LETTERS TO THE EDITOR

ON THE GRAVITATIONAL MASS OF A CHARGED PARTICLE

THE gravitational field of a charged particle, with mass m and charge e, is given by a Reissner-Nordstrom (RN) space-time

$$ds^2 = -A^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta \, d\phi^2) + A dt^2, \tag{1}$$

where

$$A(r) = 1 - 2(m/r) + (e/r)^2.$$

Florides[1] while investigating the gravitational mass of a charged particle has resorted to approximations in describing the radial motion of an uncharged particle and reached the conclusion that the effective gravitational mass ($= m$) is equivalent to the sum of the Newtonian gravitational mass ($= m_{NG}$) and mass equivalence of electromagnetic energy ($= e^2/a$).

We shall now establish the above result not taking recourse to approximation. To accomplish this, we first change a curvature coordinate r to R in such a way that $r = r(R)$ and (1) assumes the form[2]

$$ds^2 = -A(R)[dt^2 - dR^2] - r^2(R) \times [d\theta^2 + \sin^2\theta \, d\phi^2], \tag{2}$$

with

$$A(R) = dr/dR. \tag{3}$$

This new coordinate R is more convenient[2,3]. The radial speed of light is unity and hence coordinate independent whereas in the case of r, it is coordinate dependent [$= \pm A(r)$]. There also appears a closer analogy between the RN metric (2) and the Minkowski metric. Moreover if we integrate (3) and adjust a constant of integration[2], the domain $r \geq m + b$, $b^2 = m^2 - e^2$ gives place to $-\infty \leq R \leq \infty$.

The Lagrangian for a radial motion of a particle moving in the field of (2) is

$$L = -a[A(1 - \dot{R}^2)]^{1/2}, \quad \dot{R} = dR/dt, \tag{4}$$

where a is the rest mass of the particle.

Writing down the usual Lagrange's equations of motion, we arrive at[4]

$$\dot{R}^2 = 1 - A/E^2, \tag{5}$$

with aE as the total energy of the moving particle. From (5) we obtain

$$\ddot{R} = -(1/r^2)(m - e^2/r)(1 - \dot{R}^2). \tag{6}$$

The radial acceleration is always negative (with $m > e$ etc.), *i.e.*, directed towards the origin, thereby, implying the force to be of attractive nature (for large values of r).

In special relativity the generalised force is the change of canonical momentum with time[4]. By analogy with special relativity, we define the gravitational force[2,3],

$$F = \frac{d}{dt}\left(\beta \frac{dR}{dt}\right), \tag{7}$$

where β is the relativistic mass of the particle and equals to $a(1 - \dot{R}^2)^{-1/2}$.

From equations (6) and (7), we derive

$$F = -\beta(m/r^2) + \beta(e^2/r^3). \tag{8}$$

The first term on the right gives the Newtonian attraction while the second term gives repulsion which can be treated as the correction term due to the presence of the charge. Following Florides[1] for the classical analogue of the above problem, we write

$$F = -\beta\left[\frac{m_{NG}}{r^2} + \frac{e^2}{r^2}\left(\frac{1}{a} - \frac{1}{r}\right)\right], \tag{9}$$

where a is the radius of spherical body of mass m_{NG}. Comparing (8) and (9), we get the desired result

$$m = m_{NG} + e^2/a.$$

The author is thankful to Professor J. R. Rao for encouragement. He also thanks the referee for his helpful suggestions.

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July 3, 1978.

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