PROPERTIES OF THE "RELATION FIGURE" BETWEEN THE VERTICAL AND THE HORIZONTAL FIELD MAGNETIC ANOMALIES OVER A LONG HORIZONTAL CYLINDRICAL ORE BODY

D. ATCHUTA RAO AND H. V. RAM BABU

National Geophysical Research Institute, Hyderabad 500 007, India

ABSTRACT

The *relation figure* between the vertical and horizontal field magnetic anomalies due to a long horizontal cylinder is found to be a *cardioid*. The axis of symmetry of the cardioid is inclined to the coordinate axis at an angle equal to $\pi - I$ where I is the inclination of the magnetization vector in the vertical plane containing the observational line. The depth to the centre and the radius of the cylinder are directly related to a few well-defined points on the cardioid.

INTRODUCTION

If the vertical (V) and the horizontal (H) field magnetic anomalies at every point on the observational line are represented by a point in the HOV coordinate system, the resultant curve is called the *relation figure*[2]. Werner used the relation figure between the vertical and the horizontal field magnetic anomalies to evaluate the parameters of a sheet-like body. Recently Stanley and Green[1] have used the relation figure between the gradients of the gravity anomaly due to a step model and suggested a method of interpretation. In this work, we have shown that the relation figure between the vertical and the horizontal field magnetic anomalies due to a long horizontal cylinder is a *cardioid* and related its properties to the parameters of the cylinder.

LIST OF SYMBOLS USED

V = Magnetic anomaly in vertical field;
H = Magnetic anomaly in horizontal field;
i = Inclination of the geomagnetic field vector,
T = Normal level of the total field intensity,
a = Strike angle of the cylinder measured with respect to the magnetic North;
x = Distance of the point of observation from the epicentre of the cylinder;
h = Depth to the centre of the cylinder;
R = Radius of the cylinder;
K = Susceptibility contrast of the cylinder to its surroundings,
I = $\arctan(\tan i/\sin a)$,
P = $2\pi KTR^2(1-\cos^2 i\cos^2 a)^{1/2}$
$a = \dfrac{P}{2h^2}$
$\theta = \pi - I$.

THEORY

Using the above nomenclature, the expressions for the magnetic anomalies in the vertical field (V) and the horizontal field (H) components along a line perpendicular to the strike of a long horizontal cylinder (Fig. 1) magnetized due to induction are given by

$$V = P[(h^2-x^2)\sin\theta + 2xh\cos\theta]/(x^2+h^2)^2 \quad (1)$$

and

$$H = P[(h^2-x^2)\cos\theta - 2xh\sin\theta]/(x^2+h^2)^2. \quad (2)$$

Let

$$f(x)\cos\phi(x) = P\frac{h^2-x^2}{(x^2+h^2)^2} \quad (3)$$

and

$$f(x)\sin\phi(x) = P\frac{2xh}{(x^2+h^2)^2} \quad (4)$$

where,

$$f(x) = \frac{P}{x^2+h^2} \quad (5)$$

and

$$\phi(x) = \tan^{-1}\left[\frac{2xh}{h^2-x^2}\right] \quad (6)$$

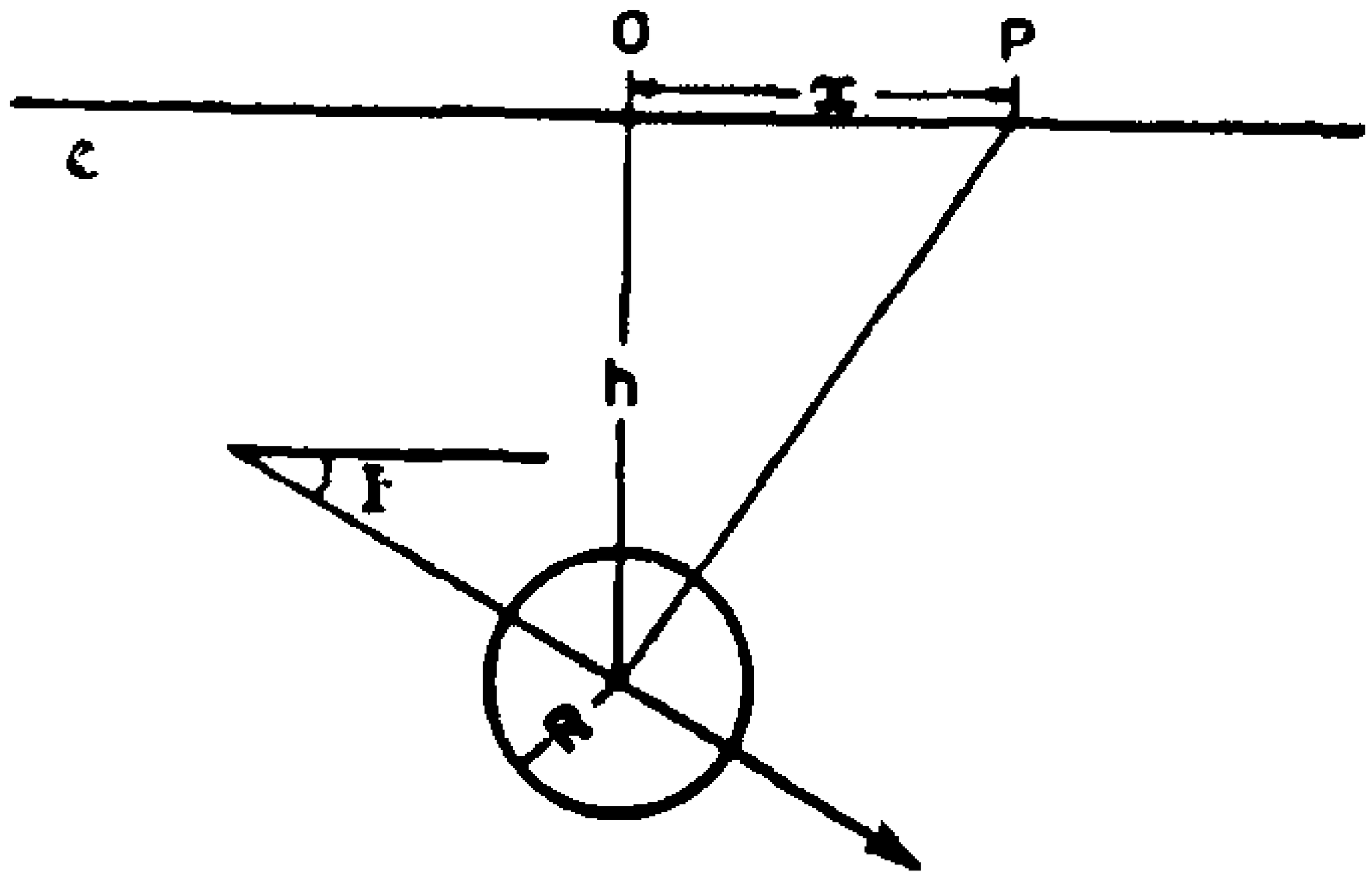

FIG. 1. Cross-sectional view of a long horizontal cylindrical ore body magnetized at an angle I.

Now equations (1) and (2) may be modified as

$$V = f(x)\sin[\phi(x)+\theta] \quad (7)$$

and

$$H = f(x)\cos[\phi(x)+\theta]. \quad (8)$$

From equations (7) and (8), it can be seen that the plot of H *versus* V in polar form will have the radius vector $f(x)$ and the azimuth $\phi(x) + \theta$. From equations (5) and (6), it can also be seen that the functions $f(x)$ and $\phi(x)$ are even and odd respectively. Hence, the plot of H *versus* V will be a symmetric curve whose axis of symmetry will be inclined to the coordinate axis at an angle θ. Rotating the coordinate axes by an angle θ, the new coordinate axes OM and ON are governed by the following relations:

$$m = f(x) \cos\phi(x) \quad (9)$$

$$n = f(x) \sin\phi(x) \quad (10)$$

where m and n are variables along the axes OM and ON respectively. From equations (3), (4), (9) and (10), we see that the plot of H *versus* V in the new coordinate system (MON) is governed by the quation

$$(m^2 + n^2 - am)^2 = a^2 (m^2 + n^2) \quad (11)$$

where

$$a = \frac{P}{2h^2}.$$

Equation (11) represents a *cardioid* (Fig. 2) whose axis of symmetry coincides with the OM axis.

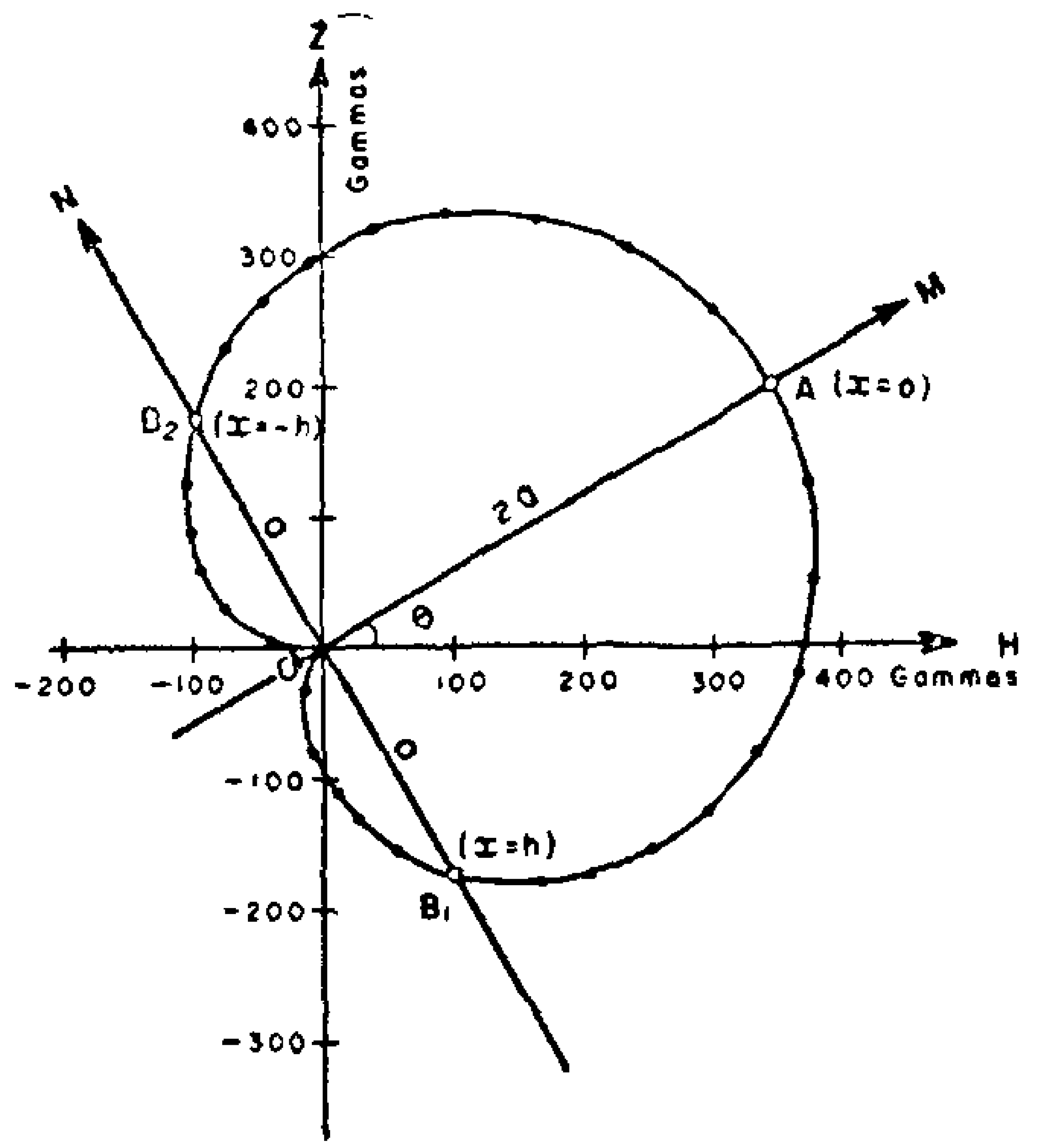

FIG. 2. Relation figure between the horizontal (H) and the vertical field (V) magnetic anomalies due to a long horizontal cylinder magnetized at an angle of 150° buried at a depth of 5 units below the observational plane. The thick dots from the origin correspond to different positions of x at which V and H values are plotted.

From equations (3), (4), (9) and (10), we derive the following properties of the cardioid.

At $x = 0$, m attains its maximum value equal to $2a$ and $n = 0$. At $x = \pm h$, $m = 0$ and $n = a$

At $x = \pm\infty$, $m = n = 0$.

From the above, we have the following four characteristic points on the cardioid (Fig. 2).

(1) The maximum value of m and the zero value of n occurs at $x = 0$ (the epicentre of the cylinder) and is denoted as point A in Fig. 2.

(2) The point at which $m = 0$ and $n = a$ (half maximum value of m) occurs at $x = h$. This point is denoted as B_1.

(3) The point at which $m = 0$ and $n = -a$ occurs at $x = -h$ and is denoted as B_2.

(4) The point where $m = n = 0$ occurs at $x = \pm\infty$ and is denoted as O.

The characteristic points of the cardioid are related to the parameters of the cylinder in the following way:

(1) *The inclination* (I) of the magnetization vector is related to the angle θ by the relation $I = \pi -$ where θ is the angle between the OH axis and the OM axis. The angle θ is reckoned positive in the anticlockwise direction from the OH axis.

(2) *The depth* (h) *to the centre of the cylinder* is equal to half of the difference between the x values where B_1 and B_2 occur.

(3) *The radius* (R) *of the cylinder* and the amplitude factor P are related by

$$P = 2\pi KTR^2 (1 - \cos^2 i \cos^2 \alpha)^{1/2}.$$

From equation (11) we have $P = 2ah^2$ and hence,

$$R = \left[\frac{ah^2}{\pi KT (1 - \cos^2 i \cos^2 \alpha)^{1/2}}\right]^{1/2}$$

In the above equation, all the terms (a, h, T, i and α) are known except K, the susceptibility contrast. Hence, assuming a proper value of K, the radius R can be evaluated.

ACKNOWLEDGEMENTS

Our sincere thanks are due to Dr. P. V. Sanker Narayan, Assistant Director, National Geophysical Research Institute, for his keen interest and encouragement in this work. We thank Dr. S. Balakrishna, Director, N.G.R.I., for his kind permission to publish this work.

1. Stanley, J. M. and Green, R., "Gravity gradients and the interpretation of the truncated plate model." In *Geophysics*, 1976, 41, 1376.
2. Werner, S., "Interpretation of magnetic anomalies at sheet-like bodies," *Sveriges Geologiska Undersokning, Ser. No. O.* 508, 1953.