That is, in the presence of gravitation the massless Dirac equation is expected to take the form

$$\gamma^{\mu}\psi_{1\mu} + (3k/8)\,\gamma^{\sigma}\gamma^{5}\psi\,.\,\bar{\psi}\gamma_{\sigma}\gamma_{5}\psi = 0.$$

It is remarkable that this has precisely the same form as the nonlinear spinor equation suggested by Heisenberg as the basis of his unified theory of elementary particles[5]. $3\kappa/8$ is Heisenberg's l_0^2 (thus $l_0 \sim 3 \times 10^{-33}$ cm).

For a vector field (omitting the mass term),

$$\mathcal{L} = -\tfrac{1}{4}\, h g^{\mu\nu} g^{\rho\sigma} f_{\mu\rho} f_{\nu\sigma}$$

where

$$f_{\mu\nu} = A_{\mu/\nu} - A_{\nu/\mu}$$
$$= A_{a\cdot\nu} h_{\mu}{}^{a} - A_{a\cdot\mu} h_{a}{}^{\nu} + A_{a}\,(\lambda_{\mu}{}^{a}{}_{\nu} - \lambda_{\nu}{}^{a}{}_{\mu}).$$

From the terms containing spin coefficients it is easy to pick out the spin tensor

$$\mathfrak{S}_{\mu\nu\rho} = f_{\mu\rho} A_{\nu} - f_{\nu\rho} A_{\mu}.$$

This is exactly the form of the spin tensor for electromagnetism deduced from the application of Noether's theorem in special relativity. However, this is not satisfactory, since the tensor $f_{\mu\nu}$ constructed here is not charge invariant. Thus in such a theory charge would not be conserved. The only way to retain charge conservation seems to be to construct the $f_{\mu\nu}$ in the Lagrangian from ordinary derivatives, in the usual way. But this gives a zero spin tensor for the spin-1 Maxwell field. Thus we have an unresolved anomaly when we try to formulate electromagnetism in the generalised theory.

The major unsolved problem in any theory which treats the Lorentz transformations as tetrad rotations rather than as coordinate transformations lies in the fact that we lose the fundamentally important Poincaré group (Inhomogeneous Lorentz group). The invariance group of the above generalised theory is a direct product of the (space-time dependent) homogeneous Lorentz group and the general coordinate transformation group. The invariance group of special relativity is the semi-direct product of the Lorentz group and the abelian translation group. Thus it is difficult to see how such a theory can be reconciled with special relativity—how the Poincaré—invariant formulation of particle physics can be regarded in some sense as a limiting case of a curved spacetime theory with a tetrad field.

Perhaps the most remarkable aspect of the generalised theory of Sciama is that the $h_a{}^{\mu}$, which is an aspect of the *reference system*, has been treated as a *dynamical field*. Nevertheless, no restriction on the tetrad is implied by this procedure (this is to be contrasted with the work of Møller, in which only tetrad fields satisfying a set of differential equations are admissible reference systems). The sharp distinction between the physics and the system of reference used to facilitate the description of the physics has been abandoned. This is true to a lesser extent in conventional general relativity: The physical fields $g_{\mu\nu}$ contain information about the coordinate system as well as information about the geometry, and the two kinds of information are quite incapable of separation in any given set of ten functions $g_{\mu\nu}$.

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"UNSOLVED PROBLEMS" IN QUARK MODEL AND HADRON SPECTRA*

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THE identification of unsolved problems in any area requires a much broader perspective and insight than is usually available to a worker pursuing a particular form of approach. Moreover, an extrapolation to the unexplored regions is inseparably tied up with the historical approach whose limitations (due to falling short of the most "logical" approach) are bound to affect the perspectives for the future. Further, an extrapolation to unsolved problems through a phenomenological level of investigation is considerably harder than through a more fundamental theoretical approach. It is with these two limitations that I venture to talk on unsolved problems in the area of resonance physics, after first offering a panoramic view of the major developments in the field.

The subject is less than a decade old. The experimental developments have been mostly in

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terms of new "bumps" in reactions, measurements of their $J^P I^G$ values, widths and branching ratios, and their classification according to theoretical ideas. The theoretical developments, which really started with SU (3) in 1961, witnessed the startling proposal of "quarks" in early 1964, which was immediately followed by SU (6), by the end of 1964. The successes of SU (6) and quarks are now old stories. Extension of this group to SU (6) XO (3), to take care of the "internal excitations", provided an elegant scheme for higher baryon states. The quark model furnishes a natural language for these internal (orbital) excitations, and has been developed considerably in terms of the harmonic oscillator model of Q–Q interactions. The classification of super multiplets in this model is represented by the symbol [SU(6); L^P; N], where N is the principal quantum number, L is the internal orbital momentum and P the parity; SU(6) stands for any one of the representations *56, 70* or *20*.

This classification has had some striking experimental successes through the identification of almost complete super multiplets with low L-values. All such states discovered so far fit in with the pattern:

(A) [*56*, $2l^+$], [*70*, $(2l+1)^-$], (plus radial excitations).

Other possible candidates (with feeble evidence) are the complementary set.

(B) *56*, $(2l+1)^-$], [*70*, $2l^+$].

However, the *20* representations do not seem to be in evidence so far, despite their predicted proximity to some of the above super multiplets of *56* and *70*.

What would be the possible significance of discovery of a *20* state, as an appropriately excited representation in the baryon spectrum? I think it is no exaggeration to say that such a discovery would constitute a powerful support for the *dynamical* quark model with H.O. interactions. In that event, one of the major unsolved problems in this field,—Bose *versus* para statistics for quarks—would have to be activised. (This question would have had a greater chance to remain in academic obscurity if the observed baryon mass spectrum were not so strongly linked with the harmonic oscillator predictions.) The shape of the nucleon's form factor would also point to the same dilemma on the statistics of quarks, in case these happen to be physical entities.

To carry the dynamical interpretation a step further (in the event of quarks being physical entities), the existence of only *56* and *70* states would imply even more "weird" structures for Q Q parirs, suppressing their interaction in all but *s*-wave states. Indeed, a purely group theoretical prediction of *56* and *70* states (without a dynamical role for quarks) seems to be rather unlikely, for one really does not know if the successive Regge recurrences implied by sets (A) and (B) can at all be described by some (non-compact) group, in preference to, *e.g.*, a dynamical manifestation of some sort of truncated QQ interactions, emphasizing merely *s*-wave interactions. What makes the picture of only *56* and *70* states (A) and (B) more appealing to theory is that it is compatible with the general requirement of exchange degeneracy, a highly desirable theoretical constraint.

To summarize the conclusion of the two previous paragraphs, a sort of dynamical interpretation, either in terms of H.O. potentials, or in terms of *s*-wave forces is strongly suggested by the nature of the available baryon spectra. In each case, para statistics is also preferred to Fermi.

Another aspect of the mystery about quarks lies in the problem of their masses. The traditional way of looking at the mass spectra in terms of the H.O. model forces the familiar conclusion of massive quarks as the basic building blocks of the hadron family. On the other hand the "parton" interpretation of "scaling" in deep inelastic scattering of electrons off nucleons (which readily tempts one to identify quarks with partons) would seem to suggest that quark masses are merely a fraction of a nucleon mass. Feynman had recently expressed the view that this represents a genuine contradiction, a resolution of which would no doubt constitute a break-through.

Further difficulties appear if the quark-parton analogy is carried to its detailed experimental implications. A favourite item of comparison is the ratio of the "scaled" quantities νW_2 for protons and neutrons as a function of the scaling variable $W = 2m\nu q^{-2}$, the parton-quark model for which predicts a value lying between 0·25 and 4·0 (under the "incoherence" assumption). The experimental ratio appears almost at the edge of the lower limit and threatens to "cross" the value 1. Should this turn out to be the case, the identification of quarks with partons would be in jeopardy.

To return to hadron spectra, there are still several important problems which I have only time to list. Thus to get a meaningful contact of the theoretical ideas with experiment, the comparisons must be made at the level of various reactions, which in turn requires the formulation of couplings of higher resonances.

These couplings, which must be formulated in a relativisticallv invariant manner, are presumably characterized by the appearance of certain form factors. The problem of these relativistic couplings in a sufficiently general way, and yet with a comprehensive enough set of unifying principles is a major unsolved problem at present. However, a comparison with experiment of several two-body reactions up to reasonably high energies ($\sim$ 10 GeV, lab) at a pedagogical level, indicates that the variation of the form factors with energy could well be a valuable pointer to a more complete theory of the future. In particular, neither the predictions of the H.O. model (which gives Gaussian shapes), nor the constraints of analyticity (which lead to some sort of exponential structures) appear to be compatible with the data which seem to require a gentler fall with energy—something more akin to an inverse power law. One wonders if this indicates some "non-compact" features in the structure of form factors. (The "inverse power law" also appears to be compatible, in principle, with scaling behaviour in the triple Regge region.)

I conclude with a speculation on the status of quarks in the hierarchy of constituents of matter, assuming for the argument that they are physical entities. At this stage of development quarks represent a sort of "third stage" in the sequence

atoms $\rightarrow$ nuclei $\rightarrow$ nucleons $\rightarrow$ quarks.

The fundamental question is then the following: "Does the process stop at the quark stage, or does it continue to infinity?" The school which believes in mathematical quarks considers this question as meaningless. A subset of this school, which believes in elementary particle democracy, has a position diametrically opposite to the quark philosophy. However, if the quarks turn out to be merely a third stage in an infinite hierarchy of constituents, then it is amusing to speculate that the "quark" and "bootstrap" points of view may not be that different after all. In any case this is a big, *unsolved*, problem today.

THE GROUP-THEORETICAL ASPECTS OF QUANTUM MECHANICS *

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THE unitary unimodular groups SU(n) occupy a position of paramount importance in the present-day physics. These groups have been used extensively to classify multi-electron states of atoms and states of nuclei. To have an idea of the importance of these groups in the theory of complex spectra one need only turn over the pages of the books written by B. R. Judd[1] on the subject. For $n = 3$, 4, 6, 12, they have been used as possible symmetries of elementary particles[2]. The experimental evidence in support of the SU(3) symmetry is now overwhelming, and some of the results obtained by using this symmetry are likely to be of permanent value. This is not quite true of the higher groups, though the SU(3) transformations form a subgroup of the larger symmetry. Nevertheless, persistent efforts are being made to gain a deeper insight into the nature of the elementary particles by using the higher groups, and it has become imperative to study the structural properties of these groups in greater detail. The mathematical problems that arise in this connection are more or less familiar and can be divided into the following broad categories:

1. Construction of state vectors.
2. Determination of the matrices of the generators.
3. Determination of the representation matrices for finite transformations.
4. Classification and construction of tensor operators.
5. Structure of the Clebsch-Gordan (CG) series.
6. Evaluation of the CG coefficients.

None of these problems appears to have been solved for general values of n. For solving problem 1 one may take[3,4], following Gel'fand, $n-1$ independent sets of variables a_j^i ($i = 1, 2, \ldots, n-1$; $j = 1, 2, \ldots, n$), each set transforming according to the basic n-dimensional representation of SU(n), and put a_j^1 in the first-row, a_j^2 in the second row, and so on, in a Young frame of $n-1$ rows. The standard Young diagrams corresponding to such a frame represent the basis states of an irreducible representation (IR) of SU(n) characterized by the row lengths m_{in}. However, in physical applications one is interested in IR's which are explicitly reduced with respect to a particular chain of subgroups, say, the canonical chain SU($n-1$) $\supset$ SU($n-2$) ... $\supset$ SU(1). To obtain such representations one must form appropriate linear combinations of the standard states. This is, in general, a difficult task and has been carried out only for values of n up to 4. The

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