show that $F_2^p(\xi) \neq F_2^n(\xi)$ and, therefore, to cancel the log divergence we must have a non-zero $H_1^p - H_1^n(\xi)$ for some ξ . But we have no data on $H_1(\xi)$ and hence no serious calculation of δm^{n-p} can really be done at present.

A new formalism for calculating δm has recently been proposed by us¹¹ where the problems mentioned in Cottingham's method have been avoided and the scaling phenomena properly taken into account. This has been made possible by employing two different kinds of dispersion relations which follow from causality of the e.m. current commutator, and not using a Wick rotation and FMDR. We first rewrite Eq. (2) in terms of the commutator $C(x) = \langle p | [j^\mu(x), j_\mu(0)] | p \rangle$ in configuration space,

$$\delta m = 2ia \int_0^\infty dt \int_0^t dr \frac{r}{r+t} C(t, t^2 - r^2) \quad (5)$$

and note that as $x^2 \rightarrow 0$ there will be divergent contributions arising from the light cone (l.c.) singularities of $C(x)$. We therefore separate the terms which are singular on the l.c. by dividing C into two parts C_s and C_R such that (i) both C_s and C_R are causal, Lorentz invariant and odd, (ii) C_R vanishes on the surface of the light cone, (iii) $\tilde{C}_R(\nu, q^2)$, the Fourier transform of C_R does not contain any singularity at $q^2 = 0$ and (iv) C_s must reproduce the high energy behaviour of the real FCA so that $\nu \tilde{C}_R(\nu, q^2 = 0)$ becomes superconvergent. These conditions leave the choice of C_s rather arbitrary, which is perhaps a blessing. Since C_s contains all the l.c. singularities, this will also include the divergent terms in the scaling region previously discussed. Eqs. (3) and (4) must now be satisfied and then the contribution of C_s to δm , denoted by δm_s is finite. To express the contribution of the rest, namely of $C - C_s = C_R$ in terms of measurable quantities, we now use two dispersion relations for $\tilde{C}_R(\nu, q^2)$. These are fixed

→ q and fixed ξ dispersion relations, both of which are unsubtracted^{10,12}. We finally obtain

$$\begin{aligned} \delta m = \delta m_s - \frac{1}{\pi} \int_0^\infty d\nu \nu \text{Re} \{f_1(\nu) - f_1^s(\nu)\} \\ + \frac{a}{\pi} \int_0^1 \frac{d\xi}{\xi} \int_0^\infty d\nu (\xi + \nu) C_R \\ \times (\nu - 2\xi\nu - \xi^2), \end{aligned} \quad (6)$$

where $f_1(\nu)$ is the real spin non-flip FCA, $f_1^s(\nu)$ the contribution of C_s to this amplitude, and the third term can be obtained from electron-scattering experiments. Note that the second integral is finite due to our condition (iv) as are all the terms in this expression.

It is not difficult to construct a C_s which will satisfy all the conditions mentioned above. We give here one example:

$$\begin{aligned} \tilde{C}_s(\nu, q^2) \\ = \int_0^\infty dS \frac{\sigma(S)}{q^2 - S} \left[\frac{3q^2}{\nu} H_1\left(\frac{S - q^2}{2\nu}\right) \right. \\ \left. + \left(2\nu + \frac{q^2}{\nu}\right) F_2\left(\frac{S - q^2}{2\nu}\right) \right] \dots \end{aligned} \quad (7)$$

where the spectral function $\sigma(S)$ is normalized to unity. We then have

$$\begin{aligned} \text{Re } f_1^s(\nu) = -a \int_0^\infty dS \sigma(S) P \int_0^1 d\xi \\ \times \frac{F_2(\xi)}{\xi^2 - (S/2\nu)^2}. \end{aligned} \quad (8)$$

With a suitable choice of $\sigma(S)$ one can now easily reproduce the asymptotic FCA. Let us mention another particular choice (for details see ref. 11) for which the formula simplifies considerably. Suppose there exists a causal function $C_{int}(\nu, q^2)$ which not only accounts for the behaviour of G in the deep inelastic region but also interpolates C in the resonance region, then one can choose this C_{int} to be C_s thus causing the last term in Eq. (6) to vanish.

This, however, does not exhaust the problems facing a δm calculation. Last, but not least, δm calculations can be ambiguous because of the ambiguity of the structure functions $W_i(\nu, q^2)$ represented by terms of the type $\epsilon(\nu) \sum_n a_n \rho_n(q^2) \theta(q^2)$, which do not contribute for spacelike q^2 . It was first pointed out by Leutwyler and Otterson¹³ that even if these polynomials do not contribute to the scaling region they can give a finite contribution to the self-energy. To see this in a simple way let us consider a commutator $C_p(\nu, q^2) = -3q^2 V(\nu, q^2)$,

where V satisfies a DGS representation with a spectral weight $\sigma(S, \beta)$ ($0 \leq S \leq \infty, -1 \leq \beta \leq 1$). Suppose now that σ has a singularity at $\beta=0$ and consider the case when $\sigma(S, \beta) = \rho(S) \delta(\beta)$. We then obtain $V(\nu, q^2) = \epsilon(q^0) \rho(q^2) \theta(q^2)$ and

$$V(x) = -i \int_0^\infty dS \Delta(x, S) \rho(S). \quad (9)$$

If the first few moments of the spectral function vanish, then there will be no contributions to the light cone singularities as is easily seen by expanding $\Delta(X, S)$. But we shall still obtain a finite contribution to δm given by

$$\delta m_p = \frac{3a}{8\pi} \int_0^\infty dS S \ln S \rho(S). \quad (10)$$

In Cottingham's method this is buried in contributions from the subtraction term, and in our method in the use of light cone dispersion relations with bad test functions. Thus even if we assume that there are no δ -function singularities in the scaling region it is not possible to calculate δm without making some additional assumption about these polynomials. We may now ask, is it possible to use the freedom inherent in the choice of C_s so that this ambiguity can be eliminated? This work is in progress and it seems that if C_{int} exists the answer is yes.

Let us now assume that nature is really simple and somehow the polynomials mentioned above do not contribute to mass difference (an obvious way out would be to speculate that these are isoscalars). A numerical calculation of δm is then in principle possible using our formalism. However, a grievous lack of data does not allow us even to make a semi-model-independent calculation of $\delta m^{\pi-\nu}$. The crucial issues are the extraction of data for the neutron from the deuteron experiments and obtaining accurate data in the scaling region, especially for $H_1(\xi)$. In our formalism the contribution of the real part of the Compton amplitude is very important. But this is extremely difficult to obtain for a wide range of energy and with any confidence. The present data are such that if we write $\text{Re } f_1^{\pi-\nu}(\nu) = A - B\sqrt{\nu}/4\pi + \dots$ for large ν then B could be anywhere between 18.3 to $6.5 \text{ GeV}^{\frac{1}{2}} \mu b$. In view of these numerous difficulties we only remark that the theoretical magnitude and sign of $\delta m^{\pi-\nu}$ is still an open question. At the same time the success of a simple calculation for $\delta m^{\pi^+-\pi^0}$ may be misleading.

Several attempts have been made in the past to develop a finite quantum electrodynamics with an indefinite metric. Here one introduces other

"shadow" particles in the theory with the same electromagnetic coupling. Depending on one's philosophy one then changes the propagators for the photon¹⁴, the electron¹⁵, the mesons or all of them from their usual forms to $-m_B^2/q^2 (q^2 - m_B^2)$, $\frac{(m_1 - m_2)(m_2 - m_3)}{\prod_{i=1}^3 (p_i' - m_i - i\epsilon)}$ or $(\mu^2 - \Lambda^2)/(q^2 - \mu^2)$, $(q^2 - \Lambda^2)$, etc., thus obviously achieving an appropriate amount of damping needed for a finite self-energy. For a numerical calculation of δm in these theories, now, we have not only to determine these unknown mass-parameters, but also have to face the same difficulties mentioned earlier (except for the divergences). Further the scattering of the "shadow" particles, which are additional complications, will also affect the mass-difference calculations.

Let us now discuss the formalism of Dashen and Frautschi¹⁶ who developed an S-matrix method to evaluate, e.m. corrections to strong interactions. The basic philosophy behind this bootstrap method is essentially the same. In this approach the mass shift of the i -th particle is basically given by¹⁷

$$\delta m_i = D_i + \delta n_j A_{ji} + \delta g_{ij} B_i \quad (11)$$

where D_i is the driving term containing diagrams with one photon exchange and the rest, the feedback terms represent the mass shifts due to changes in all other physical quantities, e.g., the masses and the coupling constants. Now in an approximate evaluation of the mass shifts there is a logarithmic infrared divergent contribution arising from the one-photon exchange term. The infrared divergence should be absent in an exact calculation. Therefore the infrared divergence of the photon exchange term has to be removed. The proper method to remove this divergence is the most important unsolved problem in this approach. Another problem is the lack of sufficient data to be used as inputs for a mass shift calculation. These include the changes in the masses, coupling constants, etc.

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WEAK INTERACTIONS*

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WEAK interactions is an exciting area to work in, as every few years, there is an unexpected surprise. For example parity violation was detected in 1957, CP violation in 1964, and $K_L \rightarrow \mu^+ \mu^-$ puzzle recently, which may lead to a revision of our ideas of weak interaction, especially CPT invariance. Further, although weak interactions seem to violate symmetries, they do possess a universality, and in fact they may also possess the greatest symmetry of all.

Weak interactions have numerous unsolved problems, and I shall mention a few of these, and then concentrate on one particular one, namely a finite theory of weak interactions. The type of problems fall roughly into two categories, those that should be explained on the known theory (i.e., V-A current-current interactions) and those that lie outside it. In the first category are:

1. Non-leptonic Weak Decays.
2. Intermediate Weak bosons?
3. Renormalizable theory.

Among the second we have,

1. CP violation,
2. $K_L \rightarrow \mu^+ \mu^-$ puzzle.

The second category, although interesting, I shall not spend much time on. CP violation in $K_L \rightarrow \pi \pi$ decay is well established and good values have been obtained for the phenomenological constants ϵ and ϵ' that characterize the decays. However,

no very basic theory has emerged. As for $K_L \rightarrow \mu^+ \mu^-$ decay, it was shown that if one assumes CP conservation then the branching ratio should be

$$R = \frac{K_L \rightarrow \mu^+ \mu^-}{K_L \rightarrow \text{all}} \geq 6 \times 10^{-9}$$

while experimentally $R < 2 \times 10^{-9}$.

If CP is given up, one can show that $K_S \rightarrow \mu^+ \mu^-$ has well-defined theoretical upper and lower bounds. The bound $K_S \rightarrow \mu^+ \mu^- / K_S \rightarrow \text{all} > 2 \times 10^{-7}$ is being explored experimentally, and one is on the verge of violating it. If this happens, either CPT or unitarity will be in trouble!

Coming back to the 1st category of problems, let us review the theory.

The present theory of weak interaction was proposed by Sudarshan and Marshak, and Feynman and Gell-Mann in (1958) and can be written as

$$H_w = \frac{G}{\sqrt{2}} \{J_\mu^+, J_\mu\}$$

where $J_\mu = V_\mu - A_\mu + l_\mu$, where V and A are hadronic currents of vector and axial type, while l_μ is leptonic current. Cabibbo has extended this to include strangeness changing decays symbolically:

$$J_\mu = \cos \theta (V - A) \Delta^{1=0} + \sin \theta (V - A) \Delta^{1=1} + l_\mu.$$

This theory is very successful in accounting for leptonic and semi-leptonic decays like

$$\begin{aligned} \mu^- &\rightarrow e^- + \bar{\nu}_e + \nu_\mu \\ n &\rightarrow p + e^- + \bar{\nu}_e \\ A &\rightarrow p + e^- + \bar{\nu}_e \end{aligned}$$

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