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The Problem of Reality in Physics.*

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1. The Present Crisis in Science.

NOWADAYS we often hear the assertion that science is passing through a crisis. Popular works and newspapers speak of a "Bankruptcy" of science, and even some excellent representatives of science express the opinion that science is developing in a wrong direction. Others declare that the very aim of science, namely, the search for truth, is wrong, or, at least, fruitless; they only attribute a value to purposes of immediate utility. Others, again, fix their attention upon the radical change of the circumstances of life under the influence of technical sciences, and, regarding the numerous effects of industrialisation which have destroyed the equilibrium of social forces, often arrive at sceptical conclusions. And one of the strictest critics of our civilisation, the recently deceased Oswald Spengler, is, according to his great work *Decline of Western Civilization*, inclined to detect certain signs of decadence in some of the most glorious achievements of modern science.

While we cannot deny that there exists a crisis in our civilisation, manifesting itself in political and social restlessness, we also cannot doubt that in science, too, there is a certain crisis. This may give a justification for devoting a few words to the nature and importance of this crisis.

Many of you may be inclined to reject at once any doubt about the value of science. And I think, to the same group would belong everybody who has merely objectively witnessed the scientific development during the last decades, as well as the majority of those who take part in scientific movements. Science has passed from one triumph to another, succeeded in observing an immense multitude of facts and in explaining them from a unitary point of view; thus we are fully justified in calling the present time a golden age of science. And, if we consider the innumerable effects of science on practical life, the part steam and electricity play in it, or even the most recent inventions such as broadcasting, the applications of various radiations, aerial traffic, and so on, we likewise arrive at the conclusion that something causing such effects must certainly possess a deep-rooted intrinsic importance. For, even if we do not agree that the value

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of science is given by its practical availability, we must acknowledge that practical effects are, though rather external, yet the more easily discernible signs of its importance.

On the other hand, science has undoubtedly undergone a very profound transformation in recent years. Many principles which were until recently regarded as incontestable axioms have lost this importance, and this can of course produce in a superficial eyewitness a feeling of uncertainty. Many excellent investigators who contributed fundamental discoveries to this transformation a few decades ago were unable to follow the further development and are now inclined to condemn it.

If, however, we examine the development of physics more attentively, we gain the conviction that the recent turn was entirely healthy and logical, without any sign of deviation or partiality, although the transformation of our ideas was very deep and we had to renounce many of them which formerly seemed evident. First of all we must emphasise that the whole development was continuous and smooth. Although it took a formidable pace during the last decades, we can say that the great discoveries followed one upon another continuously since the renaissance and particularly during the past century. We can even assert that the latest transformations, the atomic and electronic theory, the theory of relativity and the quantum theory, developed very smoothly, without deeper controversies, as if with a general consent. Struggles such as *e.g.* accompanying the Darwin theory of descent did not arise and it did not come, to the formation of great antagonistic schools. There was, of course, some contradiction, but partly of sheer outsiders and partly of physicists, who, although some of them had great merits as experimenters, were not sufficiently familiar with the language of mathematics which is so indispensable for expressing and understanding the physical contents of the new theories. All these controversies, although assuming sometimes, particularly in the case of the theory of relativity, rather grotesque forms, had only an insignificant influence on the development of science; among those possessing the necessary command of mathematics there was hardly any opposition and the foundations of the new theories now appear obvious facts in general opinion.

The theory of relativity presented itself immediately in a perfect shape, as the climax of a long evolution. The development of the quantum theory, however, was quite different. This fundamental theory of modern physics emerged as a very special hypothesis which could not be fitted into the frame of classical physics, and it grew after a long struggle to the present general theory. If leads to a modification of classical physics which is even greater than that effected by the theory of relativity, it demands a radical change in anciently-rooted ideas and presents a great number of unsolved problems. But, it has finally opened new domains and interpreted such important phenomena (I refer only to the theory of spectra and to the waves of matter), that its elimination from physics is impossible.

The continuity in the development of physics shows that the new great theories actually include the old ones; they acknowledge the validity of classical physics within the realms of observation it was meant to explain.

It follows that there is nothing in the development of physics to weaken our confidence in the healthy state of science. On the other hand, we notice that actually there is succh a distrust as a social phenomenon of to-day. Perhaps it is so because of the long continuance of the rapid development of science in a particular direction which, possibly, is not quite free from a certain onesidedness. Perhaps people are feeling that it is high time for other components of our civilisation to keep pace with the development of the exact sciences. Perhaps the effect produced by science upon the forms of life, the transformations in the production of goods, the modifications in the structure of human society are taking place too rapidly to ensue in harmony; this results in phenomena which certainly can be called critical. The problem of how to eliminate social and economic disharmony, however, cannot be solved by stopping the development of science; it must be solved, on the contrary, by the methods of science themselves. Slogans such as "stop science" and similar demagogic anti-scientific catch words which we can hear nowadays in certain countries with a glorious scientific past, certainly can cause heavy damage to the development of science; such slogans are, however, not produced by the evolution itself of science but are mere symptoms of the general crisis of our civilisation.

Whilst we have thus to reject most decidedly any superficial and harmful assertion concerning the bankruptcy of science, we cannot deny that we have to face serious problems with regard to the effects of scientific development upon life as well as to its philosophical interpretation.

This lecture is intended to give the presentation of a problem of the scientific interpretation of the world. I hope to make a contribution to the solution of this problem which is the problem of *reality* in physics, the question what is to be regarded as *real* in natural sciences. Just in these days there is an ardent polemic on this matter between "positivists" and "realists"—a polemic which, of course, does not touch the existence and the foundations of science.

2. The Notion of Reality in Common Life and in Philosophy.

As long as we take a naive, uncritical standpoint with respect to the contents of our consciousness, the problem of reality does not present itself. In this stage everything occurring in consciousness appears as real; objects and sounds, dreams and fancies. The problem of reality first emerges when, in order to acquire a unitary scientific view of the world, we begin to distinguish between essential and unessential, between permanent and transient, between "real" and "apparent". These categories are correlative and appear always simultaneously. We shall consider a few examples taken from common life in order to fix the meaning we want to attribute to these words.

If my house burns down or a friend dies, these events will have portentous consequences upon my whole life; they will greatly influence the future contents of my consciousness. If, on the other hand, these events occur in my dreams only, they will have no such consequences; I shall dwell in my house as before and I shall continue to meet my friend. The two cases are quite differently related to other events of my life: the first is of serious consequence, the second remains an isolated event scarcely related to others. When we say that a political movement has a reality, whilst another lacks reality, then we understand that the former cannot be ignored without risking serious consequences, whilst the latter can be left out of consideration without incurring a risk.

When a solid body runs against me, it will hit me and possibly hurt me. On the contrary, a shadow running against me and falling on my body will have no serious consequences. When I plunge a rod into water, it will appear angular; after removing it out of the water, it is straight again. In other cases, when a rod appears angular, it cannot be used again and will soon fall into pieces. We cannot seize a *fata morgana* or a rainbow; these do not behave as real bodies.

These examples show how we discern in common life between real and apparent, or even between different degrees of reality. We see that it is the importance or the effectiveness which serves as a measure of reality, which is thus fixed by a judgment of valuation. It is often the mere persistence in time which forces us always to reckon with a certain thing and which consequently serves as a criterion of reality.

In philosophy the notion of reality has a similar meaning.

We remember the famous simile in Plato's dialogue on the "State"; here the perceptions are compared to shadows of bodies projected on a wall by a fire behind the spectator, bodies being in movement in front of the fire. The shadows represent the realm of the "apparent", the bodies moving in front of the fire those of the "real" beings; the simile has to explain that the objects as they appear to our senses belong to the realm of the apparent, whilst the eternal and unchanging "ideas" are representations of the sphere of the real. In Indian philosophy also, real and apparent are contrasted in a striking manner. Thus in Vedanta philosophy the world of *Maya* as the realm of the apparent, of illusion, contrasts with *Brahman*, the unchanging, eternal, perfect, solely real being, the only existence which really deserves this expression.

In Buddhist scripture the things of the world are compared to "foam floating upon the Ganges" to "the *pisang* which lacks a solid framework", to a *fata morgana*, to a "bubble of water"; they are transient and unimportant. Contrasting with them there exists something defined by negative attributes only: the *Nirvana*, which, with all its negativity, plays exactly the same part as, in other systems, the highest degree of reality, the absolute reality or the Deity.

3. Realism and Nominalism.

In philosophy, a fundamental problem of any system is to determine what is to be regarded as real. Likewise, in the exact sciences the same problem is always present and passes through their history, as we shall see later. Here we shall mention two antagonistic points of view, the struggle of which runs through the entire history of philosophy and, in a modified form, can be found in physical discussions upon the question of reality. These antagonists are *realism* and *nominalism*. The idea of realism, that is, of philosophical realism, first emerged when people became conscious of the existence of abstract ideas, and recognized their importance. To-day we can hardly imagine how astonishing and admirable this perception must have been; the long-searched for Absolute Reality was thought to be found in abstract ideas. Thus, when Pythagoras began to have a present time of the actual importance of numbers, he thought them to be the true fundamental reality and made of them an object of almost religious worship. And Plato perceived Ideas as the really existent things, in contrast with the transient, alterable bodies given by sensuous perception; he founded a system of philosophy which, in a way is still existing.

The question concerning the nature and importance of ideas has since then always been on the programme of philosophy.

Opposing this realistic standpoint of Plato, there is another point of view, according to which there is no reality beyond the transient, alterable sensuous things; notions are mere names. Hence this standpoint is called "nominalism". These points of view are fighting one against another in the ideas upon the foundations of modern physics.

4. Positivism.

In the philosophy of to-day, nominalism appears in a characteristic form in the direction of epistemology which was started by the works of Ernst Mach, the Vienna physicist and philosopher, in the closest connection with the problem of the natural sciences and particularly of physics, and has become almost an official philosophy of the quantum theory of to-day. Phillip Frank is one of those who have worked out this program in detail. But also the great representatives of the quantum theory, Bohr, Heisenberg, Dirac and Schrödinger, share this way of thinking; Jordan has recently given a very clear and unexaggerated presentation of it in his newly published book. On the other hand, Planck, Laue and Sommerfeld are the chief representatives of the realistic point of view in physics.

According to Mach, our only direct data available are the sensations. Any other thing is composed of them; it is but a complex of sensations, either simultaneous or not. So are our notions, our logical functions such as judgment and so on. There is no physical reality independent of sensations; the postulate of such an independent reality is a mere logical construction which serves to express connections between our sensations, to foretell coming sensations. That my writing-desk is real, means only that some optical, haptical or thermal sensations by which its notion is defined always arise when circumstances are suitable, for example, when my eyes are opened, when a lamp is burning or when my hands are in a certain position. Beyond this nothing can be said and thus it is no problem at all but a meaningless question to ask whether the writing-desk exists when I am not observing it.

More generally, it has no sense at all to speak of the existence of a thing which cannot be observed; thus, *e.g.*, it is a senseless question whether other worlds exist which are in no connection with our world. *Nothing exists that cannot be observed*: this is the view of positivism.

The recent positivism that has arisen from quantum mechanics is much less engaged in the analysis of sensations; it considers sensations rather as symbols which can completely be replaced by pointer-readings on instruments. They *should* even be replaced by pointer-readings for the sake of exactitude and unambiguity. This is obvious to the physicist; thus a certain colour can unequivocally be described from a physical point of view by a wave-length as the result of a diffraction experiment, unless we have to do with its bearing on physiology. According to this view the fundamental facts are pointer-readings or coincidences of a pointer and a point of the graduation; the problem is to find functional relations between different pointer-readings, to infer from certain pointer-readings other ones, occurring possibly in the future.

5. Analysis of Immediate Experience.

The common point of view, however, is quite different from this. Our attention is commonly directed to the things themselves instead of our sensations of them; in common life we neglect the accidentals adhering to our sensuous impressions. When we form an idea of a thing or a person, we do not imagine a certain perspective view of the object under a certain illumination, although our impressions always refer to such a particular picture; we think, on the contrary, of characteristic features that are common to all views of the object. The art of primitive nations deals, in the first place, with characteristic features of the object: the face is represented in profile, the eyes seen in front; it is a much later stage when a singular impression is brought into consciousness by a conscious spiritual effort. We try to pick out of the variable sensuous impressions a constant kernel on which, by preference, our attention is fixed; in common life we hardly care for the immediate pure sensuous impressions which only concern the psychologist in the moment he is at work. To fix the attention on the moon, or to be conscious of a yellowish circular optical impression, are quite different things; in the first case the yellowish circle is only a sign, a symbol of an object. This difference has, of course, nothing to do with the question whether I am, in an actual case, succumbing to an illusion or not. The elucidation of these things represents an important merit of the new psychology and phenomenology. In a similar manner, the properly so-called mental activities, judgments, and so on, cannot be derived from sensuous impressions and their succession; they are actually separated from them by a wide gap. If a physicist looks upon the sensuous impressions as the only ultimate elements, he accepts an entirely obsolete and incorrect psychology. Mach already has extended the circle of the immediately given elements by including the space-time structure of the world in it, and Jordan, in his above-mentioned excellent book, points out that certain "totality conditions", Ganzheitsbedingungen, must also be taken into consideration. Nevertheless, I think that positivistically-minded physicists do not recognise clearly enough how narrow and partial their picture of the immediately given elements of the world is. It would be very advantageous for them to learn to know some important psychological and phenomenological researches concerning the question of what is immediately given to us. The works of Husserl, Stumpf, Messer, Scheler are also accessible to physicists, though they may find some difficulties in the terms used by the philosophers. So far as I know H. Weyl is the only mathematician and physicist who is aware of these problems.

6. Physical Realism.

On the other hand, the state of mind of a physicist is much nearer to that of the common man than to the positivistic theory, as soon as he deals with a concrete physical problem, instead of the philosophy of physics. His interest is then directed, not to sensations or pointer-readings which play the rôle of mere tools, but to the objects themselves. In his paper entitled "Wege der Erkenntnis" Max Planck gives a very plastic expression to this point of view. Our theories refer to the moon, to liquids and solids, to atoms, molecules and electric fields, instead of direct sensual impressions. The fundamental task of physical research is just the construction of a description of the world which is independent of our individuality and of limitations imposed by our organism. This, of course, can only be accomplished by a process of gradual development; we acquire step by step pictures of more and more profound reality. We can see this very clearly in the development of astronomy. The Ptolemaic cosmology attaches itself immediately to the observations of the curiously slung paths of planets as they appear to a terrestrial observer. On the other hand, the Copernican theory renounces a direct reference of the observed data to the terrestrial observer; it relates the planetary movements to the Sun and obtains thus very simple paths instead of the former complicated ones. By this simplification a path was opened to the establishment of fruitful hypotheses and the discovery of the laws of gravitation by Newton was made possible. Here the progress consisted of the choice of an invariant description instead of a description from the point of view of a particular observer. By this choice a mathematical description of the phenomena for any observer was rendered possible.

This process repeated itself much later in the theory of relativity and, in a more abstract form in the Dirac transformation theory of quantum mechanics.

The positivistic standpoint which endeavours to reduce everything to pointer-readings is, of course, not incorrect, as the facts can be described in this manner also; it is, nevertheless, extremely partial and even incorrect as soon as it pretends to be the only possible expression of the absolute truth. A comprehensive theory cannot be built up if nothing but the elements from which we start are permitted to figure in it. Otherwise our way of proceeding would be analogous to an effort to remove abstract ideas. Indeed, we teach children by telling them that two apples *plus* three apples equal five apples; two horses *plus* three horses equal five horses; but as soon as the child has conceived the idea of the abstract numbers, further learning on concrete examples is unnecessary.

7. Nominalism and Realism in Physics.

The antagonism between positivism and physical realism is analogous to the relation between nominalism and philosophical realism. According to nominalism and also to positivism, the only directly given data are the sensuous impressions; anything beyond them is a result of intellectual construction and is reducible to sensuous impressions. In opposition to this, realism attributes self-existent reality to notions and physical objects respectively. Here trivial contradictions result, of course if we do not discern sufficiently between physical objects and abstract notions. The notion of the number 3 does not float in the air between other things, nor exists in the time; but it has its "place" in the row of integers, *viz.*, between the numbers 2 and 4. We often hear that abstract notions can be constructed arbitrarily, whilst physical objects are given things with fixed attributes. We want to examine this question more closely. We can term a thing a Three, a Four or a Multiplication; by this, however, we only specify the notion we have in our mind, just as if we would state what physical object we intend to examine. After we have done so, we are no more free, *e.g.*, in prescribing what factors the number 6 may contain. The following arithmetical example may be useful with regard to the case of sensuous things and of physical objects also. An irrational number can be defined as the limit of certain sequences of rational numbers. Any proposition concerning irrational numbers can be interpreted as a proposition concerning such sequences; a representative positivistic philosopher, Ph. Franck, even says that an irrational number is nothing but a name and is identical with the sequence which serves to define it. This statement is true in so far as any proposition can be described by referring to the sequence only. It would, however, be partial and even false, if we would regard it as the only possible expression of the facts. We can, indeed, define an irrational number by means of an equation of which it is a solution; we can methodically deal with it on this basis as is done actually in the theory of algebraic numbers.

Quite similarly, we can express any relation between complex numbers as a relation between real numbers; we can regard a complex number simply as a system of real numbers. But we can also consider complex numbers as self-existent quantities defined by the rules they are obeying.

Correspondingly, we can regard a matrix equation in quantum mechanics as a symbolic expression of a finite or infinite number of ordinary equations. But this is one possibility only. We can also consider the matrices as defined by their operational rules and admitting an infinity of representations by means of ordinary numbers, none of which possesses a higher degree of reality than the others. In his profound book, fascinating in its ingenious exposition also, Dirac speaks of the notions of quantum mechanics as follows :

" The new theories, if one looks apart from their mathematical setting, are built up from physical concepts which cannot be explained in terms of things previously known to the student, which cannot even be explained adequately in words at all. Like the fundamental concepts (*e.g.*, proximity, identity) which every one must learn on his arrival into the world, the newer concepts of physics can be mastered only by long familiarity with their properties and uses."

The fundamental quantities of quantum mechanics, the operators, are so far from our familiar notions that we cannot wonder at the acuteness with which the question of physical reality was reopened by the quantum mechanics.

8. The Relativity of the Logical Basis.

The above-mentioned examples from arithmetic (irrational and complex numbers, matrices) show that we can regard the same fact from many points of view, which are not at all in contradiction one to another. I think this fact leads to an adequate judgment of the relation between realism and nominalism, as well as between physical realism and positivism. In a logical system it is to a certain degree arbitrary what is to be considered as belonging to the basis of the system and what as a deduction. Thus we can build up geometry with many systems of axioms which we can choose differently among the propositions, just as we can represent a vector by means of different systems of fundamental vectors.

In my opinion, the same applies to the question concerning the mutual relation of nominalism and realism and the relation between direct sensuous experience and intellectually constructed physical reality. The characteristic feature of positivism is that according to it anything which is not directly observed possesses but a lower degree of importance. In connection with this, we must remark however, that the question of what is to be considered as immediate sensuous experience is not at all so simple and unambiguous as a naive psychology might suppose. We must bear in mind that things are arranged in a space-time order and that the nature of this order is an ancient problem of philosophy. In my view, the space-time order of the things is already the result of an elementary intellectual treatment of the raw material given by pure experience ; we can call it a " natural " theory of the world. Positivism or nominalism and realism are two possible standpoints differing in the choice of the elements used for constructing a description of the world. They are both as well justified as the cosmologies of Ptolemy and Copernicus which differ only in the choice of the system of co-ordinates. We must be careful, however, not to believe that all similar philosophical and epistemological systems are true in every respect. We must, on the contrary, judge each case on its own merits, supposing that a conclusive judgment is possible at all at the present state of our knowledge.

9. Changes of the Notion of Physical Reality.

The history of physics offers numerous examples for the diversity of the ideas which at different times, served as fundaments for the description of the physical world.

Thus, in the infancy of physics, when it was not yet separated from philosophy, the central rôle was played by the doctrine of the four elements ; fire, air, water and earth. To-day, in the possession of the idea of the chemical element, one is, in the first moment, inclined to smile at this naive conception. But looking more deeply into the matter, we see that the doctrine was quite adequate to the general level of those times. It contains, essentially, the most common representatives of the three states of matter and, besides, fire as the representative of temperature, the idea of which was not yet worked out clearly enough at that time. The view that all bodies result from a suitable composition of those four elements is likewise not to be interpreted in the strict quantitative manner of modern chemistry; it means only that the state of all bodies can be expressed by means of those four "elements". The doctrine of the four elements appears thus as a primitive but by no means foolish attempt at systematising the reality. It only becomes absurd and senseless if looked upon from the present quantitative standpoint of science.

Another example is given by the cosmologies of Ptolemy and Copernicus which we have just discussed.

A particularly instructive example of changes in the idea of physical reality is offered by electromagnetism. In the classical theory of electricity and magnetism the fundamental laws refer to charges and poles, the forces acting between them being given by the well-known Coulomb laws. In this system electric and magnetic fields are of secondary importance. Energy and potential appear as important auxiliary notions which serve for expressing properties of systems of charges; their physical reality, however, is of a lower degree than that of the charges; they possess no substantiality.

The electromagnetic field and the energy appear, then, as fundamental notions and even as substances in Maxwell's electrodynamics. Here the field is not a mere auxiliary construction, determined by the distribution and motion of the charges in a certain moment; with Maxwell, the field is a self-existing entity and the charges are, in a certain sense, degraded to mere singularities of the field, to places where the lines of force join one another, the divergence being there different from zero. Energy, on the other hand, is localised and possesses a mass. Thus the view expressed first, though in a somewhat vague manner, by W. Ostwald, according to which energy is to be considered as the fundamental substance, was realized in a concrete form in electrodynamics and in Einstein's theory of relativity, representing a foundation-stone of our present conception of physics. (We can, of course, interpret the equations of Maxwell by referring them directly to electrical charges; this would, however, be rather forced.)

A more recent example is given by the quantum theory of the electromagnetic field. It was known for long that a given electromagnetic field can be developed into a Fourier series. But it was the quantum mechanics which attributed a self-existent importance to the individual terms of this Fourier series, considering them as a sort of co-ordinates and submitting them to the procedure of quantization. This idea of Dirac has then been applied to the case of waves of matter (Jordan and Wigner).

The criterian of reality suffered a thorough alteration in quantum mechanics and this process has, perhaps, not yetc ome to an end. With regard to this matter, I shall limit myself to a few hints. In the first place it turned out that the fundamental notions of classical mechanics such as the momentum, energy, etc., are to be replaced by certain operators which lead to the possible values of the corresponding quantities as they can be measured by observations. The mutual relations of the dynamical variables are also to be replaced by relations between the corresponding operators. These new relations, however, are generally not identical with the corresponding classical relations. The notion of the state of a system has also undergone incisive alterations as compared with the classical way of thinking. It turned out that in contrast to classical mechanics, we cannot attribute numerically determined values to all variables of a system simultaneously. Thus, *e.g.*, we cannot determine simultaneously both the position and the velocity of an electron. Since we can, according to the positivistic view, only speak of quantities which can actually be measured, we must say that in a given state of a system certain mechanical variables (*e.g.*, the position of an electron, the velocity of which has previously been determined exactly) actually have no sharply defined values at all. If we make a measurement which leads to a defined value of such a quantity, then we must admit that after the measurement the system is no more in the state which existed before the measurement, since in this state the measured quantity had no value at all.

If, however, the measurement refers to a quantity which has been measured immediately before, then the repetition must lead to exactly the same value. By this fact, I think, we are entitled to attribute to a state the same objective existence as in the classical theory ; the change would only refer to the manner in which the state is defined. We often hear the opinion that we cannot attribute objective characteristics to a system, since the data describing it depend on the results of measurements. This view, however, is not correct. We need a certain measurement in order to bring the system into a defined state ; after this is done, any further measurement which refers to the same quantity and thus does not alter the state of the system, leads to the same result.

We can also describe a state in the following manner. A certain measurement may have previously been performed which leaves the system in a state in which a certain quantity other than the measured one (the so-called "conjugate" quantity) has no defined value at all. Then the state is completely described if we know the probability of any possible result of a measurement of this second quantity. These probabilities are given by the solutions of the Schrödinger equation. The ψ of Schrödinger, the wave function, describes the state of a system by relating it to a given ground state. For example, if the state of a hydrogen atom is given by fixed values of the energy, of the absolute value of the angular momentum and of a component of the latter, then $|\psi|^2 \, dV$ gives the probability of finding the electron in a space element dV. This we must understand in the following manner. The measurement of the position of the electron is, in the sense explained above, incompatible with an exact measurement of the energy or momentum. In consequence of this, we destroy the state with a fixed energy and angular momentum by measuring the position, we then obtain a new state with a determined position but undetermined energy and momentum. The probability of obtaining a given position by measuring it in a system with given energy and momentum is then determined by Schrödinger's equation.

10. Summary.—The Value of Positivism.

The essential contents of our exposition may be summarised by saying that positivism and realism, and nominalism and realism respectively are both admissible points of view if they are carried through correctly ; they only differ in their starting-points. Positivism, in the form which was developed under the influence of quantum mechanics, leads to a profound change of our idea of the physical state of a system. In my opinion, however, it does not destroy the notion of the objectivity of a state ; it merely claims that, for defining a state, certain necessary measurements must have been performed. By this demand the objective existence of a state is not affected, just as the objectivity of space is not affected by the fact that when I say that a star is at the zenith, I must add the geographical position and the time the zenith is referred to.

On the other hand, I must object to the psychological doctrines concerning immediate experience as they are used in the physical literature of to-day. Similarly, I cannot agree with certain exaggerated positivistic statements, as for example with the identification of observability and existence.

In connection with this, I may return briefly to a frequently heard example of a statement which is meaningless in the positivistic sense, namely, to the possible hypothesis that a world may exist which has no connection with our world. Such a hypothesis could, of course, not be tested by observation. But we can imagine the case that the masses in the world gradually accumulate at two different places and thus the world divides into two separate parts. After this process is completed, two worlds exist which are possibly without any connection.

There is an analogy between this example and the problem of the psychological accessibility of strange persons. The feelings or impressions of other people are entirely inaccessible to a direct observation ; for example, I do not know whether the impression my friend has at seeing a certain colour is the same as my own impression or not. Here we can help ourselves with analogies only. And now I think we are on the wrong path if we infer that the existence of my friend consists merely of my having certain sensuous impressions of him.

Finally, there are parts of the world which I never can observe and yet I must suppose them as existing. I speak of the state of the world after my death. I can feel deeply concerned at the development of politics or of science after my death, although these things cannot be observed by me.

In the book which we have mentioned already, Jordan claims that it is the life of the human race instead of the individual life to which all similar problems must be related. In this case, however, we must decide to acknowledge an existence which transcends the circle of our own experience, namely, the existence of other persons which is more than a mere part of our own individual experience. I think that in doing so the dogmatic positivism falls to pieces. There survives, however, in any case the important work performed by positivism in eliminating many meaningless questions, and there survives the profound modification by the quantum mechanics of our idea of the physical state.

I think that the idea of physical reality needs a long development to appear at last in complete clarity and that this development has been strongly advanced by positivism. On the other hand, quantum mechanics most probably cannot be considered as the final stage of this development. Quantum electrodynamics and the problem of the elementary corpuscles even point at profound gaps which are yet to be filled.

Man is inclined to consider the stage he has just arrived at as the revelation of the absolute final truth; this illusion is almost a matter of course. It is good therefore to remember Newton's words:

"I do not know what I may appear to the world, to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."*

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* D. Brewster, *Life of Newton*, p. 338.