LETTERS TO THE EDITOR

NON-EXISTENCE OF AXIALLY SYMMETRIC MASSIVE COMPLEX SCALAR FIELDS IN ROSEN'S THEORY OF GRAVITATION

A BIMETRIC theory of relativity established by Rosen[1] consists of two metric tensors g_{ij} and γ_{ij} (defined at every point of space-time), the former describing the gravitational fields whereas the latter, the flat space metric, the inertial forces.

The field equations of bimetric relativity are

$$K_{ij} = -8\pi k T_{ij}, \quad (1)$$

where

$$K_{ij} = N_{ij} - \tfrac{1}{2} N g_{ij},$$

and

$$N_{ij} = \tfrac{1}{2}\gamma^{\alpha\beta}(g^{hi} g_{hj|\alpha})_{|\beta}, \quad (2)$$

where bar (|) stands for covariant differentiation with respect to

$$\gamma_{ij},\ k = (g/\gamma)^{1/2},\ g = \det|g_{ij}|,\ \gamma = \det|\gamma_{ij}|,$$

etc., and T_{ij} is the energy momentum tensor of matter or other non-gravitational fields.

In high energy physics, the complex fields find more applications. It will thus be worthwhile to study the role of these fields in the new theory of gravitation proposed by Rosen[1]. This work is the extension of our previous work communicated for publication where we have considered scalar meson fields. For our study we have taken up an axially symmetric Einstein–Rosen metric

$$ds^2 = e^{2\alpha-2\beta}(dT^2 - dR^2) - R^2 e^{-2\beta} d\phi^2 - e^{2\beta} dz^2, \quad (3)$$

where α and β are functions of R and T only. The reason for choosing this metric and its importance in general relativity, is explained by Karade[2].

The flat space-time corresponding to (3) is

$$d\sigma^2 = dT^2 - dR^2 - R^2 d\phi^2 - dz^2. \quad (4)$$

For (4), the non vanishing γ-Christoffel symbols are

$$\Gamma^2_{12} = \Gamma^2_{21} = 1/R \quad \text{and} \quad \Gamma^1_{22} = -R.$$

Now by straightforward calculations for the line element (3), we have

$$K^4_4 = 0. \quad (5)$$

From (1) and (5), we write

$$T^4_4 = 0. \quad (6)$$

The energy momentum tensor for the complex scalar field V coupled with electromagnetic field is given by

$$T_{ab} = (\bar{D}_a\bar{V}\cdot D_b V + \bar{D}_b\bar{V}\cdot D_a V) - g_{ab}(\overline{D^n}\bar{V}\cdot D_n V - m^2 V\bar{V}) - F_{ak}F^k_b + \tfrac{1}{4} g_{ab} F_{rs}F^{rs}, \quad (7)$$

where F_{ab} is a Maxwell tensor, such that

$$F_{ab} = A_{a|b} - A^*_{b|a} = A_{a,b} - A_{b,a}, \quad (8)$$

with A_a as a four potential satisfying a guage condition $\nabla^a A_a = 0$. The operator D_a and its conjugate $\bar{D}_a$ are defined by

$$\left.\begin{aligned} D_a &= \nabla_a + (4\pi)^{1/2} i\epsilon A_a,\\ \bar{D}_a &= \nabla_a - (4\pi)^{1/2} i\epsilon A_a,\end{aligned}\right\} \quad (9)$$

where ∇ indicates a covariant derivative and $i = (-1)^{1/2}$. In a charged scalar field the quantity $V\bar{V}$, the matter density, is always positive. Moreover the complex scalar field V satisfies the combined Einstein–Maxwell and Klein-Gordon equations given in Rao *et al.*[3] We do not need these equations for our purpose.

The mass and charge parameters of the field are denoted respectively by m and ϵ. The parameter ϵ is related to the fine structure constant which in turn relates to the charge of an electron.

From (6) and (7), we derive

$$\begin{aligned} &-g^{11}\bar{D}_1\bar{V}\cdot D_1V - g^{22}\bar{D}_2\bar{V}\cdot D_2V \\ &- g^{33}\bar{D}_3\bar{V}\cdot D_3V + g^{44}\bar{D}_4\bar{V}\cdot D_4V + m^2 V\bar{V} \\ &+ \tfrac{1}{2} g^{44}g^{11}g^{22}(F_{12})^2 + \tfrac{1}{2} g^{44}g^{11}g^{33}(F_{13})^2 \\ &- \tfrac{1}{2} g^{44}g^{11}g^{44}(F_{14})^2 - \tfrac{1}{2} g^{44}g^{22}g^{44}(F_{24})^2 \\ &- \tfrac{1}{2} g^{44}g^{33}g^{44}(F_{34})^2 = 0. \end{aligned} \quad (10)$$

Now all the terms on L.H.S. of (10) are positive because g^{11}, g^{22}, g^{33} are all negative and g^{44}, $\bar{D}_1\bar{V}\cdot D_1V$, $\bar{D}_2\bar{V}\cdot D_2V$, $\bar{D}_3\bar{V}\cdot D_3V$, $\bar{D}_4\bar{V}\cdot D_4V$, $V\bar{V}$ are all positive. Since the sum of all the + ve terms vanishes, each term must vanish separately, which in turn gives,

$$m = 0, \text{ etc.}$$

Hence an axially symmetric mass parameter of a complex scalar field coupled with an electromagnetic field vanishes for Einstein-Rosen metric in Rosen's bimetric theory of relativity. Moreover, it is seen that all the components of F_{ij} vanish except F_{23}. Because of the axial symmetry A_σ is independent of ϕ and Z and then from (8) follows $F_{23} = 0$. Hence the contribution of F_{ij} to T_{ij} is nil.

In the above case the axially symmetric field solutions of Rosen's theory turn out to be vacuum solutions. This is a remarkable result. In general relativity only the mass parameter of the complex scalar field vanishes (see Rao *et al.*[3]), but in bimetric relativity distribution corresponding to complex field as well as electromagnetic field yields only an empty space-time. Then follows a theorem:

'In Rosen's bimetric relativity the only possible solution of the axially symmetric fields—complex massive scalar field, Maxwell electromagnetic field is a vacuum solution.' The word axial symmetry is, however, related to E–R metric (3) and not to the most general cylindrically symmetric Marder's metric which involves three parameters.

The theorem established by us in our previous work[4] follows from the above theorem as expected.

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* In bimetric theory ordinary differentiation of general relativity is replaced by γ-differentiation.

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3. Rao, J. R., Panda, H. S. and Nayak, B. K., *Aust. J. Phys.*, 1975, 28, 353.
4. Karade, T. M. and Dhoble, Y. S., *Non Existence of the Axially Symmetric Massive Scalar Fields in Rosen's Bimetric Theory of Relativity*, Communicated for publication, 1979.

A METHOD OF DETERMINING THE SOURCE SPECTRUM OF THE RETURN STROKE OF ATMOSPHERICS

THE return stroke of a lightning discharge emits electromagnetic signals, called atmospherics that usually attain a spectral peak within the V.L.F. region, *i.e.*, 3 to 30 KHz. Source spectrum of lightning discharge has been determined by many workers using different techniques. Chapman and Mathews[1] showed that the radiant energy from the return stroke has the spectral peak around 5 KHz which has been verified by Taylor[2]. From the whistler studies, Helliwell, *et al.*[3] have obtained a spectral peak around 5 KHz. Many workers[1–8] have given different expression for the current waveforms of the return stroke which gave spectra having peak in the range of 2·5 to 11 KHz. In the present communication, a method is suggested for evaluating the source spectrum of the return stroke of the lightning discharge.

The waveforms of the atmospherics were photographed at Waltair using the atmospheric waveform recorder at night during the period March, 1976 to March, 1977 using a wide-band technique along with the direction of arrival of atmospherics by Radio Goniometer. The recorded waveforms after necessary enlargement are subjected to Fourier analysis at the intervals of I KHz using IBM 1130 digital computer to yield the amplitude in the frequency range 1–30 KHz. The source distance of atmospherics is evaluated following the methods given by Caton and Pierce[9] and Hepburn[10].

Assuming that only single dominant mode essentially contributes to the received field (Frisius and Heydt)[11], the amplitude, $F(f,\rho)$, is given by

$$F(f,\rho) = g(f) \times \frac{600}{h} \sqrt{\frac{\lambda}{r_t \sin \frac{\rho}{r_t}}} \times |\Lambda_1(f)|_{10} \frac{-A_1(f)}{20} \cdot \rho \qquad (1)$$

where $g(f)$ = source spectrum, h = ionospheric height, λ = wave-length, r_t = terrestrial radius, $|\Lambda_1(f)|$ = excitation factor of the dominant mode, ρ = distance and $A_1(f)$ is the attenuation factor at frequency f.

Further the spectral amplitude ratio, SAR, between two frequencies f_1 and f_2 is given by

$$20 \log \frac{F(f_1,\rho)}{F(f_2,\rho)} = 20 \log \frac{g(f_1)\, h(f_2) \sqrt{\lambda_1}\, |\Lambda_1(f_1)|}{g(f_2)\, h(f_1) \sqrt{\lambda_2}\, |\Lambda_1(f_2)|} - [A_1(f_1) - A_1(f_2)] \cdot \rho \qquad (2)$$

where suffixes 1 and 2 represent the parameters for frequencies f_1 and f_2. Hence for a particular atmospheric

$$\text{SAR} = \text{constant} - [A_1(f_1) - A_1(f_2)] \cdot \rho. \qquad (3)$$

The above equation shows that the SAR is a linear function of distance of travel and the constant is equal to the first term on the right hand side of equation 2. It has been proved[12] that the reflection heights $h(f_2)$ and $h(f_1)$ do not vary much in the frequency range (1 to 15 KHz). The excitation factor $|\Lambda_1(f)|$ at different frequencies is taken from the data of Wait and Spies[13].