MODIFIED COSMOLOGICAL TERM AND ITS SIGNIFICANCE

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Abstract

The importance of the modified cosmological term in the Einstein field equations and its physical interpretation as the vacuum field have been discussed. The motion of a dust cloud has been investigated and the field equations in the Newtonian approximation have been examined.

FROM the theoretical[1] as well as the observational[2] point of view it has been seen that the introduction of the cosmological term in the Einstein field equations is necessary. The cosmological term represents the 'ground-state energy of the vacuum'. It has been pointed out that the cosmical constant is a function of temperature[3], which turns out to be a function of time in a homogeneous and isotropic universe[4]. Therefore, the variation of the cosmical constant would imply the interaction of the vacuum field with the content of the universe. This gives a kind of creation or vanishing of energy in the universe. But there is no observational evidence to support it. McVittie has also pointed out that the pressure p and the density 'ρ' of a distribution of matter in a Minkowski space-time are given by[5]

$$p = \frac{\Lambda c^4}{8\pi G}, \quad \rho = \frac{-\Lambda c^2}{8\pi G} \tag{1}$$

respectively, which is unphysical on the ground that the pressure and the density are of opposite sign. Therefore, the special relativity limit of the vanishing gravitational field everywhere[6] is not satisfied by the Einstein field equations with the cosmological term. In order to avoid these drawbacks a modified form of the cosmological term has been proposed for a homogeneous and isotropic universe[7] as $\Lambda_{(ij)}\, g_{ij}$ (no summation). The quantities $\Lambda_{(ij)}$ are given by

$$\Lambda_{(ij)} = \begin{bmatrix} a & a & a & -\beta \\ a & a & a & -\beta \\ a & a & a & -\beta \\ -\beta & -\beta & -\beta & -\beta \end{bmatrix} \tag{2}$$

where a and β are invariants[8] under the coordinate transformations of the type

$$t' = t, \quad x^{i'} = x^{i'}(x^i) \quad (i = 1, 2, 3) \tag{3}$$

which is consistent with the cosmological principle[9]. Thus, the Einstein field equations with the modified cosmological term are expressed as

$$R_{ij} - \tfrac{1}{2} R g_{ij} + \Lambda_{(ij)}\, g_{ij} = -\frac{8\pi G}{C^4} T_{ij} \qquad (i, j = 1, 2, 3, 4). \tag{4}$$

When the vacuum field interacts with content of the universe then we have

$$\left[T^{ij} + \frac{C^4 \Lambda^{(ij)} g^{ij}}{8\pi G}\right]_{;j} = 0 \qquad (i, j = 1, 2, 3, 4) \tag{5}$$

Let us consider the case of a dust cloud for which the energy momentum tensor is given by

$$T^{ij} = \rho_0 u^i u^j \quad (i, j = 1, 2, 3, 4) \tag{6}$$

where ρ_0 is the proper density and u^i is the four velocity of the dust cloud. The equations (5) and (6) together yield

$$\rho_0 u^i u^j_{;j} + (\rho_0 u^i)_{;j} u^j + \left[\frac{C^4 \Lambda^{(ij)} g^{ij}}{8\pi G}\right]_{;j} = 0 \qquad (i, j = 1, 2, 3, 4) \tag{7}$$

which is decomposed into two parts as follows:

$$(\rho_0 u^i)_{;j} u^j + \left[\frac{c^4 \Lambda^{(ij)} g^{ij}}{8\pi G}\right]_{;j} = 0 \qquad (i, j = 1, 2, 3, 4) \tag{8}$$

and

$$u^i u^j_{;i} = 0 \quad (\rho_0 \neq 0) \qquad (i, j = 1, 2, 3, 4) \tag{9}$$

The expression (8) represents the conservation law for the momentum density $(\rho_0 u^i)$ of the dust cloud and the expression (9) is the geodesic equations governing the motion of the dust cloud.

Further, when the gravitational field is weak and static, we have[10]

$$g_{44} = -2\phi + C_1 \qquad (10)$$

where 'ϕ' is the Newtonial potential and 'C_1' is an arbitrary constant. The constant C_1 is determined by considering the extreme case of vanishing gravitational field, *i.e.*, $g_{44} = 1$ when $\phi = 0$. But in presence of the vacuum field the situation is different. Only the gravitational field due to the content present in the medium vanishes in the extreme case. Therefore, at any point in the space the Newtonian potential 'ϕ' would be treated as a sum of the potentials 'ϕ_m' and 'ϕ_v' due to the content and the vacuum field respectively, *i.e.*,

$$\phi = \phi_m + \phi_v \qquad (11)$$

and hence to determine the constant C_1, we consider the case of vanishing gravitational field everywhere, *i.e.*, $g_{44} = 1$ when $\phi_m + \phi_v = 0$, so that $C_1 = 1$.

The trace of the field equations (4) is given by

$$-R + (3\alpha - \beta) = -\frac{8\pi G}{C^4} T. \qquad (12)$$

Therefore, the field equations (4) can also be written as

$$R_{ij} - \tfrac{1}{2}(3\alpha - \beta) g_{ij} + \Lambda_{(ij)} g_{ij} = -\frac{8\pi G}{C^4} [T_{ij} - \tfrac{1}{2} T g_{ij}] \qquad (13)$$

$$(i, j = 1, 2, 3, 4)$$

which reduces to

$$\nabla^2 \phi + \frac{(3\alpha + \beta) C^2}{2} = 4\pi G \rho \qquad (14)$$

in the Newtonian approximation[11]. The presence of the term $(3\alpha + \beta) C^2/2$ in the equation (14) is due to the non-zero property of the vacuum. If the vacuum field behaves like a perfect fluid as suggested by N Rosen[12], then the quantity $(3\alpha + \beta) C^2/2$ would be treated as the energy density of the vacuum field. When there is vacuum field only in the medium we have $\rho = 0$, $\phi_m = 0$ the equations (11) and (14) together yield

$$\nabla^2 \phi_v + \frac{(3\alpha + \beta) C^2}{2} = 0. \qquad (15)$$

This shows that the vacuum field can be treated as a scalar meson field[13]. The potential 'ϕ_v' then satisfies the condition

$$\phi_v = -\frac{(3\alpha + \beta) C^2}{2\lambda} \; (\lambda = \text{constant}) \qquad (16)$$

whatever the case may be (the vacuum field as a perfect fluid or as a scalar meson field), the gravitational effect of the vacuum field can be observed in the Newtonian limit when $3\alpha + \beta \neq 0$. Also, in a Minkowski space-time we have $\phi_m + \phi_v = 0$, which implies that the gravitational force of attraction due to the distribution of matter is balanced by the gravitational force of repulsion due to the vacuum field. Recently it has been shown that the repulsive gravitational force due to the presence of the vacuum field in early stages of evolution of the universe plays a significant role in averting the singularity in the cosmological models[14].

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