ON THE SHAPE AND STABILITY OF FINGERS IN A DISPLACEMENT PROCESS THROUGH A FRACTURED POROUS MEDIUM

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ABSTRACT

A specific problem of fingering occurring in an immiscible displacement process through a slightly fractured porous medium is discussed here from a statistical view point. A mathematical solution has been obtained by using a perturbation technique and an expression for the average cross sectional area occupied by the fingers is obtained. It is shown that stabilization of fingers may occur in this particular case.

INTRODUCTION

It is often important for various Engineering and Hydro-geological problems to know whether the displacing fluid forms a stable front during the displacement process or whether it will spread rapidly through the displaced fluid forming an unstable front. Instead of the displacement of the front as a whole in regular form, protuberances occur that may advance through the porous medium at velocities much higher than the average front. This instability phenomenon is known as 'fingering' and occurs when a fluid of lesser viscosity displaces another of higher viscosity.

The growth and stability of fingers in homogeneous porous media was analysed from statistical view point by Scheidegger and Johnson[1], and they found that no stabilization of the fingers is possible in the statistical theory. Subsequently many authors for instance Chouke *et al.*[2], Marle[3], Verma[4,5], Venkateswarlu[6-8] have investigated the phenomenon of fingering and specific double phase flow problems in porous, and fractured media. The basic assumption underlying the present investigation is that the porous medium is slightly fractured and the fractures are randomly oriented. To make the analysis more definite we consider that the finger flow is furnished by water displacing oil from a one-dimensional fractured medium. By using a perturbation procedure, it is shown that the stabilization of fingers may occur in this particular case.

STATEMENT OF THE PROBLEM

Consider a homogeneous porous medium saturated with oil and containing randomly oriented fractures. Water at a constant velocity (V) is injected into the fractured medium. The displacement of oil by water gives rise to a well developed system of finger flow (Fig. 1). It is assumed that due to the impact of the injecting water the entire oil on the initial boundary $x = 0$ (x being measured in the direction of displacement) is displaced through a small distance. Thus at $x = 0$, $K_0 = 0$ is the boundary condition of the problem.

STATISTICS OF FINGERS

In the statistical approach only the average cross sectional area occupied by the fingers is taken into consideration while the size and shape of the individual fingers is discarded. This treatment with the introduction of notion of fictitious relative permeability becomes identical to the Buckley and Leverett description of the immiscible double phase flow (Scheidegger *et al.*[1]). In this case, the saturation of the i-th fluid (Si) is defined as the average cross sectional area occupied by it at the level x, *i.e.*, $S_i = S_i(x, t)$. As such the saturation of the displacing fluid in the porous medium represents the average cross sectional area occupied by the fingers.

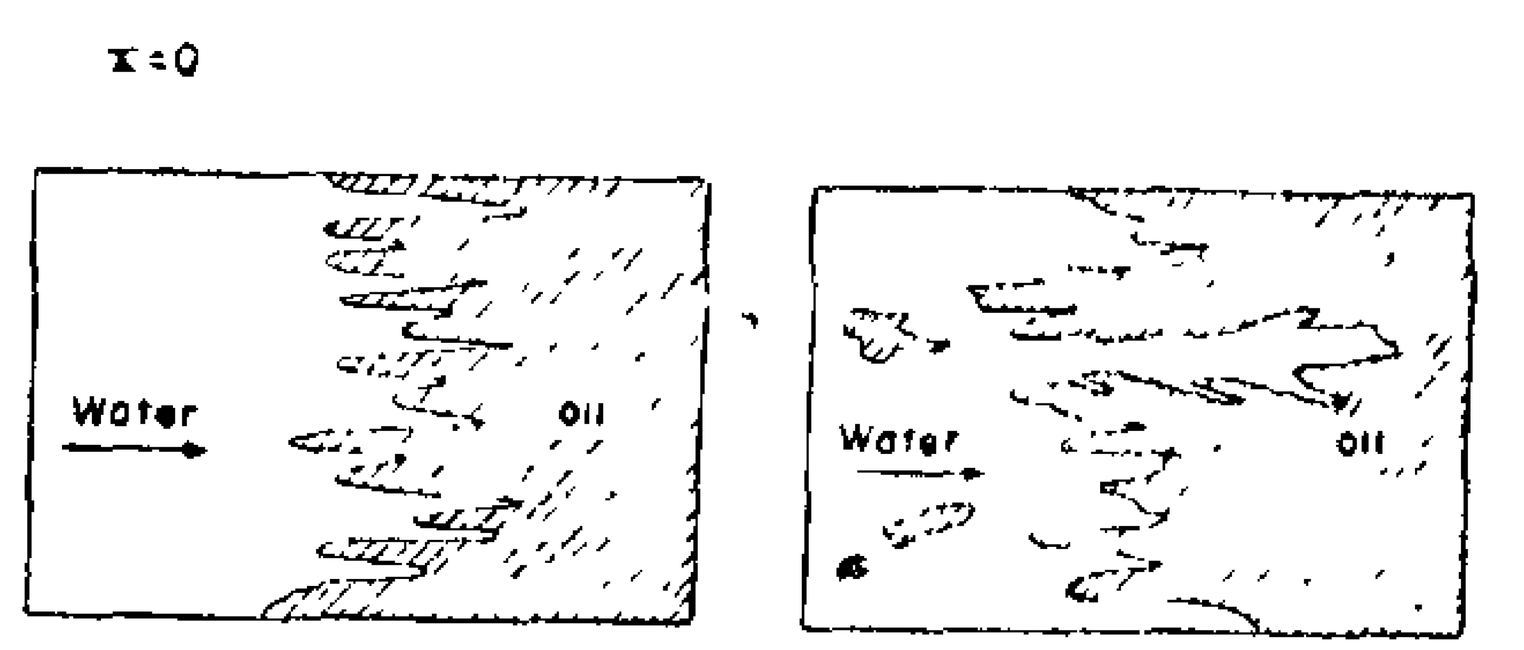

FIG. 1. Fingering in an immiscible displacement process.

Following Scheidegger and Johnson[1], we consider the fictitious relative permeability to water (K_w) and oil (K_o) as:

$$K_w = S_w, \quad K_o = S_o \tag{1}$$

where S_w and S_o denote the saturation of water and oil.

In a fractured porous medium the water entering the factures is sucked into the blocks under the capillary action. The amount of water entering the blocks in an elementary volume of the medium is known as the 'Impregnation function' $\varphi(t)$. Considering the balance of water sucked into the blocks per unit time and employing the results of

Mattax and Kyte[9], we may write the analytical value of the impregnation function $\phi(t)$ as below:

$$\phi\,[T - \tau\,(u)] = D\,(T - Rx^2)^{-\frac{1}{2}}\,;\ \tau \leqslant t \qquad (2)$$

$$u = \frac{x}{l}\,;\quad T = \epsilon t\,;\quad R = \frac{a}{l^2}$$

$$\epsilon = \frac{\sigma \cos\theta\, S^2 \sqrt{K/m_B}}{\mu_0}$$

$$a = \left(\frac{\pi A}{4q}\,\frac{S^2 g_K \sigma \cos\theta m_B \sqrt{K/m_B}}{\mu_0}\right)^2$$

The symbols are defined at the end of article.

Formulation of the Problem

Assuming the flow to be governed by Darcy's law, the seepage velocities for water (v_0) and oil (v_0) and the continuity equations for the flowing phases can be written as:

$$v_w = -\,K\,\frac{K_w}{\mu_w}\frac{\partial p}{\partial x} \qquad (3)$$

$$v_0 = -\,K\,\frac{K_0}{\mu_0}\frac{\partial P}{\partial x} \qquad (4)$$

$$m\,\frac{\partial S_w}{\partial t} + \frac{\partial v_w}{\partial x} + \phi\,[T - \tau\,(u)] = 0 \qquad (5)$$

$$m\,\frac{\partial S_0}{\partial t} + \frac{\partial v_0}{\partial x} - \phi\,[T - \tau\,(u)] = 0 \qquad (6)$$

where m and k are the porosity and permeability of the medium.

From the definition of phase saturation, it is evident that:

$$S_w + S_0 = 1. \qquad (7)$$

Combining equations 5 and 6 and using 3, 4 and 7, we have:

$$\frac{\partial}{\partial x}\left[K\,\frac{\partial p}{dx}\left\{\frac{K_w}{\mu_w} + \frac{K_0}{\mu_0}\right\}\right] = 0 \qquad (8)$$

Integrating the above equation with respect to x under the boundary condition of our problem, *viz.*,

$$K_0\,(o, t) = S_0\,(o, t) = 0\,;$$

$$-\left(K\,\frac{K_w}{\mu_w}\frac{\partial p}{\partial x}\right)_{o,t} = v_w\,(o, t) = V$$

we get.

$$\frac{\partial p}{\partial x} = -\,\frac{V}{K\left[\frac{K_w}{\mu_w} + \frac{K_0}{\mu_0}\right]} \qquad (9)$$

The values of $\delta p/\delta x$ and v_w may be used in equation 5 and write the equation of motion for the displacing phase saturation as follows:

$$m\,\frac{\partial S_w}{\partial t} - V\,\frac{\partial}{\partial x}\left[\frac{\frac{K_w}{\mu_w}}{\frac{K_w}{\mu_w} + \frac{K_0}{\mu_0}}\right] + \phi\,[T - \tau\,(u)]$$
$$= 0. \qquad (10)$$

Since K_w and K_0 are the functions of S_w we rewrite the equation of motion with the help of equation 1, as under:

$$m\,\frac{\partial S_w}{\partial t} + V\left[\frac{P}{(1 - S_w + PS_w)^2}\right]\frac{\partial S_w}{\partial x}$$
$$+\ \phi\,[T - \tau\,(u)] = 0 \qquad (11)$$

where

$$P = \mu_0/\mu_w$$

Solution by Perturbation Method

As the medium is slightly fractured, we assume the capillary suction function $\phi\,[T\text{-}\tau(u)]$ that was included in the continuity equations due to the presence of fractures is a small quantity. Therefore we use it as a perturbation parameter to solve the equation of motion for saturation. Thus neglecting $\phi\,[T\text{-}\tau(u)]$ in equation 11, we have:

$$m\,\frac{\partial S_w}{\partial t} + V\left[\frac{P}{(1 - S_w + PS_w)^2}\right]\frac{\partial S_w}{\partial x} = 0. \qquad (12)$$

The characteristic equations of equation 12 are:

$$\frac{dS_w}{dx} = 0$$

and

$$\frac{dx}{dt} = \frac{V}{m}\left[\frac{P}{(1 - S_w + PS_w)^2}\right] \qquad (13)$$

Integrating the above characteristics equations under the boundary condition that $x = 0,\ t = 0$, we have:

$$x = \frac{Vt}{m}\left[\frac{P}{(1 - S_w + PS_w)^2}\right]. \qquad (14)$$

Changing t to T and substituting the value of $[T\text{-}\tau(u)]$ from equation 2, we may write equation 11 as follows:

$$m\epsilon\,\frac{\partial S_w}{\partial T} + V\left[\frac{P}{(1 - S_w + PS_w)^2}\right]\frac{\partial S_w}{\partial x}$$
$$+\ \frac{D}{\sqrt{T - Rx^2}} = 0. \qquad (15)$$

Evgenev[10] has pointed from his experimental observations that in most cases P, the ratio of viscosity of oil to water is large and therefore we may regard $1/P$ as a small quantity. We substitute the value of T ($= \epsilon t$) as obtained from equation 14 in the last term of equation 15 and after simplifying in view of the above remark, we obtain:

$$m\epsilon\,\frac{\partial S_w}{\partial T} + V\left[\frac{P}{(1 - S_w + PS_w)^2}\right]\frac{\partial S_w}{\partial x}$$
$$+\ D\left(\frac{VP}{m\epsilon}\right)^{\frac{1}{2}}\frac{x^{\frac{1}{2}}}{(1 - S_w + PS_w)} = 0. \qquad (16)$$

This is a quasi-linear equation of motion whose characteristic equations are :

$$\frac{dx}{dt} = \frac{V}{m}\left[\frac{P}{(1 - S_w + PS_w)^2}\right],$$

$$\frac{dS_w}{dx} = \frac{D\left(\frac{VP}{m\epsilon}\right)^{\frac{1}{2}} \frac{\bar{x}^{\frac{1}{2}}}{(1 - S_w + PS_w)}}{V\left[\frac{P}{(1 - S_w + PS_w)^2}\right]}. \tag{17}$$

The equivalent form of the above equations can be written as follows :

$$D\left(\frac{VP}{m\epsilon}\right)^{-\frac{1}{2}} x^{-\frac{1}{2}}\, dx = \frac{VP}{(1 - S_w + PS_w)}\, dS_w \tag{18}$$

$$\frac{dt}{m} = \frac{1}{D}\left(\frac{m\epsilon}{VP}\right)^{\frac{1}{2}} x^{\frac{1}{2}}\,(1 - S_w + PS_w)\, dS_w. \tag{19}$$

Integration of equation 18 gives :

$$2D\left(\frac{VP}{m\epsilon}\right)^{\frac{1}{2}} x^{\frac{1}{2}} = \frac{VP}{(P-1)} \log\,(1 - S_w + PS_w) + E \tag{20}$$

where E is a constant of integration.

Similarly integrating equation 19 with the help of equation 20 we get :

$$t = \frac{m\epsilon^2}{4D^2}\frac{1}{(P-1)}\,[(1-S_w+PS_w)^2 \times \log\,(1 - S_w + PS_w)] - \frac{m^2\epsilon}{8D^2}\frac{1}{(P-1)}\,(1 - S_w + PS_w)^2 + E\,\frac{m^2\epsilon}{4D^2}\frac{1}{VP}\cdot S_w\,[1 + (1 - S_w + PS_w)] + F \tag{21}$$

where F is a constant of integration. An arbitrary functional relation between these two integrals gives the solution of equation 16.

Since the saturation S_w is defined as the average cross-sectional area occupied by the fingers, we may consider $S_w = 0$ as the criterion for investigating the stability of the fingers. Thus putting $S_w = 0$ in the equations 20 and 21, we note that definite values of x and t correspond to zero value of Sw. and this in turn implies that the stabilization of fingers is possible in the specific problem investigated.

It may be remarked here that the conclusions drawn depend on the perturbation procedure which has been used here. Notwithstanding the difficulty in using such a procedure for studying the long term behaviour of the solution (Scheideger[11]), we have adopted it due to the special nature of the medium and the particular qualitative interest of the present investigation, *viz.*, showing the occurrence of stable fingers in one case of queer permeability-homogeneity and capillary suction term.

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List of Symbols used :

- A = a constant
- g_k = saturation of blocks with water at the moment t_k
- k = permeability of the fractured system
- k_w = relative permeability to water
- k_o = relative permeability to oil
- l = mean block size
- p = viscosity ratio
- m_B = porosity of blocks
- m = porosity of the fractured system
- q = average rate of flow across the striking face
- S = mean specific surface area of the blocks
- S_w = saturation of water
- S_o = saturation of oil
- t = time
- T = weighted time
- u = mean coordinate
- v_w = seepage velocity of water
- v_o = seepage velocity of oil
- x = linear coordinate
- μ_w = viscosity of water
- μ_o = viscosity of oil
- σ = surface tension
- ϵ = a complex constant of fractured characteristics and oil viscosity
- $\phi(t)$ = impregnation function
- θ = wetting angle.

Some constants occurring in the definition of the impregnation function, *viz.*, *a*, D and R are not shown in this table.

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SOME RESULTS OF GEOPHYSICAL SURVEYS IN PONDICHERRY AND ADJOINING AREAS OF SOUTH ARCOT DISTRICT, TAMIL NADU

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ABSTRACT

The present paper deals mostly with the results of the seismic refraction surveys conducted during the Field Season 1969–70 in Pondicherry and adjoining parts of South Arcot District, Tamil Nadu for groundwater. About 122 electrical resistivity probes employing Schlumberger (Maximum AB/2–250 m) and 22 long range refraction seismic profiles (Maximum spread of 1809 m) were spaced evenly in an area of 350 Sq. Kms. The surveys have brought out valuable information on the nature of the sediments and the general disposition of the crystalline basement in and around Pondicherry.

INTRODUCTION

IN view of the drought conditions in Pondicherry State during 1969, geophysical surveys comprising seismic refraction and electrical resistivity soundings were carried out with a view to assist the tube-well drilling programme in this area. The electrical surveys were not successful in delineating the relatively more favourable granular zones within the Tertiary sandstones, cropping out around Pondicherry since they are overlain by a thick cover of laterite which is highly resistive and at times posed a severe problem of imparting current into the ground. These surveys have also failed in places where there is a thick cover of clay or very fine grained sands. However, the resistivity surveys have been useful chiefly in picking up the basement wherever shallow. Long range refraction seismic profiling was carried out to get the information on the thickness of the sediments and their nature in the deepar portions of the basin. The following account relates mostly to the results of siesmic surveys as these have been chiefly useful not only in delineating the basement topography in and around Pondicherry but also in indicating the nature of the overlying sediments for the purpose of groundwater exploration. This information on the total sedimentary column in this area should be useful contribution to the development of groundwater.

Geologically, Pondicherry is located in the well-known coastal sedimentary belt of Tamil Nadu, occupied chiefly by the Cretaceous and Tertiary rocks (Fig. 1). In view of the occurrence of tertiary coal (Lignite) around Neyveli and the possible oil bearing potential of the area, geophysical surveys were initiated in this belt nearly two decades ago (Kailasam, 1954) by the Geological Survey of India. The Oil and Natural Gas Commission has also been active in this belt for the past few years. It may be mentioned incidentally that the recent drilling results of the Geological Survey of India and Oil and Natural Gas Commission around Pondicherry were of help in the correlation of geophysical results.