Thus it follows that the space-time given by (1) is of Petrov-type I. The pressure, density, scalar field and cosmological constant are singular at

$$T = \left[\frac{1}{k}\right] \exp\left[\pi \left\{\frac{4(2\omega+3)}{(5-a^2)}\right\}^{1/2}\right]$$

The model exists for a finite time

$$\left[\frac{1}{k}\right] \leq T < \left[\frac{1}{k}\right] \exp\left[\pi \left\{\frac{4(2\omega+3)}{(5+a^2)}\right\}^{1/2}\right] \qquad (14)$$

When $\mu = 1$, the solution reduces to a simple Brans-Dicke analogue of the well known problem of perfect fluid distribution in general relativity[5].

Under the transformations

$$\bar{g}_{ij} \to g_{ij} = \frac{1}{\Phi}\bar{g}_{ij},\ \bar{T}_{ij} \to T_{ij} = \Phi \bar{T}_{ij},\ \bar{T} \to T = \Phi^2 \bar{T},$$

$$\bar{p} \to \bar{p} = \Phi^2 p,\ \rho \to \rho = \Phi^2 \rho,\ \Phi\ \Phi = e^{\Lambda}, \qquad (15)$$

$$\bar{Q} \to Q = \Phi \bar{Q},\ \bar{V}^i \to V^i = \Phi^{1/2} \bar{V}^i.$$

the solutions of the field equations (2)–(4) are changed into 1961 form of Brans-Dicke theory[1]. We now apply these transformations to the solutions obtained from field equations (2)–(4). Thus

$$\Phi = \sec^2 \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right], \qquad (16)$$

$$g_{ij} = \cos^2 \left[\left\{\frac{(5-a^2)}{(5-a^2)}\right\}^{1/2} \log(kT)\right]\ ij, \qquad (17)$$

i.e.

$$g_{11} = \cos^2 \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right] T,$$

$$g_{22} = \cos^2 \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right] T\frac{(1+a)}{2},$$

$$g_{33} = \cos^2 \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right] T\frac{(1+a)}{2},$$

$$g_{44} = -\cos^2 \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right],$$

$$8\pi p = \frac{(a^2-5)}{16T^2}\sec^6 \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right] \times \left[1 + \frac{(1-\mu)}{2\mu}\right], \qquad (18)$$

$$8\pi \rho = \frac{(a^2-5)}{16T^2}\sec^6 \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right] \times \left[1 - \frac{(1-\mu)}{2\mu}\right], \qquad (19)$$

$$V^4 = \sec \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right], \qquad (20)$$

$$Q = \frac{(1-\mu)}{4\mu}\left[\frac{(a^2-5)}{8T^2}\sec^4\right]^{1/2} \times \left[\left\{\frac{(5-a^2)}{16(2\omega+3)}\right\}^{1/2} \log(kT)\right] \qquad (21)$$

When $\mu = 1$, the cosmological term vanishes and the model, obtained from (16)–(21), reduces to plane symmetric universe with $p = \rho$ in the form of Brans-Dicke theory[1]. The model obtained in this paper is new and like other models with $p = \rho$ they may be used in the relativistic cosmology for the description of very early stages of the universe expansion.

Sincere thanks are due to Dr. T. Singh for suggesting the problem.

3 November 1982

1. Brans, C. H. and Dicke, R. H., *Phys. Rev.*, 1961 **124**, 925.
2. Endo, M. and Fukui, T., *Gen. Rel. Grav.*, 1977, **8**, 833.
3. Dicke, R. H., *Phys. Rev.*, 1962, **125**, 2163.
4. Ellis, G. F. R., *General relativity and cosmology*, (ed.) R. K. Sachs, Academic Press, New York and London, 1971, p. 117.
5. Prakash, S., *Curr. Sci.*, 1981, **50**, 78.

AN ANALYTICAL STUDY OF HIGHER ORDER BORN TERMS IN THE STATIC FIELD

N. S. Rao and H. S. Desai
Physics Department, M S University of Baroda, Baroda 390 002, India.

The closed forms of the elastic scattering amplitudes in the static field are obtained by using the Yates high energy higher order Born approximation. The differential scattering cross-section through order $(1/K_i^2)$ is obtained for elastic electron-atom scattering. Total cross-sections for the elastic scattering of the electrons by lithium atom are calculated. The results show good agreement with compared data.

In this communication we report the elastic scattering amplitudes developed for a z-electron atom. The approximation given by Yates[1] is used in these derivations. We have treated e-atom interaction as the static field[2,3] due to z-electron target. Additional advantages in this case are that all the Yates[1] elastic scattering amplitudes are simplified and one can calculate DCS (differential cross-sections), TCS (total cross-sections) very easily for any atom. One can also standardise the computer programme for the calculation of DCS and TCS for any target atom. The static potential can be defined as

$$V_{st}(r_0) = \langle \psi_i | V | \psi_f \rangle \quad (1)$$

where ψ_i and ψ_f are the initial and final state wave functions of the target atom and V is the interaction between the incident electron and the target atom. The static potential $V_{st}(r_0)$ can be obtained[2,3] for different atoms. Analytical expression for the static potential can be given as

$$V_{st}(r_0) = Z \left| \sum_{i=1}^{N} \right| R_i \left| e \frac{-Y_i r_0}{r_0} \right. \quad (2)$$

where R_i's and Y_i's are obtained from[2,3] for different atoms. The fourier form of $V_{st}(r_0)$ is given as

$$V_{st}(r_0) = \int \alpha p \, \exp(-ip.b_0) \int_{\infty}^{\infty} dp_z \, \exp(-ip_z.Z_0) \, V_{st} (p + p_z\hat{y}) \quad (3)$$

where

$$V_{st}(p+p_Z) = \frac{Z}{2\pi^2} \sum_{i=1}^{N} \frac{R_i}{(p^2+p_z^2+y_i^2)} \quad (4)$$

The aim of the present work is to study all the Yates[1] amplitudes in the static field. These amplitudes are obtained by the substitution of equation in Yates[1] approximation. The corresponding three Born terms are given as

$$f^{(1)}_{i\to f} = -2Z \sum_{i=1}^{N} \frac{R_i}{(q^2 + y_i^2)} \quad (5)$$

$$\mathrm{Im}\, f^{(2)}_{\mathrm{HEAS}} = \frac{Z^2}{\pi_2 K_i} \sum_{i,j=1}^{N} R_i R_j I_1(B_i^2, y_i^2, y_j^2) \quad (6)$$

$$\mathrm{Re}\, f^{(3)}_{\mathrm{HEAS}} = \frac{Z^2}{\pi^2 K_i} \sum_{i,j=1}^{N} R_i R_j [I_2(B_i^2, y_i^2, y_j^2) + \frac{1}{2K_i}\frac{\partial}{\partial B_i}\{I_3(B_i^2, y_i^2) - y_i^2 I_2^2(B_i^2 \cdot y_i^2 \cdot y_j^2)\}] \quad (7)$$

$$f^{(3)}_{\mathrm{HEAS}} = f_{1s}^{(3)} + f_{2s}^{(3)} \quad (8)$$

where

$$f_{1s}^{(3)} = \frac{Z^3}{2\pi^2 K_i^2} \sum_{i,j,K=1}^{N} R_i R_j R_K I_4(q^2, u, v, w,)$$

Similarly one can obtain $f_{2S}^{(3)}$ by using the present approximation and the computation procedure given by Yates[1]. The closed form of the all I_n s in the above expressions were given[1,4-6]. The elastic scattering amplitude through $(1/K_i^2)$ can be given as

$$F_{ds} = f^{(1)}_{i\to f} + \mathrm{Re}\, f^{(2)}_{\mathrm{HEAS}} + f^{(3)}_{\mathrm{HEAS}} + i\,\mathrm{Im}\, f^{(2)}_{\mathrm{HEAS}} \quad (9)$$

Total cross-section can be obtained using the optical theorem[1].

$$\sigma^{tot} = \frac{4\pi}{K_i} \mathrm{Im}\, f^{(2)}_{\mathrm{HEAS}}(0) \quad (10)$$

where

$$\mathrm{Im}\, f^{(2)}_{\mathrm{HEAS}}(0) = \frac{Z^2}{K_i}\left[\sum_{i\neq j} \frac{R_i R_j}{(y_i^2 - y_j^2)} \log \frac{B_i^2 + y_i^2}{B_i^2 + y_j^2} + \sum_{i=j} \frac{R_i^2}{(B_i^2 + y_i^2)}\right]$$

is the imaginary part of the second Born Term in the forward direction. The meaning of all the symbols are same as Yates[1].

Using σ^{tot} we have calculated TCS for lithium atom[3]. The results are given below

TABLE 1

TCS in units of (πa_0^2) for lithium atom

Incident Energy E eV	Present results	Results of Guha and Gosh[8]
50	6.295	7.511
60	5.260	6.034
100	3.152	3.379
150	2.106	2.183
200	1.577	1.614

To confirm our results these terms have been compared with the corresponding Born Terms of Yates[1] and the two types of Born terms show good agreement. The most notable observation is that when $B_i \rightarrow 0$ few of the Yates[1] integrals were divergent and cancelled with the opposite types of integrals. However, in the present studies there are no divergent integrals as can be seen from equation (10). In order to see the validity of the present approach we have calculated TCS for lithium atom using the optical theorem[7]. The TCS results are found to be in good agreement at higher incedent energies with the other data[8].

The present calculations are simpler than the Yates[1] approximation and one can calculate TCS and DCS very easily for any atom. Further work is in progress.

2 December 1982

1. Yates, A. C., *Phys. Rev.*, 1979, **A 19** 1550.
2. Cox, H. L. Jr. and Bonham, R. A., *J. Chem. Phys.*, 1967, **47**, 2599.
3. Tiett, T., *Nuovo Cimento.*, 1965, **36**, 1365.
4. Rao, N. S. and Desai, H.S., *Pramana.*, 1981, **17**, 309.
5. Yates, A. C., *Chem. Phys. Lett.*, 1974, **25**, 480.
6. Singh, S. N. and Tripathi A N *Phys, Rev.*, 1981, A **21**, 105.
7. Taylor J. R. "*Scattering Theory*"., 1972.
8. Guhas and Gosh, A. S., *Indian J. Phys.*, 1979, **B53**, 163.

GELL-MANN-OKUBO MASS FORMULA-A MODIFIED VERSION

T. N. TIWARI AND (MRS) SAROJ SHARMA*

Department of Physics, Regional Engineering College, Rourkela 769 008, India.

*Department of Physics, Deva Nagari (Post-Graduate) College, Merrut 250 002, India.

IN the absence of an exact theory of strong interactions, phenomenological theories based on approximate symmetries have played an increasingly important role in hadron physics since the early sixties. The successes of the SU(3) symmetry[1] and its chiral extension SU(3) × SU(3) are well known and need not be recounted here. For the breaking of this symmetry, the most popular and successful model in the past has been the (3,3*) + (3*,3) representation[2,3] of SU(3) × SU(3).

However, it is well known that the conventional (3,3*) + (3*,3) model cannot successfully account for certain experimental data, *e.g.*, the meson-meson scattering lengths, meson-nucleson σ — terms and the amplitute for the $\eta \rightarrow 3\pi^0$ decay. So the (8,8) representation of SU(3) × SU(3) has been proposed as a possible alternative to the more popular (3,3*) + (3*,3) model. In the present note, we have exploited the (8,8) model to derive a modified version of the famous Gell-Mann-Okubo mass formula for the pseudoscalar mesons. Our modified formula shows much better agreement with the experimental data than the original one and it reduces to the original formula in the limit of an exact SU(3) symmetry.

Our approach here is similar to that of Cicogna and co-workers[4], who used the functional method to treat the spontaneous breaking of symmetries in quantum field theory. The details of our approach, the use of the (8,8) model to derive the expressions for the masses and decay constants of the pseudoscalar mesons and our notations are explained in full details elsewhere[5].

Following the approach and notations of Ref. 4, we get the following expressions for the squared masses of the pseudoscalar mesons:

$$M^2_\pi = \frac{\sqrt{10}\, d_0 + 2\, d_8}{\sqrt{10}\, \lambda_0 + 2\lambda_8},$$

$$M^2_K = \frac{\sqrt{10}\, d_0 - d_8}{\sqrt{10}\, \lambda_0 - \lambda_8},$$

$$M^2_\eta \doteq \frac{\sqrt{10}\, d_0 - 2\, d_8}{\sqrt{10}\, \lambda_0 - 2\, \lambda_8}, \qquad (1)$$

These expressions have been derived on the assumption that the strong interaction Hamiltonian H can be written as

$$H = H_0 + H',$$
$$\text{where } H' = d_0 z_0 + d_8 z_8. \qquad (2)$$

Here H_0 is SU(3) × SU(3) invarient, while H' breaks this symmetry; d_0 and d_8 being symmetry breaking parameters. The quantities λ_0 and λ_8 in Eq (1) are the vaccum expectation values of the meson fields z_0 and z_8. This form of symmetry breaking given in Eq. (2) satisfies the requirements of the conservation of isospin and hypercharge for strong interactions.

Moreover, it can be shown[5] that the application of the PCAC (partial conservation of axial-vector currents) hypothesis gives the following expressions for the decay constants F_i of the pseudoscalar mesons π and K:

$$F_\pi = (3\sqrt{3}/10)\ (\sqrt{10}\,\lambda_0 + 2\lambda_8),$$
$$F_K = (3\sqrt{3}/10)\ (\sqrt{10}\,\lambda_0 - 2\lambda_8). \qquad (3)$$