SHORT COMMUNICATIONS

ON THE PHYSICAL MEANING OF THE NIJENHUIS TENSOR IN GENERAL RELATIVITIY

ZAFAR AHSAN
Department of Mathematics, Aligarh Muslim University, Aligarh 202 001, India.

IN the differential geometry of complex and almost complex spaces the importance of the Nijenhuis tensor is well known[1] while for general theory of relativity this tensor has been studied[2]. Recently, its role in the study of null electromagnetic field has been explored[3-5] and the integrability condition obtained for the null electromagnetic field. The vanishing of the Nijenhuis tensor is a necessary and sufficient condition for the integrability of the corresponding F-structure on the manifold. The geometric significance may be made more clear from the fact that for an almost complex manifold, the vanishing of the Nijenhuis tensor implies that an almost complex structure can be obtained from a complex structure.

In this note we look for a possible physical significance to this tensor in general relativity.

For null electromagnetic field, the Nijenhuis tensor N^{κ}_{lm} is given by[3]

$$N^{k}_{lm} = 2F^{tk} F_{lm,t} \tag{1}$$

where F_{ij} is the electromagnetic field tensor. Now in terms of 3-vectors E_α and H_α, we have

$$F_{ij} = \begin{bmatrix} 0 & H_3 & -H_2 & E_1 \\ -H_3 & 0 & H_1 & E_2 \\ H_2 & -H_1 & 0 & E_3 \\ -E_1 & -E_2 & -E_3 & 0 \end{bmatrix} \tag{2}$$

On using (1), (2) and the Maxwell's equations in 3-dimensional form, the 24 components of the Nijenhuis tensor are given by

$$N^{1}_{12} = -N^{1}_{21} = 2(F^{11} F_{12,1})$$

$$= 2\left(-H_3 \frac{\partial H_3}{\partial x^2} + H_2 \frac{\partial H_3}{\partial x^3} + E_1 \frac{\partial H_3}{\partial t}\right)$$

+ 23 other components.

These components can also be expressed as

$$\frac{1}{2} N^{\alpha}_{4\beta} \to (\vec{H}\times\nabla)_\alpha E_\beta + E_\alpha (\nabla\times\vec{H})_\beta$$

$$\frac{1}{2} N^{\alpha}_{\beta\gamma} \to (H\times\nabla)_\alpha \epsilon_{\beta\gamma\delta} \vec{H}^{\delta} + E_\alpha \epsilon_{\beta\gamma\delta} (\nabla\times\vec{E})^{\delta},$$

$$\frac{1}{2} N^{4}_{\alpha\beta} \to \epsilon_{\alpha\beta\gamma} (E.\nabla) H^{\gamma},$$

$$\frac{1}{2} N^{4}_{\alpha 4} \to (\vec{E}.\nabla) E_\alpha. \qquad (\alpha, \beta, \gamma, \delta = 1,2,3)$$

The above components are only symbolic without any regard to contravariance or covariance. In fact they are meant to represent the following.

The 9 components of $1/2\, N^{\alpha}_{4\beta}$ are the components of $-(\vec{H}\times\nabla)\vec{E} + \vec{E}$ 0 curl $\vec{H}$. The 9 components $1/2\, N^{\alpha}_{\beta\gamma}$ are the components of $(\vec{H}\times\nabla)\vec{H} + E$ 0 curl $\vec{E}$. The 3 components of $1/2\, N^{4}_{\alpha\beta}$ are the components $(\vec{E}.\nabla)\vec{H}$ and the 3 components of $1/2\, N^{4}_{\alpha 4}$ are the components of $(\vec{E}.\nabla)\vec{E}$.

It may be noted from these relations that the various components of the Nijenhuis tensor represent the variation of electric and magnetic fields in different directions, *e.g* $N^{4}_{\alpha\beta}$ and $N^{4}_{\alpha 4}$ represent the variation of magnetic field in the direction of electric field and the variation of electric field in its own direction.

Further, it can be deduced that

$$N^{\alpha}_{\alpha 4} = 2 \text{ div } (\vec{E}\times\vec{H})$$

which gives the divergence of the Poynting's vector of the electromagnetic field.

From the Ricci identity and the symmetry of F_{ij} the divergence of the Nijenhuis tensor is found to be[3]

$$N^{k}_{lm,k} = F^{tk} (F_{al} R^{a}_{mkt} + F_{ma} R^{a}_{lkt}) \tag{3}$$

This divergence vanishes for conformally and projectively-flat null electromagnetic field.

From (3) we can say that the divergence of the Nijenhuis tensor characterizes the gravitation through the Riemann tensor R_{ijkl} and electromagnetism through the electromagnetic field tensor F_{ij}, and hence it may play a role in the unification of these two fields. It is hoped that it can be used as interaction Lagrangian in evolving an unified theory, by taking the Lagrangian of the form $\mathcal{L} = M\sqrt{-g}$, where $M =$ trace $(N^{k}_{ij,k})$.

7 March 1983

1. Yano, K., *Differential geometry of complex and almost complex spaces.*, Pergamon Press, London, 1965.

2. Radhakrishna, L. and Khade, V. D., *J. Shivaji Univ.*, 1973, **6**, 57.
3. Ahsan, Z., *Studies of radiation in general relativity.*, Ph.D, Thesis, Aligarh Muslim University, Aligarh, 1978.
4. Ahsan, Z., *J. Math. Phys. Sci.*, (To appear).
5. Ahsan, Z. and Hussain, S. I., *Anali di Mat. Pure ed Appl.*, CXXVI (IV) 1980, 379.

KINETICS AND MECHANISM OF OXIDATION OF SOME SUBSTITUTED PHENYL METHYL SULPHOXIDES BY N-CHLOROPHTHALIMIDE

N. BHAVANI AND K. LILY
Department of Chemistry, Annamalai University, Annamalainagar 608 002, India.

THOUGH many N-chloro compounds such as chloramine-T, N-chlorosuccinimide, N-chlorobenzamide etc., have been used as oxidants extensively in the kinetic studies, such a study using N-chlorophthalimide (NCP) is lacking. The oxidation kinetics of various sulphoxides have already been reported[1-8]. We report in this note our results on the kinetics of oxidation of some *o*-, *m*- and *p*-substituted phenyl methyl sulphoxides with NCP in aqueous acetic acid in the presence of perchloric acid.

All the sulphoxides were prepared and purified by standard methods. Acetic acid was purified by refluxing with chromium trioxide. NCP was prepared according to the literature method[9].

Kinetic studies were carried out in 50% (v/v) acetic acid containing 0.05 M perchloric acid under pseudo-first order conditions in vessels coated on the outside with black paint. The reaction was followed by estimating the unchanged NCP by iodometric procedure[10]. Reproducible results giving first order plots ($r \simeq 0.99$) were obtained for reactions run in duplicate in each substrate and at all the temperatures studied. The pseudo-first order rate constants (k_1) were calculated by the least-square method. The stoichiometry of the reaction is 1:1 and the product obtained is identified as methyl phenyl sulphone by its m.p and the mixed m.p with an authentic sample.

The oxidation of methyl phenyl sulphoxide has been studied in detail. An increase in [sulphoxide] increases the rate. The plot of log k_1 *vs* log [sulphoxide] is linear with unit slope. Also the second-order rate constants ($k_2 = k_1/$[sulphoxide] are constant confirming a unit dependence on sulphoxide (table 1). The reaction is also found to be first order with respect to NCP as evidenced by the constancy of the first order rate constants for various concentrations of NCP (table 1). Keeping the [sulphoxide] and [NCP] constant, an incraese in the [$HClO_4$] from 0 to 0.4 M does not produce any marked change in the rate constants. The reaction rate is not altered by the addition of sodium perchlorate.

The reaction rate decreases with increase in the acetic acid content of the solvent medium. The second-order rate constants for the oxidation of methyl phenyl sulphoxide were 2.58, 2.03, 1.78. 1.34 and 0.81×10^{-2} litre mol^{-1} sec^{-1} in 20, 40, 50, 60 and 80% acetic acid respectively.

To gain more information about the nature of the transition state and the mechanism, the rates of oxidation of several *o*-, *m*- and *p*-substituted phenyl methyl sulphoxides have been studied. The data in table 2 indicate that electron-releasing substituents in the benzene ring accelerate the rate while the electron-

TABLE 1

Effect of varying reactant's concentration on the reaction rate

[NCP] $\times 10^3$ M	[$C_6H_5SOCH_3$] $\times 10^4$ M	$k_1 \times 10^4$ sec^{-1}	$k_2 \times 10^2$ lit mol^{-1} sec^{-1}
0.5	2	3.56	1.78
1	2	3.57	1.79
1.5	2	3.37	1.69
2	2	3.48	1.74
1	1	1.76	1.76
1	3	5.22	1.74
1	4	6.71	1.68
1	5	9.18	1.84

[$HClO_4$] = 0.05 M; solvent = 50% HOAC–H_2O (v/v); temperature = 35° C