LETTERS TO THE EDITOR

THERMAL BOUNDARY LAYER THICKNESS FOR FLOW PAST A PERMEABLE BED

THE thermal slip boundary condition at an interface for the flow past a permeable bed, was proposed by Rajasekhara *et al.*[1] in studying the temperature distribution for a Poiseuille flow over a naturally permeable bed. Subsequently, a correct condition was formulated by Rudraiah and Veerabhadraiah[2] in analysing the temperature distribution in Couette flow past a permeable bed in the presence of buoyancy forces. This condition has been used independently by Vidyanidhi *et al.*[3] in examining the heat transfer for Poiseuille flow past a permeable bed. In their analysis[2], they (Veerabhadraiah and Rudraiah[4]) assumed that the transition in the flow from that at the nomial surface to the Darcy velocity Q in the porous zone 2 takes place in a very thin boundary layer of thickness δ. Assuming δ to be small compared to the width of the flow h in the Zone 1, the transcendental equation in δ is solved to get the relative boundary layer thickness δ^*; this is found to be $1/\alpha\sigma$. In obtaining the temperature distribution, they[2,4] have however assumed that the boundary layer thickness is the same for both the velocity and temperature distributions.

Detailed calculations for the temperature distribution for the Poiseuille flow past a permeable bed[3] (in the absence of buoyancy forces, for simplicity) have shown that both the thicknesses are not necessarily the same.

The relevant equations[2-4], in both the zones, are as follows :

In Zone 1,

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{\partial p}{\partial x} = -\mathrm{P},$$

$$\frac{d^2\mathrm{T}}{dy^2} = -\frac{\mu}{\kappa}\left(\frac{du}{dy}\right)^2,$$

with the boundary conditions

$$u = 0,\ \mathrm{T} = \mathrm{T}_1 \text{ at } y = h,$$

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{\alpha}{\sqrt{\mathrm{K}}}[u(\mathrm{O}) - \mathrm{Q}],$$

$$\left(\frac{d\mathrm{T}}{dy}\right)_{y=0} = \frac{\beta}{\sqrt{\mathrm{K}}}[\mathrm{T}(\mathrm{O}) - \mathrm{T}_*].$$

In Zone 2,

$$\mathrm{Q} = \frac{\mathrm{K}}{\mu}\left(-\frac{\partial p}{\partial x}\right),$$

$$\frac{d^2\mathrm{T}}{dy^2} = -\frac{\mu}{\kappa}\left[\left(\frac{du}{dy}\right)^2 + \frac{u^2}{\mathrm{K}}\right],$$

with the boundary conditions

$$u = \mathrm{Q} \text{ at } y = -\delta,$$
$$\mathrm{T} = \mathrm{T}_* \text{ at } y = -\Delta. \qquad (1)$$

In addition, we require u and T are continuous across the nomial surface $y = 0$.

In terms of the non-dimensional quantities

$$\eta = y/h,\ \delta^* = \delta/h,\ \Delta^* = \Delta/h,\ \sigma = h/\sqrt{\mathrm{K}},$$

$$u' = u/ - \tfrac{1}{2}\mathrm{Q}\sigma^2,\ \theta = \frac{\mathrm{T} - \mathrm{T}_*}{\mathrm{T}_1 - \mathrm{T}_*},$$

$$\mathrm{P}r = \frac{\nu}{\kappa} \text{ (Prandtl number)},$$

$$\mathrm{E} = \frac{\rho \mathrm{Q}^2\sigma^4}{4(\mathrm{T}_1 - \mathrm{T}_*)} \text{ (Eckert number)},$$

the solutions of the foregoing equations are obtained In Zone 1,

$$u' = \eta^2 + \mathrm{A}\eta + \mathrm{B},$$

$$\theta = \mathrm{P}r.\,\mathrm{E}\left[(\beta\sigma\eta + 1)\,\mathrm{C}_1 - \left(\frac{\eta^4}{3} + \frac{2}{3}\mathrm{A}\eta^3 + \frac{\mathrm{A}^2}{2}\eta^2\right)\right]$$

where

$$\mathrm{A} = \frac{\alpha(2 - \sigma^2)}{\sigma(1 + \alpha\sigma)},\ \mathrm{B} = -\frac{\sigma + 2\alpha}{\sigma(1 + \alpha\sigma)}$$

$$\mathrm{C}_1 = \frac{(\mathrm{P}r.\,\mathrm{E})^{-1} + \left(\frac{1}{3} + \frac{2}{3}\mathrm{A} - \frac{\mathrm{A}^2}{2}\right)}{1 + \beta\sigma}$$

In Zone 2,

$$u' = -\frac{2}{\sigma^2} + \mathrm{A}_1 \operatorname{ch} \sigma\eta + \mathrm{B}_1 \operatorname{sh} \sigma\eta,$$

$$\theta = \mathrm{P}r.\,\mathrm{E}\sigma^2\left[\mathrm{C}_2\eta + \mathrm{C}_3 - \left\{\frac{2}{\sigma^4}\eta^2 + \frac{\mathrm{A}_1^2 + \mathrm{B}_1^2}{4\sigma^2}\operatorname{ch} 2\sigma\eta + \frac{\mathrm{A}_1\mathrm{B}_1}{2\sigma^2}\operatorname{sh} 2\sigma\eta\right\} + \frac{4}{\sigma^4}\left\{\mathrm{A}_1\operatorname{ch}\sigma\eta + \mathrm{B}_1\operatorname{sh}\sigma\eta\right\}\right], \qquad (2)$$

where

$$\mathrm{A}_1 = \frac{2}{\sigma^2} + \mathrm{B},\ \mathrm{B}_1 = \mathrm{A}_1 \coth\frac{1}{\alpha}$$

$$\mathrm{C}_2 = \frac{\beta \mathrm{C}_1}{\sigma} + \frac{\mathrm{A}_1\mathrm{B}_1}{\sigma} - \frac{4\mathrm{B}_1}{\sigma^3},$$

$$\mathrm{C}_3 = \frac{\mathrm{C}_1}{\sigma^2} + \frac{\mathrm{A}_1^2 + \mathrm{B}_1^2}{4\sigma^2} - \frac{4\mathrm{A}_1}{\sigma^4}$$

Neglecting the squares and higher powers of δ^*, it was shown[4] that $\delta^* = (\gamma\sigma)^{-1}$. With this value of δ^*, the eqns. (1) and (2) determine a transcendental equation in Δ^*. Under the same approximation for Δ^*, we obtain

$$\Delta^* = \frac{\sigma C_3}{\beta C_1} . \tag{3}$$

with Pr. = 0·025, E = 0·01, α = 0·1 as in ref. 2, 4 Δ^* is computed in Table 1 for different values of σ and β (Biot-number). It is found to be in fact much smaller than δ^*.

TABLE I

Computed Values of the thermal boundary layer thickness Δ^*

σ	δ^*	Δ^*	
		$\beta = 1$	$\beta = 2$
100	0·1	0·01012	0·00550
500	0·02	0·00212	0·00150
1000	0·01	0·00112	0·00100
5000	0·002	0·00032	0·00060
10000	0·001	0·00022	0·00055

Again δ^* is chosen to be unity in their numerical[1] discussion[2,4]. This appears to be a rather large value and is inconsistent with the assumption made. δ^* can at most be 0·1. In the presence of the buoyancy[2,4], Δ^* is further expected to be dependent on the parameter N_0.

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DIURNAL VARIATION OF f_0F_2 AT EQUATORIAL LATITUDES AND COUNTER ELECTROJET

ONE well documented feature of the diurnal variation of f_0F_2 at equatorial latitudes is the occurrence of two maxima, one in the morning and the other in the evening with a prominent trougeh around midday, termed as noon 'bite-out'[1-5]. The asymmetry of the bite-out, *i.e.*, the relative amplitudes of the morning and evening maxima exhibit a significant dependence on the phase of the sunspot cycle. During the periods of low sunspot activity, the ampltude of the evening maximum is prominent while during periods of high sunspot activity the morning maximum gains prominence[6,7]. The physical mechanisms responsible for this long term behaviour of the asymmetry of the bite-out are yet to be established.

It is well known that the intense band of *eastward* ionospheric currents at E-region altitudes, over and in the vicinity of the dip equator, referred to as electrojet, gives rise to the pronounced enhancement in the diurnal range of the H-component of the earth's geomagnetic field monitored on ground. The electric field (east-west) at E-region levels associated with the electrojet is transferred along the highly conducting magnetic lines of force to higher altitudes where it interacts with the horizontal geomagnetic field to set up an upward directed (during daytime) $E \times B$ drift of plasma. Thus the electrojet current strength, on which the $E \times B$ drift depends, governs to a large extent the behaviour of the equatorial F-region[8-11]. It is well established now that on occasions, designated as 'Counterelectrojet' (CEJ) events[12], the electrojet current gets reversed for a while during daytime, *i.e.*, *current is wastward*, resulting in a depression of the H-component below the mean midnight level[13]. These CEJ events are closely associated with several phenomena in the lower and upper equatorial ionosphere, the most prominent among them being the disapperance of E_{sq} traces on bottomside ionograms and reversal of the E-region horizontal drift velocitties, clearly indicating a reversal in the electrojet direction[14-16]. The probable effect of the CEJ events is to reverse or reduce the $E \times B$ vertical drift (due to a reversal of the E-region electric field) and hence to increase the F-region electron densities at and close to the dip equator[11]. The occurrence of CEJ events is noticed to be more frequent during afternoon hours under conditions of low geomagnetic and solar activity[17]. It is thus quite possible that the pronounced evening maximum in the diurnal variation of f_0F_2 at equatorial latitudes, observable during low sunspot activity, might be due to the frequent occurrence of afternoon CEJ events. In fact, such an explanation has already been put forward[18] although systematic studies have not been made. It is therefore felt worthwhile to examine this possibility.