SHORT COMMUNICATIONS

A PLANE SYMMETRIC UNIVERSE FILLED WITH PERFECT FLUID IN A MODIFIED BRANS-DICKE COSMOLOGY

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A PLANE symmetric cosmological model for perfect fluid has been derived in the Brans-Dicke theory[1]. The geometry of the universe is described by the line element

$$ds^2 = -dT^2 + T dx^2 + T\,[\tfrac{1}{2}(1+a)]dy^2 + T[\tfrac{1}{2}(1-a)]dz^2, \tag{1}$$

a being a constant. The Brans–Dicke field equations with cosmological term Q^2 can also be represented[2] in a different form under a unit transformation (UT[2]) in which length[3], time and reciprocal mass are scaled by the function $\lambda^{1/2}(x)$. The fundamental equations[2] for perfect fluid distribution go to the form

$$\bar{G}_{ij} + \bar{g}_{ij}\bar{Q} = 8\pi\left\{(\bar{\rho}+\bar{p})\bar{v}_i\bar{v}_j + \bar{p}\bar{g}_{ij}\right\} + \frac{1}{2}(2\omega+3)\quad \Lambda_i\Lambda_j - \frac{1}{2}\bar{g}_{ij}\Lambda_k\Lambda^k \tag{2}$$

$$\bar{\Box}\Lambda = \frac{8\pi\mu\bar{T}}{(2\omega+3)},\quad \Lambda = \log\Phi \tag{3}$$

$$\bar{Q} = \frac{(2\omega+3)}{4}\ \frac{(1-\mu)}{\mu}\bar{\Box}\Lambda = \frac{8\pi(1-\mu)}{4}\ \bar{T}; \tag{4}$$

where the constant μ shows how much this theory including $\bar{Q}(\Phi)$ deviates from that of Brans and Dicke and, as usual, ω is coupling constant. The barred quantities are defined in terms of $\bar{g}_{ij}$ as their unbarred counterparts are defined in terms of the unbarred metric g_{ij} and all barred operations are performed with respect to the barred metric and barred Christoffel symbols.

The pressure $\bar{p}$ and density $\bar{\rho}$ in the model (1) are given by

$$8\pi\bar{p} = \frac{(a^2-5)}{16T^2}\sec^2\left[\left\{\frac{(5+a^2)}{16(2\omega+3)}\right\}^{1/2}\log(kT)\right] + \bar{Q}, \tag{5}$$

$$8\pi\bar{\rho} = \frac{(a^2-5)}{16T^2}\sec^2\left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2}\log(kT)\right] - \bar{Q}, \tag{6}$$

Also the scalar field Λ is given by

$$\Lambda = \log\sec^2\left[\left\{\frac{(5+a^2)}{16(2\omega+3)}\right\}^{1/2}\log(kT)\right] \tag{7}$$

and

$$\bar{Q} = \frac{(1-\mu)}{4\mu}\quad\frac{(a^2-5)}{8T^2}\sec^2 \times \left[\left\{\frac{(5+a^2)}{16(2\omega+3)}\right\}^{1/2}\log(kT)\right], \tag{8}$$

where k is a constant.

The model has to satisfy the reality conditions[4] (i) $\bar{\rho}+\bar{p} > 0$ and (ii) $\bar{\rho}+3\bar{p} > 0$ which requires that

$$a^2 > 5,\ \omega < -\frac{3}{2},\quad \bar{Q} > 0\quad (i.e.\ \mu < 1) \tag{9}$$

and

$$\bar{Q} > \frac{(5-a^2)}{8T^2}\sec^2\left[\left\{\frac{(5+a^2)}{16(2\omega+3)}\right\}^{1/2}\log(kT)\right] \tag{10}$$

The coordinate system turns out to be comoving with $\bar{V}^4 = 1$. Clearly $\bar{V}_{ij}\bar{V}^j = 0$, so that the flow is geodetic. The expression for the expansion Θ for the flow vector $\bar{V}^i$ is given by

$$\Theta = \frac{1}{3T}. \tag{11}$$

The rotation tensor $W_{ij} = \bar{V}_{i;j} - \bar{V}_{j;i}$ is identically zero and the magnitude of the shear is given by

$$\sigma^2 = \frac{1}{2}\ \sigma_{ij}\sigma_{ij} = \frac{1}{48T^2}[1+3a^2] \tag{12}$$

The surviving components of the conformal curvature tensor $C_{hl}{}^{jk}$ for the line-element (1) are

$$C_{14}{}^{14} = C_{23}{}^{23} = \frac{1}{24T^2}[1+a^2]$$

$$C_{12}{}^{12} = C_{34}{}^{34} = -\frac{1}{48T^2}[1+6a+a^2] \tag{13}$$

$$C_{13}{}^{13} = C_{24}{}^{24} = -\frac{1}{48T^2}[1-6a+a^2]$$

Thus it follows that the space-time given by (1) is of Petrov-type I. The pressure, density, scalar field and cosmological constant are singular at

$$T = \left[\frac{1}{k}\right] \exp\left[\pi \left\{\frac{4(2\omega+3)}{(5-a^2)}\right\}^{1/2}\right]$$

The model exists for a finite time

$$\left[\frac{1}{k}\right] \leq T < \left[\frac{1}{k}\right] \exp\left[\pi \left\{\frac{4(2\omega+3)}{(5+a^2)}\right\}^{1/2}\right] \tag{14}$$

When $\mu = 1$, the solution reduces to a simple Brans-Dicke analogue of the well known problem of perfect fluid distribution in general relativity[5].

Under the transformations

$$\bar{g}_{ij} \to g_{ij} = \frac{1}{-}\bar{g}_{ij},\ \bar{T}_{ij} \to \bar{T}_{ij} = \Phi \bar{T}_{ij},\ \bar{T} \to \bar{T} = \Phi^2 \bar{T},$$

$$\bar{p} \to \bar{p} = \Phi^2 p,\ \rho \to \rho = \Phi^{\bar{2}} \rho,\ \Phi\ \Phi = e^{A}, \tag{15}$$

$$\bar{Q} \to Q = \Phi \bar{Q},\ \bar{V}^i \to V^i = \Phi^{1/2} \bar{V}^i.$$

the solutions of the field equations (2)–(4) are changed into 1961 form of Brans-Dicke theory[1]. We now apply these transformations to the solutions obtained from field equations (2)–(4). Thus

$$\Phi = \sec^2 \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right], \tag{16}$$

$$g_{ij} = \cos^2 \left[\left\{\frac{(5-a^2)}{(5-a^2)}\right\}^{1/2} \log(kT)\right]\ ij, \tag{17}$$

i.e.

$$g_{11} = \cos^2 \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right] T,$$

$$g_{22} = \cos^2 \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right] T^{\frac{(1+a)}{2}},$$

$$g_{33} = \cos^2 \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right] T^{\frac{(1+a)}{2}},$$

$$g_{44} = -\cos^2 \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right],$$

$$8\pi p = \frac{(a^2-5)}{16T^2} \sec^6 \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right] \times \left[1 + \frac{(1-\mu)}{2\mu}\right], \tag{18}$$

$$8\pi \rho = \frac{(a^2-5)}{16T^2} \sec^6 \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right] \times \left[1 - \frac{(1-\mu)}{2\mu}\right], \tag{19}$$

$$V^4 = \sec \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right], \tag{20}$$

$$Q = \frac{(1-\mu)}{4\mu} \left[\frac{(a^2-5)}{8T^2} \sec^4\right] \times \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right] \tag{21}$$

When $\mu = 1$, the cosmological term vanishes and the model, obtained from (16)–(21), reduces to plane symmetric universe with $p = \rho$ in the form of Brans-Dicke theory[1]. The model obtained in this paper is new and like other models with $p = \rho$ they may be used in the relativistic cosmology for the description of very early stages of the universe expansion.

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AN ANALYTICAL STUDY OF HIGHER ORDER BORN TERMS IN THE STATIC FIELD

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The closed forms of the elastic scattering amplitudes in the static field are obtained by using the Yates high energy higher order Born approximation. The differential scattering cross-section through order $(1/K^2_i)$ is obtained for elastic electron-atom scattering. Total cross-sections for the elastic scattering of the electrons by lithium atom are calculated. The results show good agreement with compared data.

In this communication we report the elastic scattering amplitudes developed for a z-electron