LETTERS TO THE EDITOR

A PLANE SYMMETRIC UNIVERSE FILLED WITH PERFECT FLUID

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A PLANE symmetric cosmological model for perfect fluid has been derived. The geometry of the universe is described by the line element

$$dS^2 = -dT^2 + T dx^2 + T^{1/2(1+a)} dY^2 + T^{1/2(1-a)} dZ^2 \quad (1)$$

a being a constant. Considering the Einstein's field equations for perfect fluid distribution

$$R^k_i - \tfrac{1}{2} R\, \delta^k_i + \Lambda\, \delta^k_i = -8\pi\left[(\epsilon + p) V_i V^k + p\delta^k_i\right] \quad (2)$$

the distribution in the model (1) is given by

$$8\pi p = \frac{5}{16T^2} - \frac{a^2}{16T^2} - \Lambda \quad (3)$$

$$8\pi\epsilon = \frac{5}{16T^2} - \frac{a^2}{16T^2} + \Lambda. \quad (4)$$

The model has to satisfy the reality conditions (Ellis[1]) (i) $\epsilon + p > 0$ and (ii) $\epsilon + 3p > 0$ which requires that

$$a^2 < 5 \text{ and } \Lambda < \frac{5 - a^2}{8T^2}. \quad (5)$$

The coordinate system turns out to be comoving with $V^4 = 1$. Clearly $V^4_{;j}$, $V_j = 0$, so that the flow is geodesic. Following the method outlined by Tolman[2] the red shift in the model is given by

$$\frac{\lambda + \delta\lambda}{\lambda} = \frac{\left[T_1^{1/4(a-1)} + U_z\right]}{T_2^{1/4(a-1)}\left[1 - U^2\right]^{1/2}} \quad (6)$$

where U is the velocity of the source at the time of emission and U_z the z-component of the velocity. The expression for the expansion θ for the flow vector V^i is given by

$$\theta = \frac{1}{T}. \quad (7)$$

The rotation ω is identically zero and the magnitude of the shear is given by

$$\sigma^2 = \tfrac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{48T^2}\left[1 + 3a^2\right]. \quad (8)$$

Using $\frac{1}{\rho}\rho_{,\mu} V^\mu = \frac{1}{3}\theta$, the scalar of linear dimension vector ρ^μ is given by

$$\rho = KT^{1/3} \quad (9)$$

K being a constant. The surviving components of the conformal curvature tensor C^{jk}_{hi} for the line-element (1) are

$$C^{14}_{14} = C^{23}_{23} = \frac{1}{24T^2}\left[1 + a^2\right]$$

$$C^{12}_{12} = C^{34}_{34} = -\frac{1}{48T^2}\left[1 + 6a + a^2\right] \quad (10)$$

$$C^{13}_{13} = C^{24}_{24} = -\frac{1}{48T^2}\left[1 - 6a + a^2\right].$$

Thus it follows that the space-time given by (1) is of Petrov type 1. It is also to be noted that the model starts expanding from its singular state at $T = 0$ and its expansion decreases with time till its geometry is that of a flat space-time.

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ON SPHERICALLY SYMMETRIC ELECTROMAGNETIC FIELD DISTRIBUTION

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IN a general spherically symmetric (SS) space-time domain with the line element

$$ds^2 = -A dr^2 - B d\Sigma^2 + C dt^2 \quad (1)$$
$$d\Sigma^2 = d\theta^2 + \sin^2\theta\, d\phi^2.$$

Takeno[1] has shown that a SS vector field A_i must have the structure

$$A_i = (A_1, 0, 0, A_4), \quad (2)$$

where A_1 and A_4 are functions of (r, t) only. This result implies that F_{14} is the only surviving component of a SS electromagnetic field provided we adopt the following definition of a SS electromagnetic field. Here F_{ij} stands for Maxwell electromagnetic skew tensor.

Definition

An electromagnetic field is SS if its 4-potential A_i is SS in the sense of Takeno, *i.e.*, it has the structure given in (2), in a suitable gauge.

Note that the above result (that only $F_{14} \neq 0$ in a SS F_{ij}) is consistent with the fact that the non-zero components of a general second rank skew-tensor F_{ij} are given by[1]

$$F_{14} = f_1, F_{23} = f_2 \sin \theta,$$

where f_1 and f_2 are functions of (r, t) only.

Since the electromagnetic 4-potential A_i admits arbitrary gauge transformations, we note that in a general gauge, the structure of a SS A_i is given by

$$A_i = \left(A_1 - \frac{\partial f}{\partial x^1}, -\frac{\partial f}{\partial x^2}, -\frac{\partial f}{\partial x^3}, A_4 - \frac{\partial f}{\partial x^4}\right). \quad (3)$$

Evidently the special structure given in (2) is preserved under the subclass of gauge transformations with $f = f(r, t)$.

It is interesting to note that the above definition of SS elctromagnetic fields can be used in general relativity to simplify the proof of Birkhoff's theorem in the presence of electromagnetic fields. It is easy to see that in the presence of a SS electromagnetic field, the non-vanishing components of the electromagnetic energy tensor T_{ij} are given by

$$T_{11} = -I^2/2C, T_{22} = r^2 I^2/2AC,$$
$$T_{33} = T_{22} \sin^2 \theta, T_{44} = I^2/2A. \quad (4)$$

where $F_{14} = I$.

For the general SS line-element (1), the field equations

$$R_{ij} = -8\pi T_{ij} \quad (5)$$

and the energy tensor (4), yield, on integration,

$$A = A(r) \quad (6)$$

and

$$AC = H(t), \quad (7)$$

where H is a function of t alone. Changing t to a new variable T such that

$$T = \int [H(t)]^{1/2} dt \quad (8)$$

and using (7) we get

$$ds^2 = -A(r) dr^2 - r^2 d\Sigma^2 + [A(r)]^{-1} dT^2 \quad (9)$$

which evidently is static. This establishes Birkhoff's theorem in the presence of a SS electromagnetic field.

In classical text-books[2,3], while obtaining the Reissner-Nordstrom (RN) solution, F_{14} is shown to be only non-vanishing component of F_{ij} by assuming the space-time to be static. On the other hand, if we assume the electromagnetic field to be SS, as we have described earlier, it is not necessary to assume the space-time to be static.

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D–X SYSTEM OF COPPER BROMIDE MOLECULE

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Copper bromide molecule is known to have four band systems, namely, A system (5100–4600 Å), B system (4600–4200 Å), C system (4600–3900 Å) and D system (4000–3700 Å) (see refs. 1–2). All these systems involve ground state ($X^1\Sigma^+$) as the lower state. The D system of this molecule was first observed in emission by Rao and Rao[2]. The rotational structure in the (2, 0) and (0, 1) bands of this system was analysed by Rai *et al.*[3] Since the two bands analysed do not involve any common vibrational level, the correctness of the analysis could not be verified. Moreover the structure reported by Rai *et al.*[3] appears to be quite diffuse (due to large slit width) creating more doubt in the analysis. Therefore it is thought worthwhile to restudy the structure of this system in continuation to our previous work on copper bromide[4,5]. We have photographed the structure of (0, 0), (1, 1), (0, 1), (0, 2) and (2, 0) bands under high resolution and the details of this study are given here.

The bands were excited using a 125 watt microwave oscillator. A 99·9% pure (Riedel) copper bromide sample was used. The bands were found developing well when the colour of the discharge was deep violet. The bands were photographed in the second order of a 10·6 m concave grating spectrograph with 0·33 Å/mm dispersion. An exposure of 15 hours