SHORT COMMUNICATIONS

PERIOD BETWEEN BURSTING IN TURBULENT SHEAR FLOW: INTERMEDIATE SCALING

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THE period T between bursts in wall-bounded turbulent shear flows has been studied; yet there is no agreement whether period scales with inner or outer variables[1].

Black[2] proposed that T scales with inner variables implying that T^+ ($=Tu_\tau^2/\nu$ where u_τ is the friction velocity and ν is the molecular kinematic viscosity) is a universal number[3-5]. The uncertainties in the measurements of T are well known; however, it is doubtful whether the entire variation of T^+ (for pipe data[3,6] from 145 to 485 and for constant pressure boundary layer data[7] from 70 to 500) could be ascribed to them. From the analysis of these data it was concluded[6,7] that T^+ has a systematic dependence on Reynolds number.

Rao *et al*[7] proposed that T scales with outer variables and that $U_\infty T/\delta$ (where U_∞ is the velocity at the edge of the boundary layer $y \simeq \delta$) is a universal number, which is now widely adopted as 5. However, the situation is not so simple as the measured values of $U_\infty T/\delta$ vary[8] from 2.5–10. The variations are commonly ascribed[9] to the uncertainties in measurements of T.

Recently Bandyopadhyay[8] has shown that the variations of $U_\infty T/\delta$ are not entirely due to uncertainties in the measurements of T, but show a systematic variation implying that $U_\infty T/\delta$ is not a universal number and its value depends on the flow field. It was concluded[8] that both inner and outer layer variables play active roles during the bursting process. The findings of Rao *et al*[7] that the burst rate per unit span scales with neither inner nor outer, but mixed variables (inner for spatial length and outer for time), show the role of coupling of the inner and outer layers during the bursting process. Further, Offen and Kline[10] observed that during the bursting process the low speed wall streaks behave like a sub-boundary layer (within the conventional turbulent boundary layer) growing in a manner that is reminiscent of the development of a conventional laminar boundary layer near the leading edge of a flat plate.

Recently, in between the classical inner and outer layers an intermediate layer has been proposed[11-15]. The length scale of intermediate layer

$$\Delta = (\nu\delta/u_\tau)^{1/2}, \tag{1}$$

is the geometric mean of the classical inner and outer length scales. In the intermediate layer the Reynolds stress is maximum and the velocity distribution is governed by a half-defect law whose behaviour is logarithmic with universal slope and intercept[12-15]. For a turbulent boundary layer $\delta/L = u_\tau/U_\infty$ (where L is the scale of flow variations in the streamwise direction), and relation (1) may also be written[13,14] as

$$\Delta = (\nu L/U_\infty)^{1/2}, \tag{2}$$

which is analogous to the length scale of the conventional laminar boundary layer.

In the light of the fact that during the bursting process both inner and outer layers play significant roles[5,6] and the resemblance of the intermediate layer scale (2) with the sub-boundary layer of Offen and Kline[10], it is instructive to non-dimensionalize T by the intermediate length scale Δ and velocity scale u_τ as

$$\Gamma = u_\tau T/\Delta = T^+ R_\tau^{-1/2} \tag{3}$$

where $R_\tau = u_\tau(\delta/\nu)$ is the frictional Reynolds number and δ is the boundary layer thickness or radius of the pipe as appropriate. Equation (3) implies that the appropriate time scale is the geometric mean of the time scales for inner and outer layer.

For the pipe/channel flow the burst rate T^+ data[16-19], displayed against R_τ in figure 1, show that T^+ increases with R_τ. The behaviour of the data is different from $T^+ =$ constant (inner scaling) and $T^+ R_\tau^{-1} =$ constant (outer scaling), both lines being displayed in figure 1. The half-slope line (3) is also displayed in figure 1. Although the data points are scattered about the half-slope line, the data may be represented by the relation

$$T^+ = B\, R_\tau^{1/2},\ B = 8 \pm 2. \tag{4}$$

For the constant pressure turbulent boundary layer the data[7,20-23] are displayed in figure 2. The high Reynolds number data point[7,24] is not included as it does not correspond to the bursting frequency[26]. A half-slope line

$$T^+ = 7.0\, R_\tau^{1/2}, \tag{5}$$

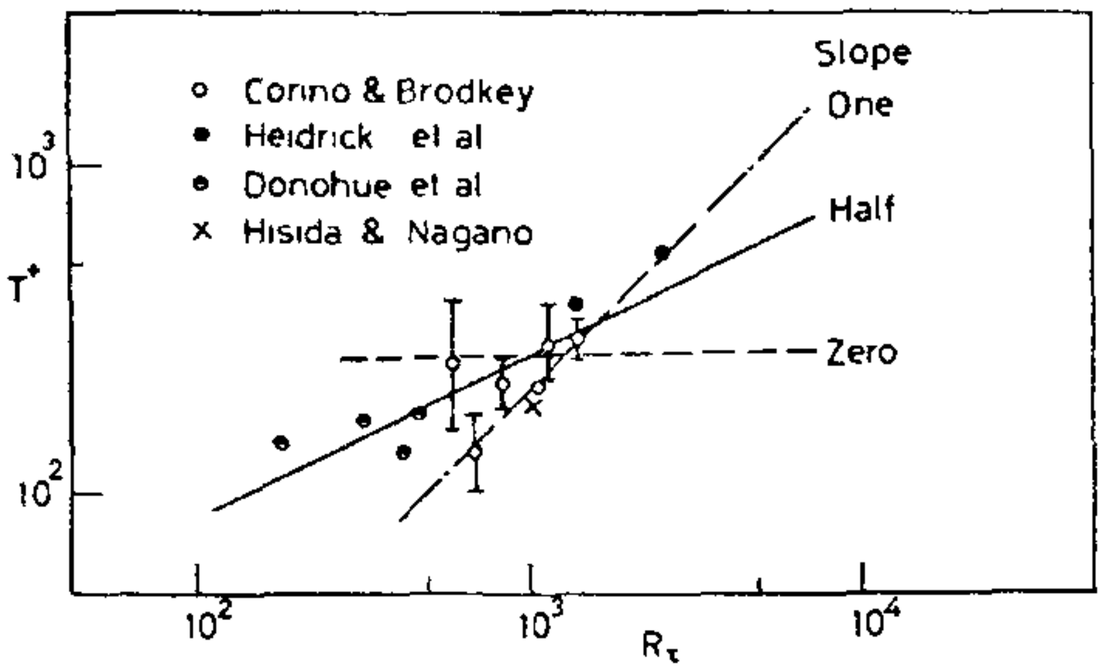

Figure 1 Burst rate data for turbulent pipe/channel flow.

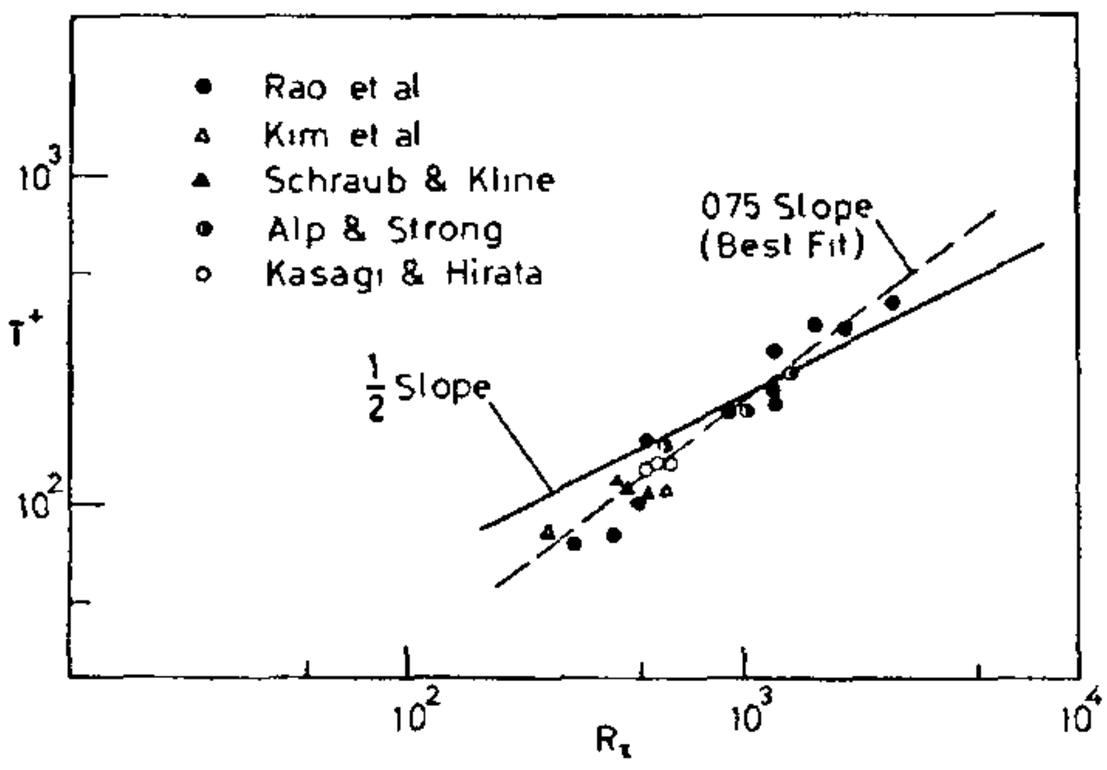

Figure 2 Burst rate data for turbulent boundary layer with constant pressure.

and a line ($T^+ = 1.15\ R_\tau^{0.75}$) corresponding to the best fit to the data, are also displayed in figure 2. The best fit line is close to the proposal of Rao *et al*[7] implying outer scaling for T. This would imply that the burst rate in boundary layers is different from burst rate in fully developed duct flows. It is well known, however, that the mean velocity and Reynolds stress profiles in the two flows (boundary layer and duct) scale with the same characteristic variables and therefore there is apparently no reason that the burst period in two flows should scale differently. On the other hand the intermediate scaling relation (5) would not be inconsistent with data for $R_\tau \geqslant 600$; below this value systematic departure is observed. As these measurements are at low Reynolds number the possibility of a Reynolds number effect is not ruled out[7, 21]. It remains to be seen whether the scatter in data from the intermediate scaling (5) in figure 2 is due to uncertainties in measurements of T. Clearly there is need for better data especially at high Reynolds number.

Conclusions

1. There is prolonged controversy whether the burst scales with inner or outer variables. On first sight the intermediate scaling may be taken as a compromise between inner and outer scaling.
2. The intermediate layer formed by the interactive association of the inner and outer layers is associated with the maximum of Reynolds stress, and the characteristics of its length scale are analogous to the sub-boundary layer of Offen & Kline modelling the turbulent bursting process. Therefore the intermediate length scale may be a more appropriate candidate for the scaling of bursts when compared to inner and outer scaling.
3. For fully developed duct flows the burst rate data obey the intermediate scaling. For boundary layers the evidence for intermediate scaling is inconclusive and it remains to be seen whether the scatter in the existing boundary layer data in terms of intermediate scaling is due to uncertainties in the measurement of T. There is need for better data especially at higher Reynolds numbers.

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TOPOLOGY OF OXY-ORGANIC COMPOUNDS

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GRAPH theory[1] has, recently, been applied to various branches of theoretical organic chemistry. Several authors have discussed the molecular topology of several types of organic compounds[2-5]. The molecular graph[1] is a set of vertices and edges, where the vertices denote the atoms and the edges denote the chemical bonds. In a graph of an organic compound the vertices represent the carbon atoms C and the hetero atoms X, and the edges represent the C–C, carbon-carbon bonds and C–X, the hetero bonds respectively. The hetero atom X may be an O, N, or S atom. In our previous work[4, 5] we have defined the root mean square value of the topological distances W_{rms} of a molecular graph. The index W_{rms} was found to reflect the changes in physical properties of the organic compounds in a homologous series[4, 5]. In a recent communication[6] we have defined the edge-weighted graphs and sum of the topological distances[7] from these graphs of purine and pyrimidine bases. In the present paper we have calculated the W_{rms} values of oxy-organic compounds and correlated them with their boiling points.

The edge-weighted graph[6] G_{we} is defined as a graph in which, some real numbers known as weights are associated with its edges. In a weighted molecular graph the weights associated with the edges correspond to the k values of the Hückel parameter[8] for the chemical bonds. The Hückel parameter k is defined by the relationship[8],

$$\beta_{cx} = k\beta_{cc}, \qquad (1)$$

where β_{cc} is the resonance integral of a C–C bond and β_{cx} is that of a hetero bond. The value of k reflects the difference between the resonance integrals of a C–C bond and the C–X hetero bond (X may be O, N, or S).

In the edge-weighted graph the topological distance[7] d_{ij} between two vertices i and j, is defined as the distance which is associated with a minimum weight[6]. Thus, we can define a distance matrix D associated with the graph[4]. The distance matrix D of a graph is a real $N \times N$ matrix (N is the total number of vertices) the elements of which d_{ij} are the topological distances between the vertices i and j. The root mean square value of the topological distances W_{rms}, is defined by[4],

$$W_{rms} = \left[\frac{1}{N(N-1)} \sum_{ij} d_{ij}^2\right]^{1/2}. \qquad (2)$$

Constructing the edge-weighted graphs and taking appropriate k values for C–C, C–O, and C=O bonds from the well-known results of Hess and Schaad[9], we have calculated the $W\overline{W}$ and W_{rms} indices[4] for a series of oxo-organic compounds *e.g.*, alcohols, ethers, aldehydes, ketones and carboxylic acids. The topological indices for oxy-alcohols are reported in table 1. The W_{rms} values for series of compounds in each class are correlated with the experimental boiling points. It has been observed that the boiling points of the oxy-organic compounds containing normal alkyl groups, vary linearly with the index W_{rms}. Table 2 reports the boiling points (BP) of the corresponding homologous series in each class of compound as a function of W_{rms}, along with the standard deviation of the correlation in each case. The linear correlation for normal oxy-alcohols, is shown in figure 1. These correlations show that the physical property of the oxy-organic compound containing a normal alkyl group, depends directly on its topology. There are deviations from linear correlations for the compounds having