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## A NOTE ON THE APPLICATION OF HILBERT TRANSFORM FOR THE TRANSFORMATION OF GEOMAGNETIC ANOMALIES DUE TO TWO-DIMENSIONAL BODIES

D. ATCHUTA RAO AND H. V. RAM BABU

National Geophysical Research Institute, Hyderabad 500 007, India

### ABSTRACT

In the magnetic method of geophysical prospecting, the anomalous magnetic field of the earth is measured choosing either the vertical, the total or the horizontal field magnetometer. Interpretation of the anomalous field may be carried out in a number of ways which can be improved if the measurements in two or three components are available. Geological structures such as sheet-like bodies, dykes, faults, etc., causing magnetic anomalies, may be approximated to be two-dimensional. For two-dimensional bodies, it is shown here that the magnetic anomaly in any one component can easily be transformed into the other by means of Hilbert transform. The relations for such transformations, derived using a thin sheet model are presented here. These relations are applicable for magnetic anomalies over all two-dimensional bodies.

### INTRODUCTION

It is the usual practice in magnetic prospecting to take measurements of the anomalous magnetic field of the earth choosing one particular component (total, vertical or horizontal) of interest. The magnetic anomaly thus measured is interpreted in terms of sub-surface geological structures assumed to be responsible for the anomaly. Magnetic interpretation can be carried out in several ways and may be improved if the measurements are available in two or three components<sup>1</sup>. Geological structures such as dykes, thin sheets, faults, etc., extending infinitely in the strike direction are generally approximated to two-dimensional bodies. For two-dimensional bodies, the magnetic anomaly in the vertical and the horizontal components form a Hilbert transform pair<sup>2</sup>. Hence, the

magnetic anomaly in the vertical (horizontal) component can be obtained from the horizontal (vertical) component using Hilbert transform. For two-dimensional bodies it is shown here that the magnetic anomaly in any component can be transformed into the other using the fundamental relationship among them and the Hilbert transform. The relations for computing these components are derived using a thin sheet model. These relations are applied on a field example.

### THEORY

Let there be a thin sheet (Fig. 1), extending infinitely along its strike and is magnetized due to induction only. The expressions for the vertical ( $\Delta V$ ), horizontal ( $\Delta H$ ) and total ( $\Delta T$ ) field magnetic anomalies

at any point P(x) along a line perpendicular to its strike are given by

$$\Delta V = A \frac{h \cos \theta + x \sin \theta}{x^2 + h^2} \quad (1)$$

$$\Delta H = A \sin \alpha \frac{h \sin \theta - x \cos \theta}{x^2 + h^2} \quad (2)$$

and

$$\Delta T = Ab \frac{h \sin (\theta + I) - x \cos (\theta + I)}{x^2 + h^2} \quad (3)$$

where,

$h$  is the depth to the top of the sheet,

$$A = 2KTb \sin \delta$$

$$b = (1 - \cos^2 i \cos^2 \alpha)^{1/2}$$

$$I = \arctan (\tan i / \sin \alpha)$$

$$\psi = I - \delta$$

in which,  $\delta$  is the dip of the sheet,  $k$  is the susceptibility contrast of the sheet to its surroundings,  $T$  is the base level of the total magnetic field intensity,  $i$  is the inclination of the geomagnetic field vector and  $\alpha$  is the strike of the body measured clockwise with respect to the magnetic North.

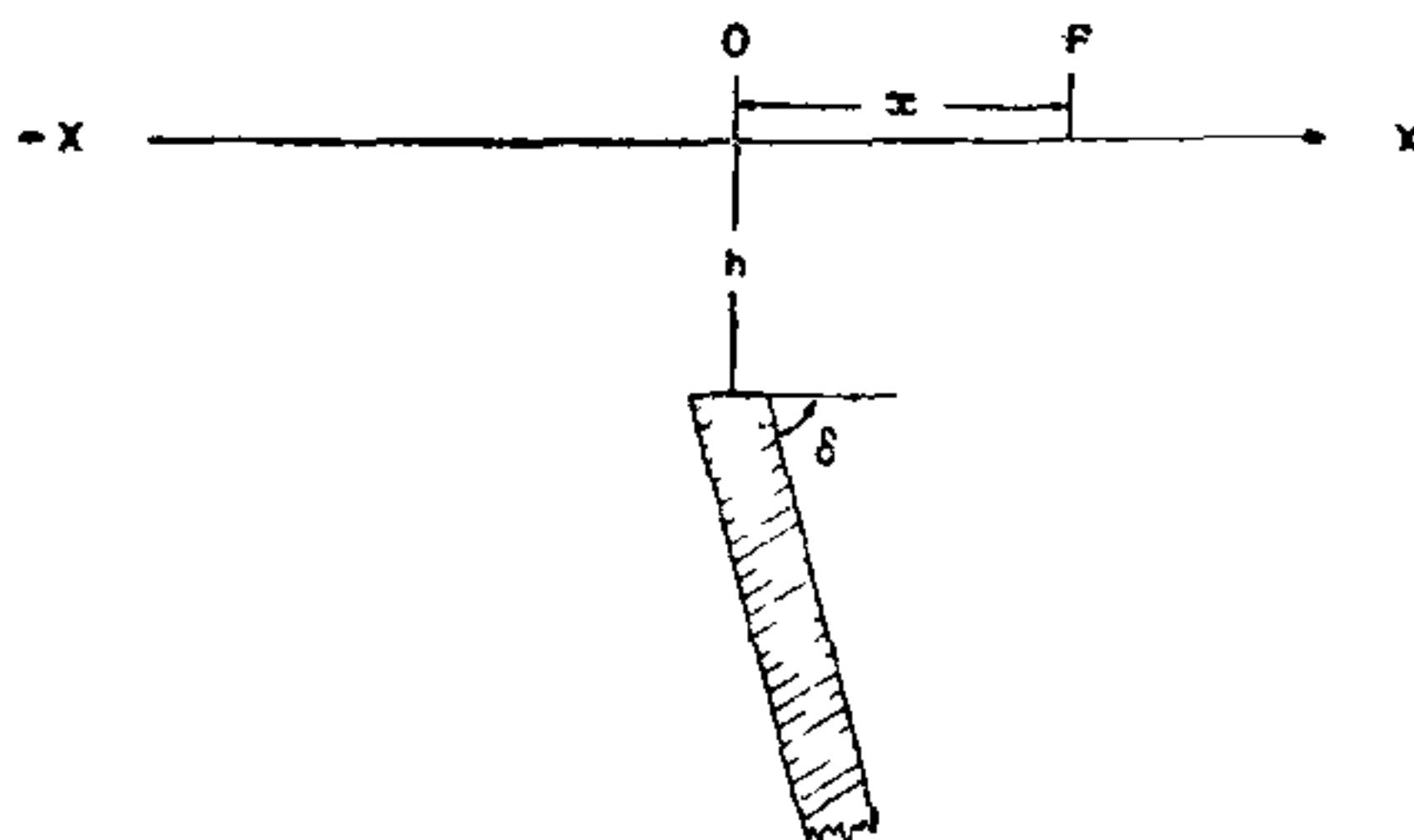


FIG. 1. Cross-sectional view of a thin sheet model.

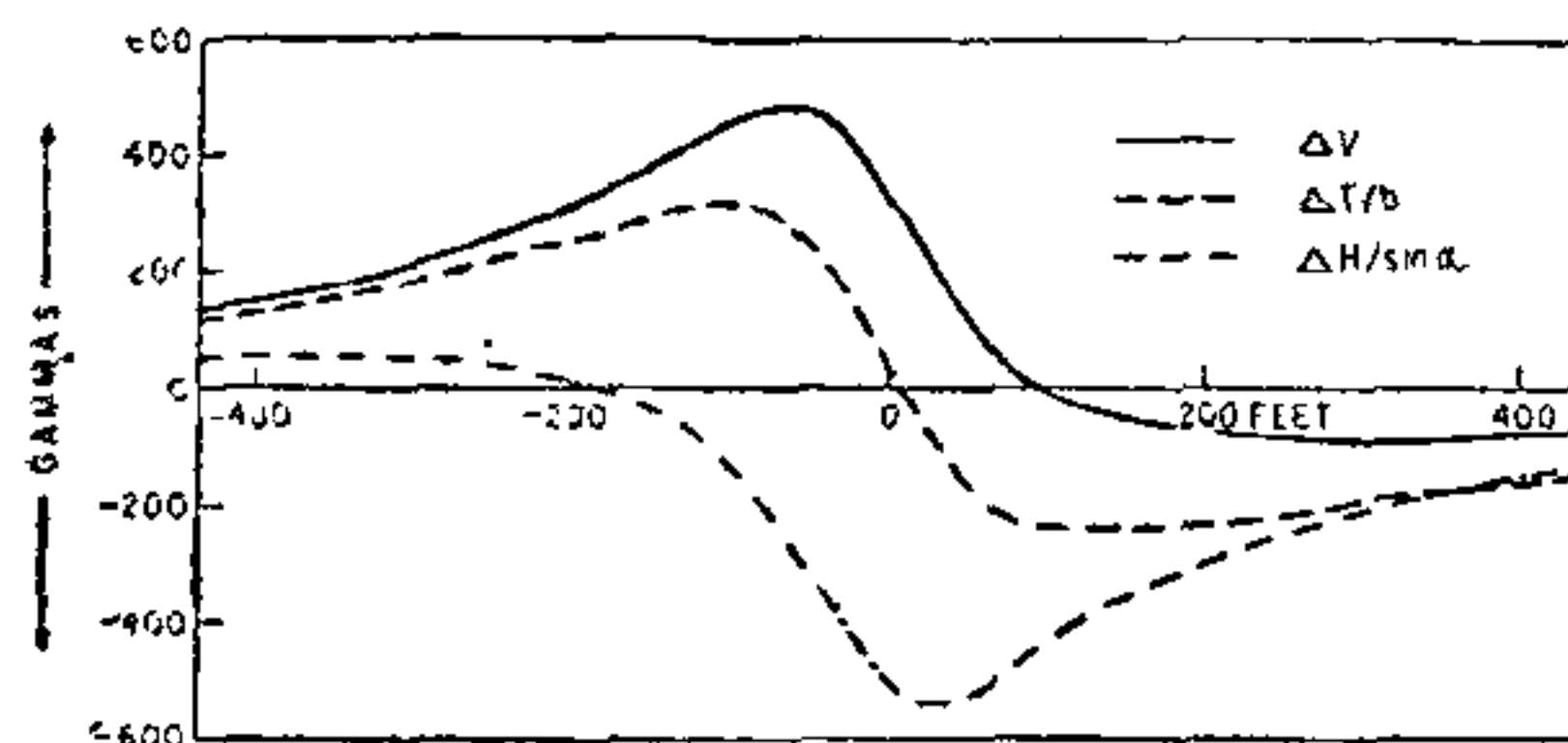


FIG. 2. Curves showing the measured vertical ( $\Delta V$ ) magnetic anomaly over the Pima Copper mine in Arizona, analysed by Gay<sup>4</sup> and the transformed horizontal ( $\Delta H$ ) and total ( $\Delta T$ ) magnetic anomalies.

Transformation from one component into the other of the anomaly functions  $\Delta V$ ,  $\Delta H$  and  $\Delta T$  defined in equations (1), (2) and (3) may be carried out using Hilbert transform. At present, we treat the Hilbert transform as an operator which transforms sine func-

tions into cosine functions and vice versa<sup>2</sup>. From the Table of Integral Transforms<sup>3</sup>, we have the following Hilbert transform pairs:

$$\left. \begin{aligned} \int_{-\infty}^{+\infty} \frac{x}{x^2 + h^2} dx &= -\frac{h}{x^2 + h^2} \\ \int_{-\infty}^{+\infty} \frac{h}{x^2 + h^2} dx &= \frac{x}{x^2 + h^2} \end{aligned} \right\} \quad (4)$$

Substituting the above transform pairs in equations (1), (2) and (3) we get the Hilbert transforms of  $\Delta V$ ,  $\Delta H$  and  $\Delta T$  as

$$\Delta V_H = A \frac{x \cos \theta - h \sin \theta}{x^2 + h^2} = -\Delta H \operatorname{cosec} \alpha \quad (5)$$

$$\Delta H_H = A \sin \alpha \frac{x \sin \theta + h \cos \theta}{x^2 + h^2} = \Delta V \sin \alpha \quad (6)$$

and

$$\Delta T_H = Ab \frac{h \cos (\theta + I) + x \sin (\theta + I)}{x^2 + h^2} \quad (7)$$

respectively.

From equations (5) and (6) we see that the vertical and the horizontal components of the magnetic anomaly due to a thin sheet are related by Hilbert transform and hence when either of the components is known the other can be computed. Now we know the basic relation among the three components of the magnetic anomaly, viz.,

$$\Delta T = \Delta V \sin i + \Delta H \cos i. \quad (8)$$

Using the relations given in equations from (5) to (8) we establish the following simple relations for transforming one anomaly component into the other.

#### (1) Transformation of $\Delta V$ to $\Delta H$ and $\Delta T$

From equation (5) we see that the horizontal ( $\Delta H$ ) component can be obtained directly from the Hilbert transform of  $\Delta V$ , i.e.,

$$\Delta H = -\Delta V_H \sin \alpha. \quad (9)$$

After obtaining  $\Delta H$ , we can compute  $\Delta T$  from  $\Delta V$  and  $\Delta H$  using the relation given in equation (8).

#### (2) Transformation of $\Delta H$ to $\Delta V$ and $\Delta T$

In equation (6) we found that the vertical component can be obtained directly from the Hilbert transform of  $\Delta H$  as

$$\Delta V = \Delta H_H \operatorname{cosec} \alpha. \quad (10)$$

Therefore after obtaining  $\Delta V$ , the total field component  $\Delta T$  can be computed using the relation given by equation (8).



### (3) Transformation of $\Delta T$ to $\Delta V$ and $\Delta H$

Using equations (5), (6), (7) and (8), we deduce the following relations among  $\Delta V$ ,  $\Delta H$ ,  $\Delta T$  and  $\Delta T_H$ :

$$\Delta V = (\Delta T \sin I + \Delta T_H \cos I)/b \quad (11)$$

$$\Delta H = \sin \alpha (\Delta T \cos I - \Delta T_H \sin I)/b. \quad (12)$$

Hence, from equations (11) and (12), the vertical and the horizontal components can be computed from the total component and its Hilbert transform.

The relations given in equations (8), (9) and (10) are fundamental to all the two-dimensional bodies. Bhattacharyya and Leu<sup>1</sup> have shown that the relations given in equations (5) and (6) are true for all two-dimensional bodies. Therefore, the transformations can be carried out using the above relations even in the case of other two-dimensional bodies.

### Computation of Hilbert Transform

The practical approach of obtaining the Hilbert Transform  $F_H(x)$  of any function  $F(x)$  using the Fourier Transform  $F(\omega)$  of  $F(x)$  has been suggested by Green<sup>5</sup> which involves the following procedure.

The Fourier Transform  $F(\omega)$  can be broken up into a series of sine and cosine terms of known amplitude. Let  $a_0, a_1, a_2, \dots, a_n$  be the amplitudes of the cosine terms and  $b_1, b_2, b_3, \dots, b_n$  be the amplitudes of the sine terms. Now produce a waveform made of cosine terms with amplitudes  $0, b_1, b_2, b_3, \dots, b_n$  and sine terms of amplitudes  $-a_1, -a_2, -a_3, \dots, -a_n$ ; then, the synthesized waveform is the Hilbert transform of the original waveform. It amounts to an interchange of the amplitude coefficients of the sine and cosine terms of the Fourier transform with a change in sign of the sine terms. The following three steps are involved in the computation.

- (1) Calculate the Fourier transform of the profile,
- (2) Set the negative frequencies to zero,
- (3) Take the inverse Fourier transform of the modified Fourier transform.

The resulting output gives the Hilbert transform.

### PRACTICAL EXAMPLE

The relations given in equations (8) and (9) are used to transform a vertical field magnetic anomaly ( $\Delta V$ ) over Pima copper mine in Arizona, taken from figure 10 of Gay's paper<sup>4</sup>. The  $\Delta V$  anomaly has been transformed into the total ( $\Delta T$ ) and the horizontal ( $\Delta H$ ) anomalies using the above relations and are shown in Fig. 2 along with the original  $\Delta V$  anomaly. Gay<sup>4</sup> interpreted the anomaly comparing it with the standard curves presented by him. The transformed  $\Delta T$  and  $\Delta H$  anomaly curves are found to fit well with the corresponding standard theoretical curves expected over the same body.

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## A STUDY ON THE LYSOZYME PATTERN IN MURINE LEUKAEMIA AND LYMPHOMAS

SUKTA DAS, ARUN BASU, AMAR MITRA, ASISH CHATTERJEE AND SANTOSH MITRA

Chittaranjan National Cancer Research Centre, 37, S.P. Mookerjee Road, Calcutta 700 026, India

**L**YSOZYME, a low molecular weight basic protein, is a non-specific mucolytic enzyme which is widely distributed in nature—both in the plant and in the animal kingdoms. Its presence in the majority of animal tissues and fluids has been demonstrated as early as 1922 by Fleming<sup>1</sup>. Recently, lysozyme determination in blood and urine has proved to be a valuable diagnostic tool in tubular kidney disease<sup>2</sup>, in

sarcoidosis<sup>3</sup>, certain tumors of the central nervous system<sup>4</sup> and specific types of leukaemias<sup>5,6</sup> in man. Similar studies on animals, however, are relatively few.

In an attempt to gain insight into the biological role of lysozyme in malignant neoplasia, determination of serum levels of this enzyme in a variety of tumor bearing mice was undertaken, and the changes that occurred during the development of the tumors noted.