LETTERS TO THE EDITOR

INVARIANT CHARACTERIZATION FOR NULL ELECTROMAGNETIC FIELDS IN GENERAL RELATIVITY*

A NECESSARY and sufficient condition for a projectively-flat space-time to represent a null electromagnetic field is given.

Prasanna[2] has defined an invariant $A = R_{hijk} F^{hi} F^{jk}$ characterizing the gravitation through the curvature tensor R_{hijk} and the electromagnetism through the field tensor F_{ij}. It is hoped that such an invariant will play an important role in formulating a satisfactory unified theory of electromagnetism and gravitation. In terms of this invariant, Prasanna has given a necessary and sufficient condition for a conformally flat space-time to represent a null electromagnetic field.

In this note, using this invariant, we shall prove the following.

Theorem: A projectively-flat space-time represents a null electromagnetic field if $A = 0$ and conversely.

Proof: Since the space-time is projectively-flat, therefore, by definition the projective curvature tensor is zero. Hence, we have

$$R_{hijk} = \tfrac{1}{3}(g_{hk} R_{ij} - g_{hj} R_{ik}). \tag{1}$$

The invariant $A = R_{hijk} F^{hi} F^{jk}$ thus becomes

$$A = \tfrac{1}{3}(g_{hk} R_{ij} - g_{h} R_{ik}) F^{hi} F^{jk}. \tag{2}$$

But for null electromagnetic fields, it is known that[1]

$$R_{ij} = 2F_{ip} F^{p}_{j}. \tag{3}$$

Therefore (2) becomes

$$A = \tfrac{2}{3}(F_{ip} F^{p}_{j} F^{j}_{h} - F_{iq} F^{q}_{k} F^{k}_{h}) - \tfrac{4}{3}(F_{ip} F^{p}_{j} F^{j}_{h}) \tag{4}$$

as p, q, j and k are dummy indices. Also, for null electromagnetic fields, we have[1]

$$F^{3} = 0, \quad \text{or} \quad F_{ip} F^{p}_{j} F^{j}_{h} = 0. \tag{5}$$

From (5), we have $A = 0$.

Now conversely, since $A = 0$, therefore $F_{ip} F^{p}_{j} F^{j}_{h} = 0$. Hence the field is null. This completes the proof of the theorem.

It is known that the plane wave electromagnetic fields are always null, therefore we have

Corollary: The projectively-flat space-time represents the field of a plane electromagnetic wave if the invariant $A = 0$.

Remark: The conharmonic and concircular curvature tensors are defined as

$$L_{hijk} = R_{hijk} - \tfrac{1}{2}(g_{ij} R_{hk} - g_{hj} R_{ik} + g_{hk} R_{ij} - g_{ik} R_{hj}). \tag{6}$$

$$M_{hijk} = R_{hijk} - \frac{R}{12}(g_{hk} g_{ij} - g_{hj} g_{ik}). \tag{7}$$

From (6) and (7) using (3) and (5) it can easily be proved that the above theorem also holds for conharmonically—and concircularly—flat space-times.

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ROTATIONAL ANALYSIS OF THE A-X SYSTEM OF THE CuCl MOLECULE

CuCl is known to have seven band systems all in the visible region and all degraded to red. Ritschl[1] made detailed investigations of the A, B, C, D and E systems of CuCl in absorption. Bloomenthal[2] photographed the F system under low dispersion. A thorough investigation of the coarse structure of all the systems was made by Rao and Brody[3]. Rao and Brody[3] showed that all these systems involve the same lower level which is the ground $^1\Sigma$ state of CuCl. The rotational analysis of the different systems of this molecule is complicated due to the fact that the natural CuCl is a mixture of four isotopic species ($^{65}Cu\,^{37}Cl$, $^{65}Cu\,^{35}Cl$, $^{63}Cu\,^{37}Cl$, $^{63}Cu\,^{35}Cl$ with abundance ratio 1 : 2 : 3 : 7). Taking the isotopic molecule $Cu^{65}Cl^{35}$ Rao *et al.*[4] analysed the rotational structure in the B, C, D, E and F systems of this molecule and confirmed the nature of the states. The A system of CuCl which is weak relative to the other systems and is packed with a great deal of overlapping of structure due to the closely lying bands of its sequences could not be analysed by these workers. Nair and Upadhya[5] photographed the A system in the 1st order of a 10·6 m

concave grating spectrograph with a dispersion of 0·7 A/mm and calculated the constants for the A and X states using the head origin separation relation. We have succeeded in photographing this system in the 2nd order of the same grating spectrograph at higher resolution (dispersion of 0·33Å/mm) and have analysed the rotational structure in the (0,1), (0,0) and (1,0) bands of the system in detail. In the present communication we report the results of our analysis.

A spec pure sample of CuCl (Ridel make) was taken in an electrodeless discharge tube made up of quartz and the bands were excited by a 125 watt 2450 mc/sec. Raytheon microwave oscillator. The overlapping of structure due to sequence bands and the less abundant isotopes was avoided to some extent by adjusting the time of exposure. As the system is very weak an exposure of 30 hours on ORWO rapid 400 ASA panchromatic film was found necessary for recording the structure with good intensity. Iron arc spectrum was used as the comparison spectrum. The probable error in the measurement is estimated to be $\pm 0{\cdot}04\ cm^{-1}$ for unblended lines.

The rotational structure in the (0,1) (0,0) and the (1,0) band is well resolved and was found suitable for measurement. The (0,0) band shows two heads, R and Q, while the other two bands reveal four heads —the R and Q due to $Cu^{63}Cl^{35}$ and the R and Q due to $Cu^{65}Cl^{35}$ (the Q head of 1,0 band of $Cu^{63}Cl^{35}$ is however masked by a strong copper line). In all the bands three branches R, Q and P are found to be present of which the Q branch is the most intense. Though near the head there is some overlapping the structure is well resolved towards higher J. A portion of the (0,0) band is shown in Fig. 1. The presence

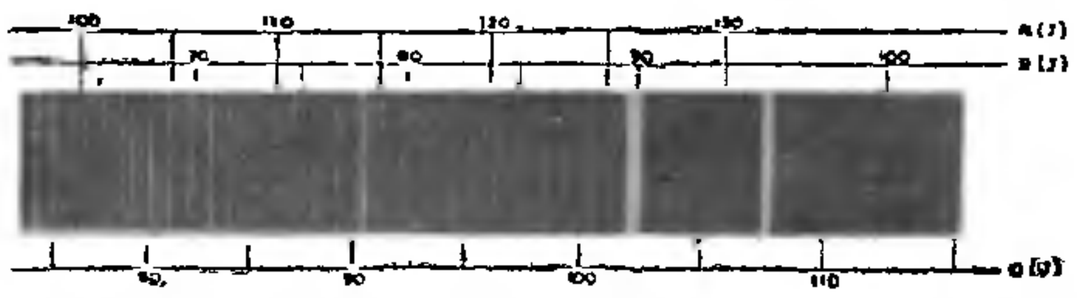

FIG. 1. A part of the (0,0) band of the A–X system of the CuCl molecule.

of two heads and three branches indicate that the transition of this system must involve $\Delta\Lambda = \pm 1$. It is well known from vibrational analysis that the lower state of the A system of this molecule is its ground state which is a $^1\Sigma$ state. The A state is therefore assigned as a $^1\Pi$ state and the transition as $^1\Pi$–$^1\Sigma$. The Λ doubling is found to be appreciable and shows significantly as a combination defect. The procedure of analysis and J assignment adopted is the same as suggested by Herzberg[7]. The lower state constants obtained from the present analysis agree excellently with the microwave values given by Manson *et al.*[8]. The various constants determined are tabulated in Table I.

TABLE I

Molecular constants of the A–X system of the CuCl molecule

	Upper state	Lower state
Be (cm^{-1})	0·1679	0·1783
De (cm^{-1})	$1{\cdot}1 \times 10^{-7}$	$1{\cdot}3 \times 10^{-7}$
αe (cm^{-1})	0·0009	0 0010
q	0·0007 (5)	..
ν_{01} (cm^{-1})	18579·90	
ν_{00} (cm^{-1})	18994·10	
ν_{10} (cm^{-1})	19403·09	

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THE CRYSTAL STRUCTURE OF GLYCINE ORTHOPHOSPHATE

THIS is a preliminary note reporting the atomic coordinates in the crystal structure of glycine orthophosphate.

Single crystals of glycine orthophosphate ($NH_2CH_2COOH \cdot H_3PO_4$) were grown from a saturated aqueous solution containing glycine and orthophosphoric acid in stoichiometric proportions. The crystal data are as follows : $a = 9{\cdot}63$, $b = 7{\cdot}89$, $c = 9{\cdot}24$ Å, $\beta = 114°$, $V = 641{\cdot}4$ Å^3, F.W. = 173·06 gm, $D_{mea} = 1{\cdot}78$ gm. cm^{-3}, $D_{cal} = 1{\cdot}79$ gm. cm^{-3} $Z = 4$, μ (CuKα) = 37·41 cm^{-1}. Space group : $P2_1/C$.

The three-dimensional intensity data (hKl, K = 0 to 7, $\mu r = 0{\cdot}37$; *hk*L, L = 0 and 1, $\mu r = 0{\cdot}36$) were