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WEAK INTERACTIONS*

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WEAK interactions is an exciting area to work in, as every few years, there is an unexpected surprise. For example parity violation was detected in 1957, CP violation in 1964, and $K_L \to \mu^+ \mu^-$ puzzle recently, which may lead to a revision of our ideas of weak interaction, especially CPT invariance. Further, although weak interactions seem to violate symmetries, they do possess a universality, and in fact they may also possess the greatest symmetry of all.

Weak interactions have numerous unsolved problems, and I shall mention a few of these, and then concentrate on one particular one, namely a finite theory of weak interactions. The type of problems fall roughly into two categories, those that should be explained on the known theory (*i.e.*, V–A current-current interactions) and those that lie outside it. In the first category are:

1. Non-leptonic Weak Decays.
2. Intermediate Weak bosons?
3. Renormalizable theory.

Among the second we have,

1. CP violation,
2. $K_L \to \mu^+ \mu^-$ puzzle.

The second category, although interesting, I shall not spend much time on. CP violation in $K_L \to \pi\pi$ decay is well established and good values have been obtained for the phenomenological constants ϵ and ϵ' that characterize the decays. However, no very basic theory has emerged. As for $K_L \to \mu^+ \mu^-$ decay, it was shown that if one assumes CP conservation then the branching ratio should be

$$R = \frac{K_L \to \mu^+ \mu^-}{K_L \to all} \geqslant 6 \times 10^{-9}$$

while experimentally $R < 2 \times 10^{-9}$.

If CP is given up, one can show that $K_S \to \mu^+ \mu^-$ has well-defined theoretical upper and lower bounds. The bound $K_S \to \mu^+ \mu^- / K_S \to all > 2 \times 10^{-7}$ is being explored experimentally, and one is on the verge of violating it. If this happens, either CPT or unitarity will be in trouble!

Coming back to the 1st category of problems, let us review the theory.

The present theory of weak interaction was proposed by Sudarshan and Marshak, and Feynman and Gell-Mann in (1958) and can be written as

$$H_W = \frac{G}{\sqrt{2}} \{J_\mu^+, J_\mu\}$$

where $J_\mu = V_\mu - A_\mu + l_\mu$, where V and A are hadronic currents of vector and axial type, while l_μ is leptonic current. Cabibbo has extended this to include strangeness changing decays symbolically;

$$J_\mu = \cos\theta\,(V - A)^{\Delta S=0} + \sin\theta\,(V - A)^{\Delta S=1} + l_\mu.$$

This theory is very successful in accounting for leptonic and semi-leptonic decays like

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$
$$n \to p + e^- + \bar{\nu}_e$$
$$\Lambda \to p + e^- + \bar{\nu}_e.$$

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However, there are difficulties with non leptonic decay like

$$\Lambda \rightarrow p + \pi^-$$
$$K_1 \rightarrow \pi^+ + \pi^-.$$

Here the rate comes out wrong if you use the theory naively.

Instead if one retains only the $\Delta I = \frac{1}{2}$ part of the current-current theory and drops the sin θ factor, one gets a better agreement. $\Delta I = \frac{1}{2}$ rule can also be obtained by adding terms to H_w that involve only hadronic currents. This seems rather *ad-hoc*. One may also invoke octet enhancement, though it is not clear how one quantitatively establishes this on the present theory. A Hamiltonian that works rather well is due to Sakurai,

$$H_w = \frac{G}{\sqrt{2}} d_{8\alpha\beta} J_\alpha{}^\mu J_\beta{}^\mu.$$

Relation of this to the original H_w is not clear. This then constitutes an unsolved problem.

The hypothesis of intermediate boson in weak interactions is an old one. In electromagnetism we have a field associated with the force (*i.e.*, photon). In strong interaction, although we do not have a final theory, most low energy scattering can be accounted for by means of forces generated by meson exchange. In analogy the weak interaction is supposed to be mediated by a field (intermediate weak boson).

Most people believe this to be a spin one boson. The effective weak interaction is then

$$H_{eff.} = \frac{g^2}{(2\pi)^4} \int d^4q \frac{\left(g_{\mu\nu} + \frac{q_\mu q_\nu}{m_w{}^2}\right)}{q^2 + m_w{}^2} \times \int e^{iqx}\, TJ_\mu(x)\, J_\nu(0)\, d^4x.$$

For $m_w{}^2 \gg q^2$ we have the usual theory of

$$\frac{G}{\sqrt{2}} = \frac{g^2}{m_w{}^2}.$$

The intermediate beson can also be spin O, if the theory is described by Kemmer-Duffin theory.

$$\mathcal{L} = -\bar{\psi}(\beta_\mu \partial^\mu + m)\psi + g\bar{\psi}\beta_\mu{}^\mu \psi J^\mu$$

where β_μ are 5×5 matrices, and for free fields

$$\psi_a = \left(\delta_{a\mu} \frac{\partial_\mu \phi}{\sqrt{m}} + \delta_{a4}\phi\sqrt{m}\right) \quad \begin{matrix} a = 0, 1, 2, 3, 4 \\ \mu = 0, 1, 2, 3. \end{matrix}$$

The effective Hamiltonian is

$$H_{eff.} = \frac{g^2}{(2\pi)^4} \int d^4q \left(g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2 + m_w{}^2}\right) \times \int e^{iqx}\, TJ_\mu(x)\, J_\nu(0)\, d^4x,$$

However, these theories are as divergent as the basic Hamiltonian, and thus are no improvement over contact interaction. The offending term in the vector boson propagator is the $q_\mu q_\nu / m^2$. If this can be got rid off by some means and replaced by $q_\mu q_\nu / q^2$, then the theory would be well behaved. Such propagators come in gauge theories like electromagnetism. However, gauge invariance normally causes masslessness of the vector field. Is there a way to have a guage theory, and at the same time massive particles? It turns out if you have a guage invariant theory, with the gauge symmetry *spontaneously broken*, then vector particles can have a mass, at the same time preserving the renormalizability of the theory. To see how this is done we take a simple example of a theory with U(1) symmetry.

Consider the Lagrangian

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - h(\phi^* \phi)^2$$

which is invariant under the transformation $\phi \rightarrow e^{i\alpha}\phi$. For $m^2 < 0$ the Lagrangian has spontaneously broken solutions characterized by ground state given by $\partial V / \partial \phi = 0$, where $V = m^2|\phi|^2 + h|\phi|^4$. Thus $\langle \phi \rangle \neq 0$ and it is easy to see that the solution has two fields ϕ_1 and ϕ_2, one is massless and the second with positive definite mass.

If now a vector gauge field is introduced, we have a new gauge invariant Lagrangian

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu}{}^2 + (D_\mu \phi)^* D^\mu \phi - m^2 \phi^* \phi - h(\phi^* \phi)^2$$

where

$$D_\mu = \partial_\mu - ieA_\mu.$$

Now one can make a rotation,

$$\phi = \frac{\rho}{\sqrt{2}} e^{i\theta(x)}$$
$$B_\mu = A_\mu + \frac{1}{e} \partial_\mu \theta(x)$$

and write

$$\mathcal{L} = -\tfrac{1}{4} B_{\mu\nu}{}^2 + \tfrac{1}{2}(\partial_\mu \rho)^2 + \tfrac{1}{2} e^2 (B_\mu)^2 - \tfrac{1}{2} m^2 \rho^2 - \tfrac{1}{4} h \rho^4$$

this theory for $m^2 < 0$, again has $\langle \rho \rangle \neq 0$, and defining $\rho' = \rho - \langle \rho \rangle$, one gets a theory with massive vector particle, and a massive scalar particle. The field $\theta(x)$ has completely disappeared.

The new model thus obtained is still renormalizable. The method can be used to make models of weak interactions. An example is a model of leptons with just weak interactions based on SU(2) group.

Define

$$L = \frac{(1 + \gamma_5)}{2} \begin{pmatrix} \nu_e \\ e \end{pmatrix}$$

$$R = \left(\frac{1 + \gamma_5}{2}\right) e^-,$$

Gauge field $\vec{W}_\mu$ and scalar doublet

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$\mathcal{L} = -\bar{L}\gamma_\mu D^\mu L - \bar{R}\gamma_\mu D^\mu R - \tfrac{1}{4}(\vec{W}_{\mu\nu})^2 + \tfrac{1}{2}|D_\mu\phi|^2 - G_e(\bar{L}R\Phi) + \bar{R}\phi^+ L) - m^2|\phi|^2 - h|\phi|^4$$

where

$$D_\mu = \partial_\mu - ig\,\vec{t}\cdot\vec{W}_\mu.$$

For $m^2 < 0$ the spontaneous broken solutions of this model leads to the usual weak interactions, and an additional $e + \nu \to e + \nu$ scattering due to massive $W_\mu{}^\circ$. Weinberg has constructed models including electromagnetism and strong interactions. Such models are currently under investigation.

GRAVITATIONAL THEORY IN THE PRESENCE OF SPINORS *

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ABSTRACT

The theory of spinor fields in interaction with gravitation is described. The massless Dirac equation is modified by the interaction and becomes identical in form with the nonlinear spinor equation that proves the basis of Heisenberg's Unified Field Theory. When applied to electromagnetism the theory leads to an unresolved difficulty connected with charge conservation and the spin-density of electromagnetism.

IN special relativity, the Lorentz transformations, interpreted as *rigid rotations of a Cartesian* coordinate system in flat specetime, are fundamental; the basic equations of physics are constructed in such a way that their form is independent of the choice of Cartesian coordinate system. That is they are required to be invariant under Lorentz transformations. Thus the most general admissible equation is a *spinor* equation.

In the presence of gravitation the introduction of a Cartesian reference system is no longer possible, we are forced from the outset to work with general (curvilinear) coordinates. The requirement of Lorentz invariance has to be abandoned in favour of general covariance. The representations of general coordinate transformations are tensor densities; the generalisation of a Lorentz-invariant tensor equation to a generally covariant equation is straightforward, and simply consists of assigning 'weights' to tensor field and replacing derivatives by covariant derivatives. However, there is no such procedure that can be applied to spinor equations. Thus, when Dirac discovered the special-relativistic equation for the electron, which is a spinor equation, the problem immediately arose of how do describe mathematically an electron in a gravitational field. Can we assign a meaning to a spinor when spacetime is not flat, and can we construct a generally covariant Dirac equation?

The solution given by Schrödinger was to regard a spinor as *invariant* for coordinated changes, and to regard the Lorentz group that is represented by Spinor transformations as the group of rotations of a set of orthonormal reference vector fields, not a coordinate transformation. Thus, choosing four orthonormal vectors $h_a{}^\mu$ $(a = 1, 4)$ at each spacetime point (a 'tetrad'),

$$h_a{}^\mu h_a{}^\nu g_{\mu\nu} = \eta_{ab},\ \eta_{ab} = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix}$$

and

$$h_\mu{}^a h_\nu{}^b \eta_{ab} = g_{\mu\nu}$$

where $h_\mu{}^a$ is the inverse of the matrix $h_a{}^\mu$. Change to a new tetrad is given by

$$\bar{h}_a{}^\mu = \Lambda_a{}^b h_b{}^\mu$$

where $\Lambda_a{}^b$ is a (spacetime-dependent) Lorentz matrix. Let γ^a be the four usual Dirac matrices. Then $\gamma^\mu = h_a{}^\mu \gamma^a$ satisfy

$$\tfrac{1}{2}(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu) = g^{\mu\nu}$$

and the spinor representation L of the Lorentz matrix Λ is given by

$$L^{-1}\gamma^a L = \Lambda_b{}^a\gamma^b.$$

The derivative $\partial_\mu\psi$ of a spinor transforms under tetrad rotation to

$$L\partial_\mu\psi + (\partial_\mu L)\psi$$

so that a spinor connection Γ_μ has to be introduced, with transformation law defined so that the 'covariant' derivative $\psi_{;\mu} = \partial_\mu\psi - \Gamma_\mu\psi$ transforms simply to $L\psi_{;\mu}$. The equation $\gamma^\mu\psi_{;\mu} + m\psi = 0$ is

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