SHORT COMMUNICATIONS

SOME GENERALIZED FRIEDMANN MODELS

ABDUSSATTAR
Department of Mathematics, Banaras Hindu University, Varanasi 221 005, India.

RECENT observations with microwave back-ground radiation and cosmic abundances of He^4 and D suggest that the universe must have been slightly inhomogeneous and anisotropic in the past though its present large scale structure is fairly well described by the Friedmann models[1]. In this note we present an anisotropic and inhomogeneous generalization of the Friedmann models with flat space-sections as probable models of the early universe.

The necessary conditions for the cylindrically symmetric metric of Marder,

$$ds^2 = A^2(dt^2 - dx^2) - B^2 dy^2 - C^2 dz^2 \qquad (1)$$

where the metric potentials A, B, C are functions of x and t to represent a perfect fluid distribution and the explicit expressions for the pressure and density in terms of the curvature components have been obtained by Singh and Abdussattar[2]. Here we consider a special case of this metric when the metric potentials A and B being equal are functions of t alone and C is a function of x and t. This makes $f_2 = 0$. In order that the coordinate system[2] may be comoving we require $F = 1$ *i.e.* $f_2 = 0$ (see ref. 2 eqn (1.13)). This gives $C = [M(x) + N(t)]A$ where M and N are arbitrary functions of x and t respectively. Equations (1.9) and (1.10) (see ref. 2) imply $M_{11} = 0$ and $2A_4N_4 + AN_{44} = 0$. Integrating these equations we get

$$M = Px + Q \quad \text{and} \quad N = K\int \frac{1}{A^2}\,dt + L$$

where P, Q, K, L are arbitrary constants of integration. The metric (1) can be written as

$$ds^2 = A^2 \left[(dt^2 - dx^2 - dy^2) - \left(1 + \alpha\int \frac{1}{A^2}\,dt + \beta x\right)^2 dz^2\right] \qquad (2)$$

where α, β are arbitrary constants and A is some function of the time t. Suppose the transformation $T = \int A\,dt$ gives t as some function of T say $\psi(T)$ then $A(t) = A(\psi(T)) = R(T)$ and

$$\int \frac{1}{A^2}\,dt = \int \frac{1}{R^2(T)}\frac{dt}{dT}\,dT = \int \frac{1}{R^3(T)}\,dT$$

Thus the line-element (2) can be expressed as

$$ds^2 = dT^2 - R^2\left\{(dx^2 + dy^2) + \left(1 + \alpha\int \frac{1}{R^3}\,dT + \beta x\right)^2 dz^2\right\}$$

where R is some function of the cosmic time T. If we take $\alpha = \beta = 0$ the metric reduces to the Friedmann models with flat space-sections.

The distribution in the model is given by

$$8\pi p = -\frac{\dot{R}^2}{R^2} - \frac{2\ddot{R}}{R} \qquad (4)$$

$$8\pi\rho = \frac{3\dot{R}^2}{R^2} + \frac{2\alpha R}{R^4\left(1 + \alpha\int \frac{1}{R^3}\,dT + \beta x\right)} \qquad (5)$$

where an overhead dot($\cdot$) denotes differentiation with respect to time T. The volume expansion θ and the shear σ for the models (3) are obtained[3] as

$$\theta = \frac{3R}{R} + \frac{\alpha}{R^3 \quad 1 + \alpha\int \frac{1}{R^3}\,dT + \beta x}, \qquad (6)$$

$$\sigma^2 = \frac{\alpha^2}{3R^6\left(1 + \alpha\int \frac{1}{R^3}\,dT + \beta x\right)^2} \qquad (7)$$

A representative length l given by the equation

$$\frac{l^*}{l} = \frac{1}{3}\theta$$

which represents[3] the volume behaviour of the fluid is obtained as

$$l = \chi(x) R \left(1 + \alpha\int \frac{1}{R^3}\,dT + \beta x\right)^{1/3}$$

where $\chi(x)$ is an arbitrary function of x. The pressure and density in terms of the Hubble parameter H, the deceleration parameter q and the shear σ can be expressed as

$$8\pi p = -H^2(1 - 2q) - \sigma^2, \qquad (8)$$

$$8\pi\rho = 3H^2 - \sigma^2 \qquad (9)$$

where

$$H = \frac{l^*}{l} \quad \text{and} \quad q = -\left(\frac{l^{**}}{l}\right) \frac{1}{H^2}$$

The reality[3] conditions $(\rho + p) > 0$ and $(\rho + 3p) > 0$ demand that $H^2(1+q) > \sigma^2$ and $3qH^2 > 2\sigma^2$.

The explicit form of the function R can however be determined by assuming an equation of state. For example in the case of incoherent fluid distribution $p = 0$ gives $R = (aT+b)^{2/3}$ where a and b are arbitrary constants of integration. For this value of R from equations (5), (6) and (7) we get

$$8\pi\rho = \frac{4a^2}{3(aT+b)^2} \times \left[1 + \left(\frac{\alpha}{(1+\beta x)a(aT+b) - \alpha}\right)\right] \tag{10}$$

$$\theta = \frac{a}{(aT+b)} \times \left[2 + \left(\frac{\alpha}{(1+\beta x)a(aT+b) - \alpha}\right)\right] \tag{11}$$

and

$$\sigma^2 = \frac{\alpha^2 a^2}{3(aT+b)^2\{(1+\beta x)a(aT+b) - \alpha\}^2} \tag{12}$$

From the above equations we see that the relative rate of the fluid shear σ^2/θ^2 and the density contrast $\partial/\partial x(\log\rho)$ both tend to zero as $T \to \infty$ which indicate that the model ultimately evolves to the isotropic and homogeneous structure.

24 February 1982; Revised 21 August 1982

1. Wagoner, R. W., in *Confrontation of cosmological theories with observational data*, *IAU Symp.*, 1974, **63**, (ed.) M. S. Longair (Reidel).
2. Singh, K. P. and Abdussattar, *J. Phys. A: Math. Nucl. Gen.*, 1973, **6**, 1090.
3. Ellis, G. F. R., in *General Relativity and Cosmology*, 1971, (ed.) R. K. Sachs (Academic Press).

ZIRCONS IN QUARTZOFELDSPATHIC GNEISS FROM KOTTAYAM DISTRICT, KERALA

NARAYANASWAMY AND A. M. NAIR

Centre for Earth Science Studies, Trivandrum 695 010, India.

ZIRCONS are physically as well as chemically resistant to geological processes, and, therefore, their morphology has been statistically studied for years to interpret the history and petrogenesis of rock formations[1-4]. The present study is based on the fact that the morphological characters of zircons are reflections of the varied physicochemical environments of formation of the rocks. The paper pertains to the gneissic rock associated with the migmatised charnockites and allied gneisses in the Western Ghat section at Mundakkayam in Kottayam district, Kerala.

LOCATION AND GEOLOGICAL SETTING

The rock under discussion has been collected from the banks of Manimala river at Mundakayam (76° 53′ 5″: 9° 32′ 15″) in Kottayam district Kerala. The area in and around this location belongs to the Precambrian terrain containing migmatised charnockites and its variants as the major lithological units, and also granitic gneisses, quartzofeldspathic gneisses and lenses of pink feldspathic granites. These rocks are cut across by dolerite and gabbro dykes. The quartzofeldspathic gneiss is conformable in trend to the adjoining charnockitic variants. The general trend is nearly N-S with 55° easterly dip.

PETROGRAPHY

The rock is leucocratic, medium to coarse-grained and gneissic in texture. It contains quartz and feldspar; biotite is the dominant ferromagnesian mineral which imparts the gneissic fabric.

Texturally, the rock is medium to coarse grained and inequigranular, showing feeble gneissose texture. Quartz is anhedral and feldspar dominates over quartz. The plagioclase feldspar ranges in composition from oligoclase to andesine. Antiperthitic and myrmekitic growths are also observed. Feldspars seem to be of two generations, the earlier one exhibits well-developed twinning and the latter one is devoid of twinning. The major mafic constituents are green and brown biotite. Accessories include zircon, apatite and opaques.

STUDIES ON ZIRCONS

Recovery of zircons: Zircons are separated from the gneiss in accordance with the standard separation techniques[5]. The rock sample is crushed to pass through 80 mesh (ASTM) taking precautions to avoid fine crushing. This rock powder is washed and by panning the heavy concentrates are obtained. Magnetic fractions are then removed with a hand magnet and the concentrates are treated with dilute hydrochloric acid with a few drops of stannous chloride to remove the iron stains. It is further treated with bromoform (Sp-gr.2.9) to get rid of the lighter minerals. The fraction is fed to the Frantz Isodynamic magnetic separator (25° forward tilt and 15° side tilt) to obtain a pure zircons from the non-magnetic fraction. Final treatment with