VECTOR-MATRIX REPRESENTATION OF BOOLEAN ALGEBRAS AND APPLICATION TO EXTENDED PREDICATE LOGIC (EPL)—Part II†

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6. NON-MATRIX AND MATRIX CONNECTIVES IN EPL

(*a*) *Unary connectives,* E, N, M, L, *for affirmation and negation*:

As shown in Table 4, Section 5(c), these operators permute the four states $\forall$, Φ, Λ, $\exists$, among themselves, and similarly permute the two states Σ and Θ between themselves and Δ and $\varnothing$ within the pair. The algebraic equations that define these transformations, for the operator in $\mathsf{aZ}=\mathsf{b}$ (for E = affirmation, N = negation, M = complementation and L = ellation:), are

$$\mathsf{Z}=\mathsf{E}\ :\ a_\gamma=b_\gamma,\ a_\delta=b_\delta,\ a_\epsilon=b_\epsilon, \tag{20a}$$

$$\mathsf{Z}=\mathsf{N}\ :\ a_\gamma=b_\epsilon,\ a_\delta=b_\delta,\ a_\epsilon=b_\gamma, \tag{20b}$$

$$\mathsf{Z}=\mathsf{M}:\ a_\gamma^c=b_\gamma,\ a_\delta^c=b_\delta,\ a_\epsilon^c=b_\epsilon, \tag{20c}$$

$$\mathsf{Z}=\mathsf{L}\ :\ a_\gamma^c=b_\epsilon,\ a_\delta^c=b_\delta,\ a_\epsilon^c=b_\gamma. \tag{20d}$$

It is readily verified that $\mathsf{N}^2=\mathsf{M}^2=\mathsf{L}^2=\mathsf{E}$, so that all three have the nature of *negating* a term to which they are applied, but in three different ways.

(*b*) *Binary connectives* E and G:

The unary "affirmation" operator E is also the binary "equivalence" operator, for which the matrix form

$$|E|=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \tag{21a}$$

is the best representation for practical use. Thus, if

$$(\mathsf{a}\Leftrightarrow\mathsf{b})=\mathsf{aEb}=(a\,|\,E\,|\,b)=(1\ 0)=\mathrm{T} \tag{21b}$$

then a and b have the same 3-vector canonical form. However, just as in SNS, this operator does not have the property of giving F if $\mathsf{a}\not\Leftrightarrow\mathsf{b}$. For this purpose, we require the "agreement" operator[5] G, similar to the SNS G. We also define this operator G in EPL, similar to SNS, by the equations

$$(\mathsf{aG}=\mathsf{b})\Longleftrightarrow a_\gamma=b_\gamma,\ a_\delta=b_\delta,\ a_\epsilon=b_\epsilon \tag{22}$$

The binary operator G gives T if the three components of the two 3-vectors are *all* alike, and F otherwise. It is useful for designing the canonizer and the standardizer discussed in the next section. For lack of space, we shall not give the Boolean algebraic expression for $(a\,|\,G\,|\,b)=\mathsf{c}$.

(*c*) *Canonizer and Standardizer*

We utilize the formulae in Tables 7(a) and 7(b) for this purpose. Taking Table 7(a), it can be verified that the application of the canonizer Z gives

$$\mathsf{q}'\mathsf{Z}=\mathsf{q} \tag{23}$$

where $q'=(q'_\gamma\ q'_\delta\ q'_\epsilon)=(\gamma'\ \delta'\ \epsilon')$ of the standard form $(\zeta)\,(\gamma'\ \delta'\ \epsilon')\,(\alpha\ \beta)$, and $\mathsf{q}=(\gamma\ \delta\ \epsilon)$. The nature of Z is as in Column 3 of Table 7(a).

For the standardizer, we use the information taken in for the canonizer in the reverse direction. We suppose that the canonical state $\mathsf{q}=(\gamma\ \delta\ \epsilon)$ is given, and we require (ζ) and $(\alpha\ \beta)$, given $\mathsf{q}'=(\gamma'\ \delta'\ \epsilon')$ of the standard form $(\zeta)\,(\gamma'\ \delta'\ \epsilon')\,(\alpha\ \beta)$. This is done as follows.

(i) Calculate $\mathsf{n}_1, \mathsf{n}_2, \mathsf{n}_3, \mathsf{n}_4$, equal to qZ for Z = E, N, M, L respectively, and find out the SNS state of $(\gamma_j\,|\,G\,|\,q')=\mathsf{g}_j$.

(ii) If g_j is T, for some j = 1, 2, 3 or 4, then the standardized state is given by the six elements, in the row corresponding to j, in Table 7(b).

(iii) If all g_j are F, ζ can be taken to be 1, and we calculate $\mathsf{q}\vee\mathsf{q}'=\mathsf{q}''$ and then the SNS state of the relation $(\mathsf{q}''\,|\,E\,|\,q)$, (one of T, F, D, X) gives the state of s. These are summarized in Table 7(b).

The proofs of these are reserved for a detailed paper.

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(*d*) *Boolean operator connectives* U *and* V *in BA-3*

We have already seen that these two operators U and V in logic, respectively correspond to the Boolean "sum" ($\oplus$) and the Boolean "product" ($\otimes$) in Boolean algebra. They have been defined already for *n*-vectors and used for SNS in [1], corresponding to BA-2. Similarly, the operator V used in a'Va" in BA-3, brings out the state that is *common* to two state-vectors a' and a". For example, if a' = ∀ = (1 0 0), and a" = ∃ = (1 1 0), then a'Va" is (1 0 0) $\otimes$ (1 1 0) = (1 0 0), *i.e.* "for all". (Note the analogy to the concept of "intersection" in set theory. However, the connective "and" in QPL has other interpretations, representable by 64 possible matrices ∀ (*i*, *j*), *i*, *j* = 1 to 8, (as shown in Section 6(f) below) in EPL).

On the other hand, if the logical operation has the property of taking as true the information provided by either one of the two sources, (as with rumour), then we must employ the connective operator U. (Note the connection, in this case, with "union" in set theory. Here again "or" in QPL is described in full, only by a set of 64 O(*i*, *j*) operators—see (f) below).

(*e*) *Singular matrix connectives* $S_{\lambda\mu}$:

As we have already seen, the input and output vectors in EPL are all canonical 3-vectorsof the type $a \equiv (a_\gamma a_\delta a_\epsilon)$ and $b = (b_\gamma b_\delta b_\epsilon)$ so that a general matrix connective between them is representable by 3 × 3 Boolean matrix $|Z|$. Any matrix connective Z in EPL is thus representable by $|Z|$, which is a sum of matrices $|S_{\lambda\mu}|$ as

$$|Z| = \sum_{k=1}^{K} |S_k(\lambda_k, \mu_k)|, \quad K \le 9 \tag{24}$$

The matrix $|S_{\lambda\mu}|$ has a component $S_{\lambda\mu} = 1$ (λ, μ = One of γ, δ, ϵ) and 0 otherwise, and may be called the "singular matrix" $|S_{\lambda\mu}|$ and a general 3 × 3 Boolean matrix is a sum of *at most* nine such singular matrices. Thus, the "and" relation between (a_γ 0 0) and (0 0 b_ϵ) is

$$(a_\gamma | S_{\gamma\epsilon} | a_\epsilon) = (c_\alpha c_\beta) \tag{25}$$

and this will give c ≡ T only for a ≡ ∀ and b = Φ and c = F, for all the other eight out of the nine possible combinations of the basic states.

Because of this, the result of any *unary relation* $\langle a | Z | = \langle b |$ or any *binary relation* $\langle a | Z | b \rangle$ = c is a Boolean direct sum of the application of 9 or less singular matrices. However, we shall discuss below particular 3 × 3 matrices, since they have a direct *logical* interpretation in EPL.

(*f*) *Sixty four operators each of types* A *and* O *and their relation to* I *and* J:

In logic, one often gets a relation like ∀(*x*) & ∃(*y*) = c. An examination of the relational matrix for the connective "and" in this shows that if the "and" is between a *q*(*i*) and a *q*(*j*), then the matrix $|A(i, j)|$ has

TABLE 7

Algorithmic Table for the Canonizer and Standardizer

(a) Canonizer q'Z = q

Sign ζ	State of sentence s (α β)	Canonizer Z
1	T (1 0)	E
1	F (0 1)	N
0	T (1 0)	M
0	F (0 1)	L
Any	D (1 1)	D*, q = Δ
Any	X (0 0)	X*, q = ∅

(b) Standardizer, yielding ζ and (α β), given q and q'

Value of j for g_j = T	Standardized state		
	(ζ)	(γ' δ' ε')	(α β)
1	1	qE	T (1 0)
2	1	qN	F (0 1)
3	0	qM	T (1 0)
4	0	qL	F (0 1)
None	1	qVq' = q" q"Eq = s (α β)	

These are defined in subsection (vi), but we can straightaway write q = Δ = (1 1 1) in the former case, and q = ∅ = (0 0 0) in the latter.

non-zero elements for those λ for which the elements of $q(i)$ are non-zero and for those μ for which the elements of $q(j)$ are non-zero. Considering our case, ∀ corresponds to $i = 1$, and only $q_\gamma(1)$ is non-zero and ∃ corresponds to $j = 6$ (of Table 5), with $q_\gamma(6), q_\delta(6)$ non-zero. Then the corresponding matrix for ∀(1, 6) has only $A_{\gamma\gamma}$, $A_{\gamma\delta}$ non-zero out of its 9 components. Thus,

$$|A(1,6)| = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \text{ for } (\forall \ \& \ \exists) \quad (26)$$

In the same way 64 A(i, j)'s can be formulated, for the 64 "and" relations, q(i) A(i, j) q(j) with $i, j = 1$ to 8 (for specific examples, see Section 7). The corresponding matrix $|A(i, j)|$ is, as in (26), the "outer product" of $|q(i)\rangle$ and $\langle q(j)|$ *via.* the Boolean product operator ⊗ of BA-1, giving

$$|A(i,j)| = |q(i)\rangle \otimes \langle q(j)| \quad (27)$$

Table 8(a) illustrates the rule for the outer product. Similarly, the EPL "or", illustrated by O(1, 6) in Table 8(b), has for its matrix, the Boolean "outer sum" of $q(1)$ and $q(6)$. Hence, in general,

$$|O(i,j)| = |q(i)\rangle \oplus \langle q(j)| \quad (28)$$

Proofs of (27) and (28) are omitted for lack of space.

TABLE 8

Generation of $|A(1, 6)|$ and $|O(1, 6)|$ as the outer product and sum of $\langle q(1)|$ and $\langle q(6)|$

(a) $|A(1, 6)| = |q(1)\rangle \otimes \langle q(6)|$

q(1) \ q(6)	1	1	0
1	1	1	0
0	0	0	0
0	0	0	0

(b) $|O(1, 6)| = |q(1)\rangle \oplus \langle q(6)|$

q(1) \ q(6)	1	1	0
1	1	1	1
0	1	1	0
0	1	1	0

Using just the definitions (27) and (28) for A(i, j) and O(i, j), all the interrelations between "and", "or", "if" and "only if" are derivable, if we take over the definition of the complement $|Z^c|$ of $|Z|$ from the general theory of Section 2. The series of equations from (29) to (32) all follow purely from 3 × 3 Boolean matrix algebra, using the above correspondence with connective operators A and O of EPL, and by taking over the inter-relations between A, O, I, J of SNS[1].

We shall use the notation $q(i^c)$ also for $q^c(i)$, in which the relation between i and i^c is as in (29), if the serial numbers in Table 5 are used for $q(i)$ and $q(j)$ in (27) and (28).

$$i^c = i + 1, \ j^c = j + 1, \text{ if } i \text{ and } j \text{ are odd}$$
$$i^c = i - 1, \ j^c = j - 1, \text{ if } i \text{ and } j \text{ are even} \quad (29)$$

Then, the eight binary connective A, A^c, O, O^c, I, I^c, J, J^c of classical logic have the interrelations given in (30 to 32) (not all are listed, but only the more essential ones). Thus,

A(i, j) ≡ O$^c(i^c, j^c)$ (First de Morgan relation) (30a)

A$^c(i^c, j^c)$ ≡ O(i, j) (Second de Morgan relation) (30b)

For the implications I(i, j) ("if", in the forward direction) and J(i, j) ("only if" in the forward direction), we have

I(i, j) ≡ O(i^c, j) ≡ A$^c(i, j^c)$ (31a)

J(i, j) ≡ O(j^c, i) ≡ A$^c(j, i^c)$ (31b)

I(i, j) ≡ J(j, i) (Contrapositive form) (31c)

Similarly, for the denials of the relations I and J, we obtain Eqns (32a, b).

I$^c(i, j)$ ≡ O$^c(i^c, j)$ ≡ A(i, j^c) (32a)

J$^c(i, j)$ ≡ O$^c(j^c, i)$ ≡ A(j, i^c) (32b)

Two of the A's require special mention, namely D = A(7, 7) and X = A(8, 8). D has the property of converting any vector $q(i)$ into $q(7) = (1\ 1\ 1)$ by the unary operation $\langle q(i)|D|$, while X does the opposite, namely converting all vectors $q(i)$ into $q(8) = (0\ 0\ 0)$ by the formula $\langle q(i)|X|$. These matrices have been used in Table 7.

In addition to these, we must mention the matrix representations of E and N, which are used quite often.

$$|E| = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \ |N| = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad (33)$$

Thus, out of the 2^9 (= 512) possible 3 × 3 Boolean matrices, only 130 are used for the connectives of EPL to serve as representations of commonly utilized logical relations. The others could be used for rare occasions, *via.* the singular matrix sum representation for any 3 × 3 Boolean matrix. Some of them have good logical sense, *e.g.*

$$|R| = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (34)$$

makes b always ∀ in $aR = b$, irrespective of the state of a; but we shall not describe any of these.

7. Application of Vector-matrix Algebra to EPL

In this last section, we have considered the algebra of the logical connectives of EPL. These are however, applicable only to the canonical form of QPL terms. We shall now consider the use of these matrices for unary and binary relations of EPL, but with only one variable. The extensions to more variables, and to the cases where the sentence s itself contains more than one term connected by logical relations, can be made, but they are not discussed in this paper.

(a) Binary relations:

It has not been recognized so far, that the 3×3 matrix that represents a particular relation (*e.g.* "and") is not unique, but has 64 variations in the BA-3 representation—$A(i, j)$, $i, j = 1$ to 8. Thus, taking $A(1, 6)$ illustration (see Table 7), it has the necessary property that it will give

$$(a \mid A(1,\ 6) \mid b) = T \quad (35a)$$

only if

$$\langle a \mid \epsilon \langle q(1) \mid (= \forall) = (1\ 0\ 0)$$
and $\quad (35b)$
$$\langle b \mid \epsilon \langle q(6) \mid (= \exists) = (1\ 1\ 0)$$

This agrees with our intuitive concept of the "and" relation and, for $aA(1,\ 6)b = c$ we have for any input states of $a(= q(k))$, and $b(= q(l))$, the equations

$$\langle q(k) \mid A(1,\ 6) \mid q(l) = c_\alpha \quad (36a)$$
$$\langle q(k) \mid A^c(1,\ 6) \mid q(l) = c_\beta \quad (36b)$$
$$c = (c_\alpha\, c_\beta) \quad (36c)$$

giving the SNS state of the relation c.

The same process can be applied for $O(i, j)$ (and for $A^c(i, j)$ and $O^c(i, j)$ also). Taking *e.g.* $O(1,\ 6)$, we can verify from the matrix that

$$(a \mid 0(1,\ 6) \mid b) = T \quad (37a)$$

only if

$$\langle a \mid \epsilon \langle q(1) \mid (= \forall) = (1\ 0\ 0)$$
or $\quad (37b)$
$$\langle b \mid \epsilon \langle q(6) \mid (= \exists) = (1\ 1\ 0)$$

which again agrees with our ideas regarding the logical connective "or", as applied to QPL.

It is quite likely that binary relations of the type c & d, where one is an SNS term and the other is a OPL term can occur. In such cases, we use 2×3 matrices $A(T, j)$ or $A(F, j)$ defined exactly as in (27). In this, $\langle q(i) \mid$ becomes one of the basic SNS 2-vectors $T(= (1\ 0))$ or $F(= (0\ 1))$ as required, while $\langle q(j) \mid$ is an EPL 3-vector. A similar procedure is adopted for $A(i, T)$ and $A(i,\ F)$. We shall not pursue this further, but an example is given in Section 7(c).

The reversal of a binary relation gives a unary relation as explained in Section 3, and therefore we shall not discuss binary reverse connectives, but only unary connectives.

(b) Unary connectives:

The most important unary connective is implies (⟹) $\equiv I(i, j)$. The straightforward way of obtaining this matrix is by its equivalent form $A^c(i, j^c)$. That this has the necessary logical properties is seen as follows. We know that the SNS "implies" (I) gives for $aI = b$ the consequences $a_T \longmapsto b_T$, $a_F \longmapsto b_D$. In the same way, if we take, for example, $aI(1,\ 6) = b$, then $(a = \forall) \longmapsto (a = \exists)$, and $(a = \neg\forall = \Lambda) \longmapsto (b = \Delta)$. The matrix $\mid I(1,\ 6) \mid = \mid A^c(1,\ 5) \mid$ is readily seen to have these properties. It is also seen that the analog, in EPL, of the SNS equivalence $(\langle a \mid I \mid = \langle b \mid \Longleftrightarrow \langle b \mid J \mid = \langle a \mid)$, where $I^t = J$, is (31c).

With these preliminaries, we shall work out a practical example to indicate how facile the matrix representation of EPL and SNS is for working out actual problems.

(c) The audience-concert-crowd problem:

The problem given below employs unary and binary QPL connectives, an SNS to QPL connective, and also the canonizer and standardizer. The problem is first stated in words, then in our notation for logic, and finally in the vector-matrix formalism. The reversal of the steps is, however made completely in the matrix formalism, except for the last step of translating the result into the form required. It is obvious that all the steps are computerizable.

Problem

(i) If not all the audience seats are unoccupied, and some of the musicians are present, the concert will go on.

(ii) If the concert takes place, all windows will be opened, otherwise all windows are shut.

(iii) If some windows are open, the verandah will be partly or fully filled with people listening to the music.

(iv) There are no people found in the verandah. Prove that, if, in addition, all seats are occupied, no musicians have come.

We write the three steps of the problem in the notation of EPL in Table 9.

TABLE 9

QPL problem in standard notation

	Symbols	Logical equations in standard form
A	Seats s x; Musicians m y Concert c	$\neg(\Phi x)$ (sx) & $(\exists y)$ (my) = c
B	Window w z	c $\Longrightarrow (\forall z)$ (wz), $\neg$c $\Longrightarrow (\Phi z)$ (wz)
C	People p u	$(\exists z)$ (w z) $\Longrightarrow (\exists u)$ (pu) (Note: $(\exists u) \equiv (\forall u) \oplus (\Sigma u)$)

(α) *Boolean-algebraic notation*

The relevant equations are (38), (39), (40) and we explain them below.

Part A

$$(0\ 1)\ (0\ 0\ 1)\ (1\ 0) \overset{C}{\Longrightarrow} (1\ 1\ 0) = q(6) \text{ for s} \qquad (38a)$$

$$(1\ 0)\ (1\ 1\ 0)\ (1\ 0) \overset{C}{\Longrightarrow} (1\ 1\ 0) = q(6) \text{ for m} \qquad (38b)$$

$$(\exists s)\ \mathrm{A}(6, 6)\ (\exists m) = \mathrm{c};\ |A(6, 6)| \equiv \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (38c)$$

Part B

$$\mathrm{c}R = \mathrm{w}, \qquad |R| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (39a)$$

$$(1\ 0) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = (1\ 0\ 0);$$

$$(0\ 1) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = (0\ 0\ 1) \qquad (39b)$$

Part C

$$(\exists w)\ \mathrm{I}(6, 6) = (\exists p);\ |I(6, 6)| = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad (40a)$$

Verify:

$$(1\ 1\ 0) \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = (1\ 1\ 0) \qquad (40b)$$

$$(1\ 1\ 0) \overset{S}{\Longrightarrow} (1)\ (1\ 1\ 0)\ (1\ 0) \qquad (40c)$$

We first canonize the inputs for Part A of Table 9 in (38a, b), and then select the right $\mathrm{A}(i, j)$ to connect the canonical inputs to give (c_α c_β), as shown in (38c). Next we formulate the SNS-QPL connective "imply" of Part B by a 2 × 3 matrix. For c = T(1 0), w = $\forall$ = (1 0 0), and for c = F = (0 1), w = Φ = (0 0 1). Hence the relational matrix R of (39a) is obtained for the relations in this Part B, and it is verified that it has the required properties in (39b).

Part C of Table 9 is straightforward, since it is a unary relation, and both input and output are already in the canonical form. The canonical equation for this, and the relevant matrix I(6, 6) are shown in (40a and b). If necessary, the ($\exists$p) can be standardized to ($\exists$ u) (p u) as in (40c).

(β) *Reversal of the steps in Boolean algebraic notation*

In reverse, the two inputs are:

(i) "No persons are in the verandah", which in canonical BA form, is

$$(\Phi\ p) = (0\ 0\ 1) \text{ for p.} \qquad (41a)$$

and

(ii) "All seats are occupied", which in canonical BA form is,

$$(\forall\ s) = (1\ 0\ 0) \text{ for s.} \qquad (41b)$$

We shall simply reverse the canonical equations in (40), (39) and (38), and write them as (42c, b, a) respectively, in that order, to represent the reversals of Parts C, B, A of Table 9.

$$\langle p \mid I^t(6,\ 6) \mid = \langle w \mid \text{ (3 vector)} \tag{42c}$$

$$\langle w \mid R^t \mid = \langle c \mid \text{ (2-vector)} \tag{42b}$$

$$\langle s \mid A \mid = \langle m \mid \text{ (3 vector), if } c = T$$
and
$$\langle s \mid A^c \mid = \langle m \mid \text{ (3 vector), if } c = F \tag{42a}$$

Now $\langle p \mid = (0\ 0\ 1)$, so that $\langle p \mid I^t(6,\ 6) \mid$ is

$$(0\ 0\ 1)\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 0 & 0 & 1\end{bmatrix} = (0\ 0\ 1) \text{ for } \langle w \mid \tag{43}$$

Putting $\langle w \mid = (0\ 0\ 1)$ in (42b), we get

$$(0\ 0\ 1)\begin{bmatrix}1 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix} = (0\ 1) \text{ for } \langle c \mid \tag{44}$$

$$\langle c \mid A^c \mid = \langle m \mid \tag{45a}$$

In this, by (41b), $\langle s \mid = \langle 1\ 0\ 0 \mid$ in QPL, and taking the complement of $\mid A(6,\ 6) \mid$ in (38c) for $\mid A^c \mid$, we obtain

$$(1\ 0\ 0)\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 1\\ 1 & 1 & 1\end{bmatrix} = (0\ 0\ 1) = \langle m \mid \tag{45b}$$

Standardization of $\langle m \mid = (0\ 0\ 1)$ is straightforward, and $\mathsf{m} = (\Phi y)\ (\mathsf{m} y)$, "No musicians are present" (QED).

Concluding Remarks

Thus, we have effectively converted the axioms and rules of EPL into vector-matrix equations, associated with the usual logical functions (NOT, AND, OR, XOR (of BA-1)) available in a computer. As a matter of fact, just like MATLOG for SNS, it is not at all difficult to write a program in FORTRAN IV for all that we have discussed in this paper. Our technique is therefore eminently practical and suitable for expanded application to more complicated formulae in SNS and QPL.

On the purely theoretical side, the most interesting aspect is the introduction of the new basic state "some" (Σ), which is quite enigmatical for the common man. It was so for the Jaina philosophers in India in the B.C.'s, and they gave this indefinite state the name "*avaktavya*" (indescribable) [6] along with the simple words "true" and "false" for the two definite states[1]. Godel's demonstration[7] that any theory in PL, which is large enough, must contain theorems in this state (neither provable, nor disprovable) is therefore not an enigma for epistemology as it appears to be at first sight, but a necessary consequence of the structure of QPL, when extended into EPL. If the state Σ is needed for the completeness of QPL so as to be isomorphic to BA-3, there must be some statements in any system of logic isomorphic to BA-3, which possess the property of this Σ state—namely that of neither complete truth, nor complete falsity, both of which are unprovable. That this is true of some theorems in *any theory making use of QPL* is the beauty of Godel's theorem[4].

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Note added in proof

1. After this article was sent to press Ref. 8 has come to the notice of the author. This book on "Many-valued Logic" has several interesting examples of three-valued logics—namely due to Lukasiewicz, Bochvar and Kleene, in which an intermediate state (called I) is introduced in addition to two truth values T and F. However, the exact behaviour of I in the three examples is quite different. On examination, it is found that Kleene's I has all the properties of D in SNS logic and Bochvar's interpretation of I has all the properties of X in SNS logic. In a particular way, the three-valued systems are complete in the forward direction; but no reverse relation is dealt with by the above authors. When this is also done, the logical system becomes complete only if we take into account all the *four* states T, F, D and X.

   Thus, our approach, of developing the isomorphism between BA-2 and propositional calculus, is, in a way, a *generalization* of some of the special approaches to multi-valued logic available in the literature. A more detailed account of this, along with further studies on accepted systems of multi-valued logic, will be published in due course.

2. The program of the 1983 International Symposium on Multi-valued Logic (Kyoto, May 23–25, 1983) has just now (April 10, 1983) come to the attention of the author. In this, there is an invited address by M. Goto, S. Kao and T. Ninomiya on "Synthesis of Axiom Systems for the Three-valued Predicate Logic by means of the Special Four-valued Logic" (Preprint not available). Our results for predicate logic are even more general, in that it uses an Eight-valued Logic isomorphous to BA-3 ($2^3 = 8$), which has the Four-valued Logic isomorphous to BA-2 as a sub-algebra.