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FUNCTIONAL REGRESSIONS IN FISHERY RESEARCH

In linear regression situations arising in fishery biology, Ricker[1] has recommended the use of a functional regression, when both the variates are subject to error of measurement or inherent variability or both. In such cases the regression line is obtained by finding the line which minimizes the sum of the products of the vertical and horizontal distances of each point from the line. Ricker refers to the estimate of the regression coefficient thus obtained as the 'GM regression'.

If, instead of sum of products, the sum of squares of both the horizontal and vertical distances of each point from the line is considered, another line with slope very close to the GM regression is obtained. Let the regression line of y on x be

$$y = a + bx. \tag{1}$$

If P be any point (x,y) such that PM and PN are the vertical and horizontal distances from the line $(x, a + bx)$ and $[(y - a/b, y)]$ are the coordinates of M and N respectively. Obviously, the distances PM and PN are $(y - a - bx)$ and $[(y - a/b) - x]$, Minimizing $PM^2 + PN^2$ is same as minimizing MN^2, MN being the hypotenuse of the right-angled triangle PMN. Minimization of MN allows the line to shift itself towards the point, which is ideally required.

Considering all such n points as P, the quantity

$$\sum_{i=1}^{n} \left\{ (y_i - a - bx_i)^2 + \left(\frac{y_i - a}{b} - x_i\right)^2 \right\} \tag{2}$$

is to be minimized with respect to a and b. Differentiating (2) with respect to a and equating to zero gives, after simplification,

$$\sum_{i=1}^{n} y_i = na + b \sum_{i=1}^{n} x_i \tag{3}$$

Similarly differentiation with respect to b leads to the simplified form

$$b(b^2 - 1) \sum_{i=1}^{n} y_i x_i - ab(b^2 - 1) \sum_{i=1}^{n} x_i - b^4 \sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} (y_i - a)^2 = 0. \tag{4}$$

If from (3) substitutions are made for a, (4) simplifies to

$$b^4 \left[\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n} \right] - b^3 \left[\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n} \right] + b \left[\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n} \right] - \left[\sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n} \right] = 0,$$

which may be replaced by

$$S_x^2 b^4 - S_{xy} b^3 + S_{xy} b - S_y^2 = 0, \tag{5}$$

where S_x^2, S_y^2, S_{xy} are respectively the variances of x and y and the covariance between them.

Employing the method of iteration, starting with the estimate of GM regression as a trial value will lead to a solution of (5) which being the slope of the line under consideration.

In the case of GM regression since the estimate involves only the standard deviations of the variables, the pairing up of x and y has no effect. But, since the values come from a bivariate distribution, the association of the variables has to be given due consideration when finding the functional relationship. Here, this is guaranteed by the involvement of covariance in the esmating equation (5).

The regression of x on y is also the same, but the slope will be $1/b$ since the axes are reversed. This is obtained by interchanging the expressions for variances of x and y in the esimating equation (5) and solving.

The variance of this regression is same as that of the corresponding GM regression.

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