SHORT COMMUNICATIONS

MHD FLOW OF A DUSTY GAS THROUGH A HEXAGONAL CHANNEL

H. C. SHARMA and MUKESH GUPTA*

Department of Mathematics, Agra College, Agra 282 002, India.

* *Postal address; D28/284, Gokulpura, Agra 282 002, India.*

THE atttention of many research workers[2-7] in fluid mechanics has been directed towards studying the influence of dust particles on the motion of fluids in the past few years. Recently, the flow of a viscous incompressible gas with uniform distribution of dust particles through a hexagonal channel with arbitrary time varying pressure gradient has been studied[3].

The present investigation aims at extending the analysis of Gupta and Varshney[3] in the case of a transverse magnetic field.

Formulation and solution of the problem

Using a rectangular cartesian coordinate system (x, y, z) such that z-axis is along the axis of the channel and the cross-section of the channel is formed by the straight lines:

$$y = \pm\frac{a\sqrt{3}}{2} \quad \text{and} \quad y \pm \sqrt{3}x = \pm a\sqrt{3}. \tag{1}$$

The gas and particle velocities $u(x, y, t)$ and $v(x, y, t)$, respectively, are in z-direction.

Under the transformations

$$x_1 = y,\ x_2 = y - \sqrt{3}x \quad \text{and} \quad x_3 = y + \sqrt{3}x,$$

the non-dimensional momentum equations[3] after applying a constant transverse magnetic field of strength H_0 appear in the form:

$$\frac{\partial u}{\partial t} = f(t) + \left(\frac{\partial^2}{\partial x_1^2} + 4\frac{\partial^2}{\partial x_2^2} + 4\frac{\partial^2}{\partial x_3^2} + 2\frac{\partial^2}{\partial x_1 \partial x_2} + 2\frac{\partial^2}{\partial x_1 \partial x_3} - 4\frac{\partial^2}{\partial x_2 \partial x_3}\right)u + \beta(v-u) - Mu, \tag{2}$$

$$\partial v/\partial t = L(u - v), \tag{3}$$

where $\beta = (f/r) = N_0 K a^2/\rho v$, $f = N_0 m/\rho$, $r = mv/Ka^2$, $L = 1/\gamma$, $M = \sigma B_0^2 a^2/\rho v$ (magnetic field parameter) are dimensionless constants and $-\delta p/\delta z = f(t)$, an arbitrary function of time, $B_0 (= \mu_e H_0)$ the component of electromagnetic induction, σ the electrical conductivity, μ_e the magnetic permeability and other symbols have their usual meanings[3].

Non-dimensional initial and boundary conditions are:

(i) $t \leqslant 0,\ u(x_1, x_2, x_3, t) = 0 = v(x_1, x_2, x_3, t)$, every where in the channel, (4)

(ii) $t > 0,\ u(x_1, x_2, x_3, t) = 0 = v(x_1, x_2, x_3, t)$

$$\text{at } x_1 = \pm\sqrt{3}/2,\ x_2 = \pm\sqrt{3} \text{ and } x_3 = \pm\sqrt{3}. \tag{5}$$

Due to symmetry of motion about $x_1 = 0$, $x_2 = 0$ and $x_3 = 0$, we shall consider the motion in the region $x_1 \geqslant 0$, $x_2 \geqslant 0$ and $x_3 \geqslant 0$ and accordingly the boundary conditions (5) become

$$u(x_1, x_2, x_3, t) = 0 = v(x_1, x_2, x_3, t),$$
$$\text{at } x_1 = \sqrt{3}/2,\ x_2 = \sqrt{3} \text{ and } x_3 = \sqrt{3}, \tag{6}$$

and

$$\frac{\partial u}{\partial x_1} = 0 = \frac{\partial v}{\partial x_1} \text{ at } x_1 = 0;$$
$$\frac{\partial u}{\partial x_2} = 0 = \frac{\partial v}{\partial x_2} \text{ at } x_2 = 0;$$
$$\frac{\partial u}{\partial x_3} = 0 = \frac{\partial v}{\partial x_3} \text{ at } x_3 = 0. \tag{7}$$

To solve the problem by using the technique of integral transforms, we use finite Fourier cosine transforms and inversion formula[3]. Multiplying (2) and (3) by $[\cos(P_p x_1) \cdot \cos(Q_q x_2) \cdot \cos(R_r x_3)]$ and integrating within the limits 0 to $\sqrt{3}/2$, 0 to $\sqrt{3}$ and 0 to $\sqrt{3}$ and using conditions (6) and (7), also using finite Fourier cosine transforms, we get

$$\partial U/\partial t = \frac{(-1)^{p+q+r}}{P_p Q_q R_r} f(t) - b_{pqr} U + \beta(V - U) - MU, \tag{8}$$

$$\partial V/\partial t = L(U - V), \tag{9}$$

where U, V, b_{pqr}, P_p, Q_q and R_r have the same values as in Gupta and Varshney[3].

We apply Laplace transforms to (8) and (9) under the transform initial conditions $U = 0 = V$ at $t = 0$, we get

$$s\bar{U} = \frac{(-1)^{p+q+r}}{P_p Q_q R_r} \bar{f}(s) - b_{pqr}\bar{U} + (\bar{V} - \bar{U}) - M\bar{U}, \tag{10}$$

$$s\bar{V} = L(\bar{U} - \bar{V}), \tag{11}$$

where $\bar{U}$, $\bar{V}$ and $\bar{f}(s)$ are Laplace transforms of the respective quantities.

Now to obtain u and v in the case of constant pressure gradient, first solve (10) and (11) for $\bar{U}$ and $\bar{V}$ and then invert Laplace transforms by convolution theorem and after that applying inversion formulae for finite cosine transforms and then putting $f(t) = C$, where C is an absolute constant, and on simplifying, we get

$$u = \frac{16C}{3\sqrt{3}} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} \frac{(-1)^{p+q+r}}{P_p Q_q R_r (b_{pqr} + M)} \times \left[1 - \frac{1}{(\alpha_1 - \alpha_2)}\{(\alpha_1 + M + b_{pqr}) \exp(\alpha_2 t) - (\alpha_2 + M + b_{pqr}) \exp(\alpha_1 t)\}\right] \times \cos(p_p x_1) \cos(Q_q x_2) \cos(R_r x_3), \quad (12)$$

$$v = \frac{16C}{3\sqrt{3}} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} \frac{(-1)^{p+q+r}}{P_p Q_q R_r (b_{pqr} + M)} \times \left(1 - \frac{\alpha_1 \exp(\alpha_2 t) - \alpha_2 \exp(\alpha_1 t)}{\alpha_1 - \alpha_2}\right) \times \cos(P_p x_1) \cos(Q_q x_2) \cos(R_r x_3) \quad (13)$$

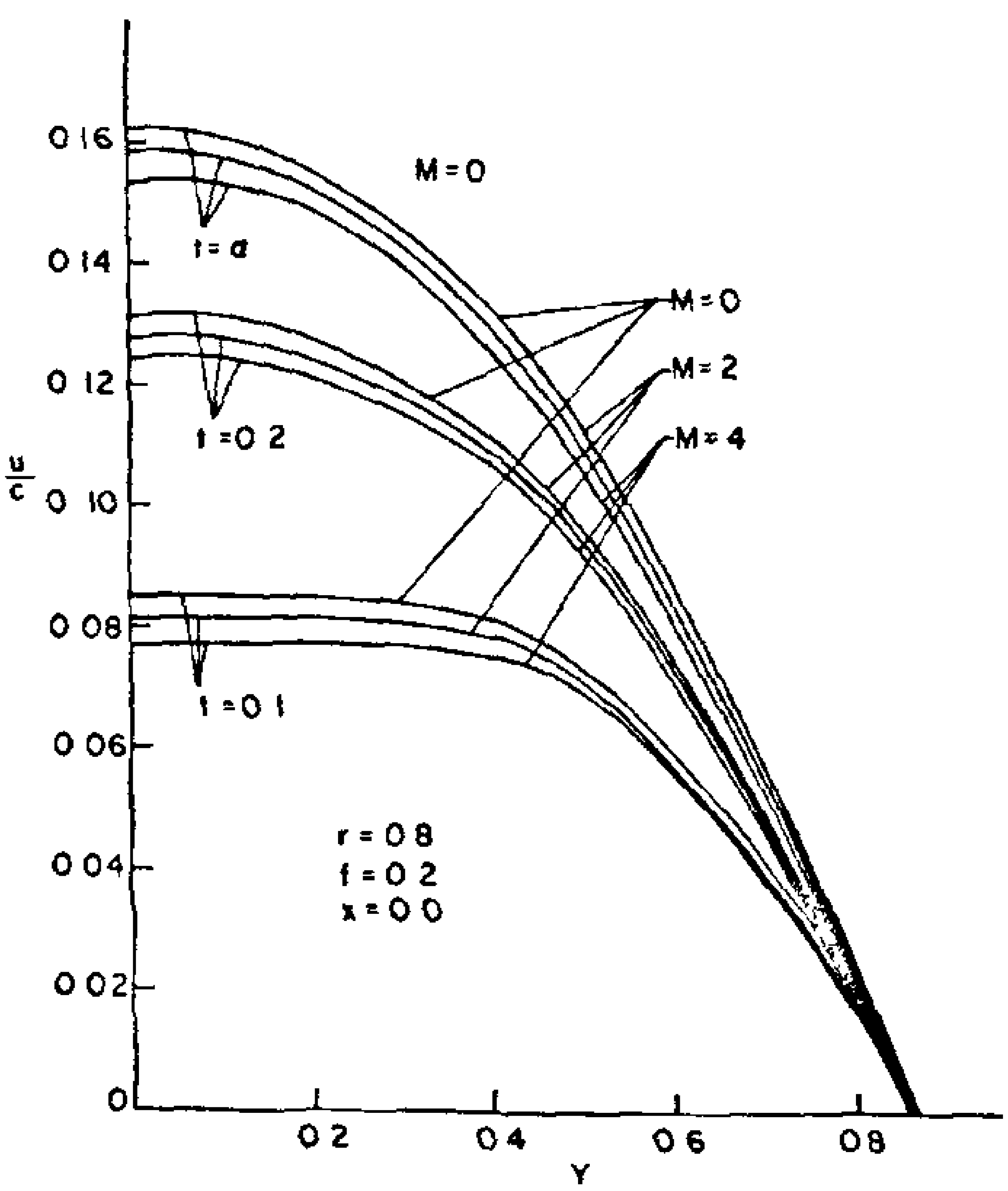

Figure 1. Velocity profiles of gas for constant pressure gradient C, for different values of M and T.

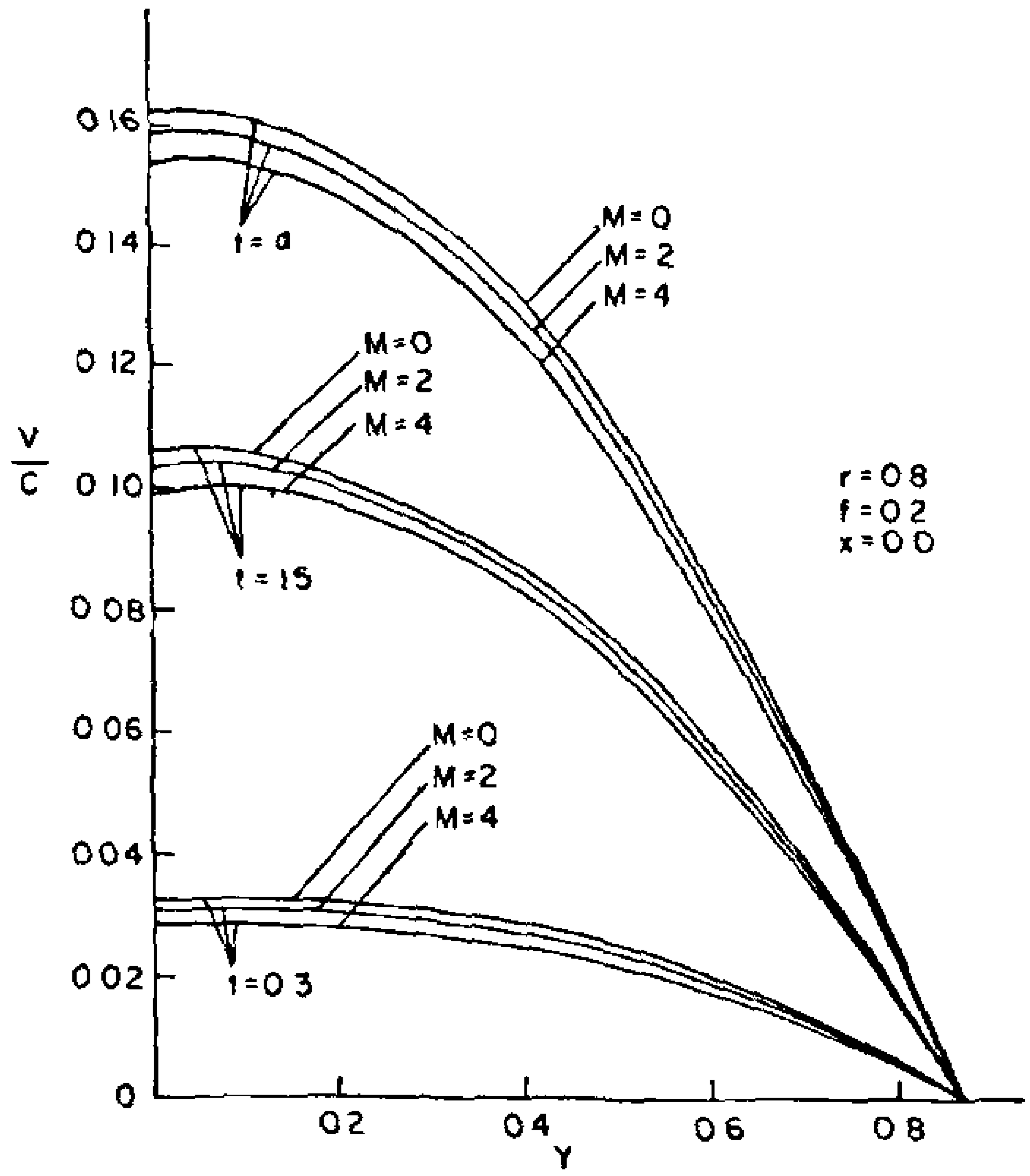

Figure 2. Velocity profiles of particles for a constant pressure gradient C, for different values of M and T.

Deduction

When magnetic field parameter $M \to 0$, the problem reduces to that considered by Gupta and Varshney[3].

Conclusion

We have plotted u and v for different values of t and M with $r = 0.8$ and $f = 0.2$ under constant pressure gradient, as shown in figures 1 and 2. It is clear that u and v decrease with the increase of M. Thus the implementation of a magnetic field is a device for laminar flow control, while u and v increase with the lapse of time and have a maximum on the axis of the channel. It is also evident from the figures that the gas moves faster than the particles.

Authors are highly thankful to the learned referee for his critical comments and suggestions.

31 March 1984; Revised 17 July 1984

1. Churchill, R. V., *Fourier series and boundary value problems*, McGraw Hill, New York, 1951.
2. Gupta, M. and Sharma, H. S., *Proc. ISTAM, 23rd Congress*, 1978, 278.

3. Gupta, M. and Varshney, O. P., *J. Engg. Appl. Sci.*, 1983 (in press).
4. Gupta, R. K. and Gupta, S. C., *ZAMP*, 1976, **27**, 1, 119.
5. Michael, D. H., *Proc. Camb. Philos. Soc.*, 1965, **61**, 569.
6. Michael, D. H. and Norey, P. W., *Q. J. Mech. Appl. Maths*, 1968, **21**, 375.
7. Saffman, P. G., *J. Fluid Mech.*, 1962, **13**, 120.

LIGHT SCATTERING FROM $(NH_4)_2ZnBr_4$ IN THE INCOMMENSURATE PHASE

A. G. K. MOORTHY*, P. CHANDRA SEKHAR and A. K. BHATNAGAR

Department of Physics, Indian Institute of Science, Bangalore 560012, India.

** School of Physics, University of Hyderabad, Hyderabad 500 134, India.*

$(NH_4)_2ZnBr_4$ belongs to the $(NH_4)_2ZnCl_4$[1] family of crystals which exhibit successive phase transitions and incommensurate phases[2]. X-ray structural studies[3] have shown that this crystal has an incommensurate phase in the range 395–432 K. Cs_2ZnCl_4, another member of the family, also shows a high temperature phase transition which is probably to an incommensurate phase[4]. This paper reports light scattering intensity and Raman spectroscopic studies of the high temperature incommensurate phase in $(NH_4)_2ZnBr_4$. Preliminary Raman spectroscopic studies, particularly in the low temperature ferroelectric phase[3], have been reported elsewhere[5,6].

Polycrystalline samples of $(NH_4)_2ZnBr_4$ were grown at room temperature (20–25°C) by slow evaporation of the aqueous solution of NH_4Br and $ZnBr_2$ in stoichiometric proportions. Unpolarised light scattering intensity and Raman spectroscopic measurements were made, using techniques described in an earlier communication[4].

Figure 1 shows the total intensity of light scattered at 90° from the sample as function of temperature. A very large increase in the vicinity of 400 K shows striking evidence of the incommensurate phase.

Figures 2 and 3 show the results of Raman spectroscopic studies on the same sample. The peak heights of three different Raman lines (figure 2), corresponding to internal modes of the $ZnBr_4^{2-}$ ion, versus temperature show a large increase in the region corresponding to the incommensurate phase and the integrated areas

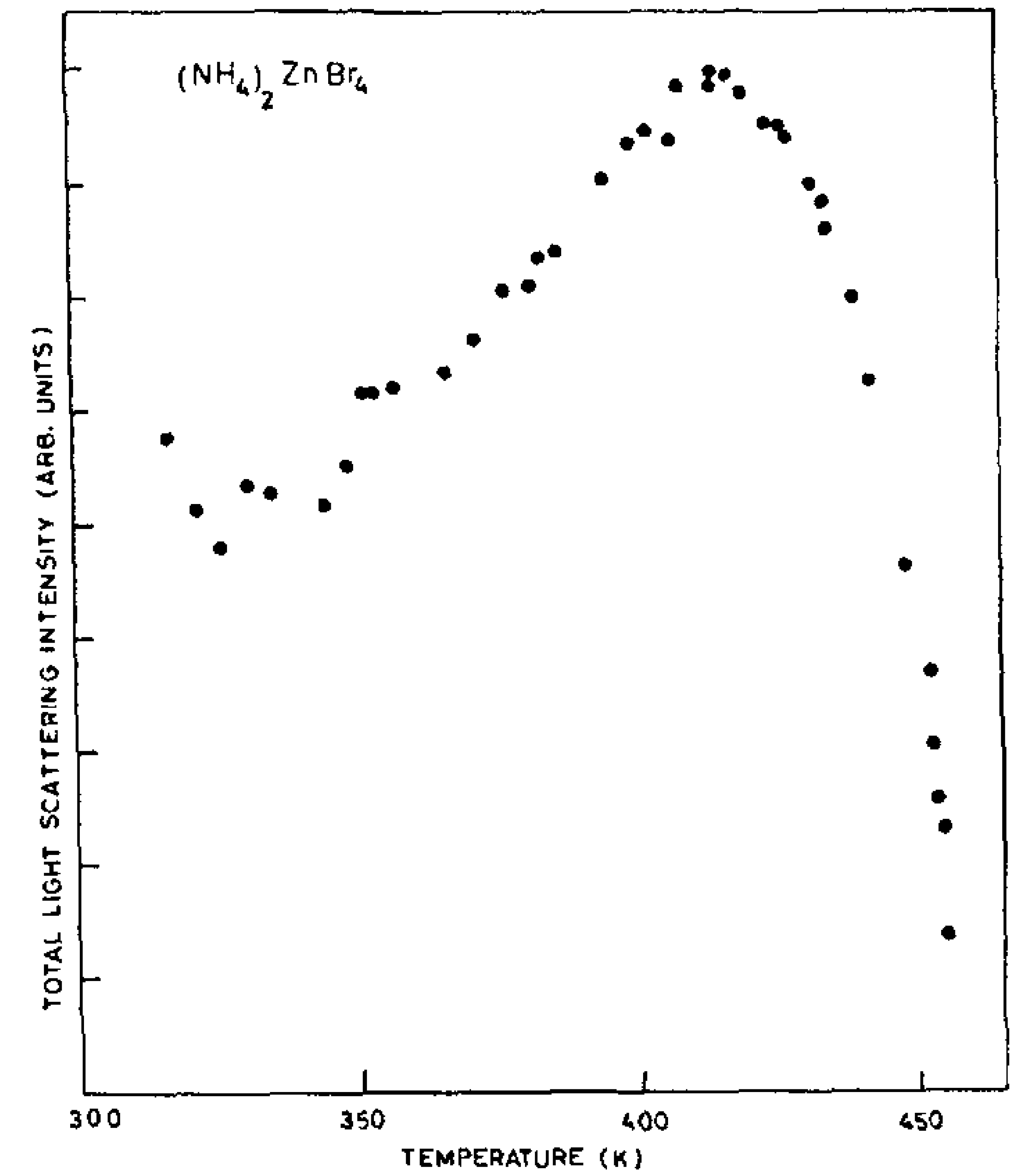

Figure 1. Total intensity of light scattering *vs* temperature

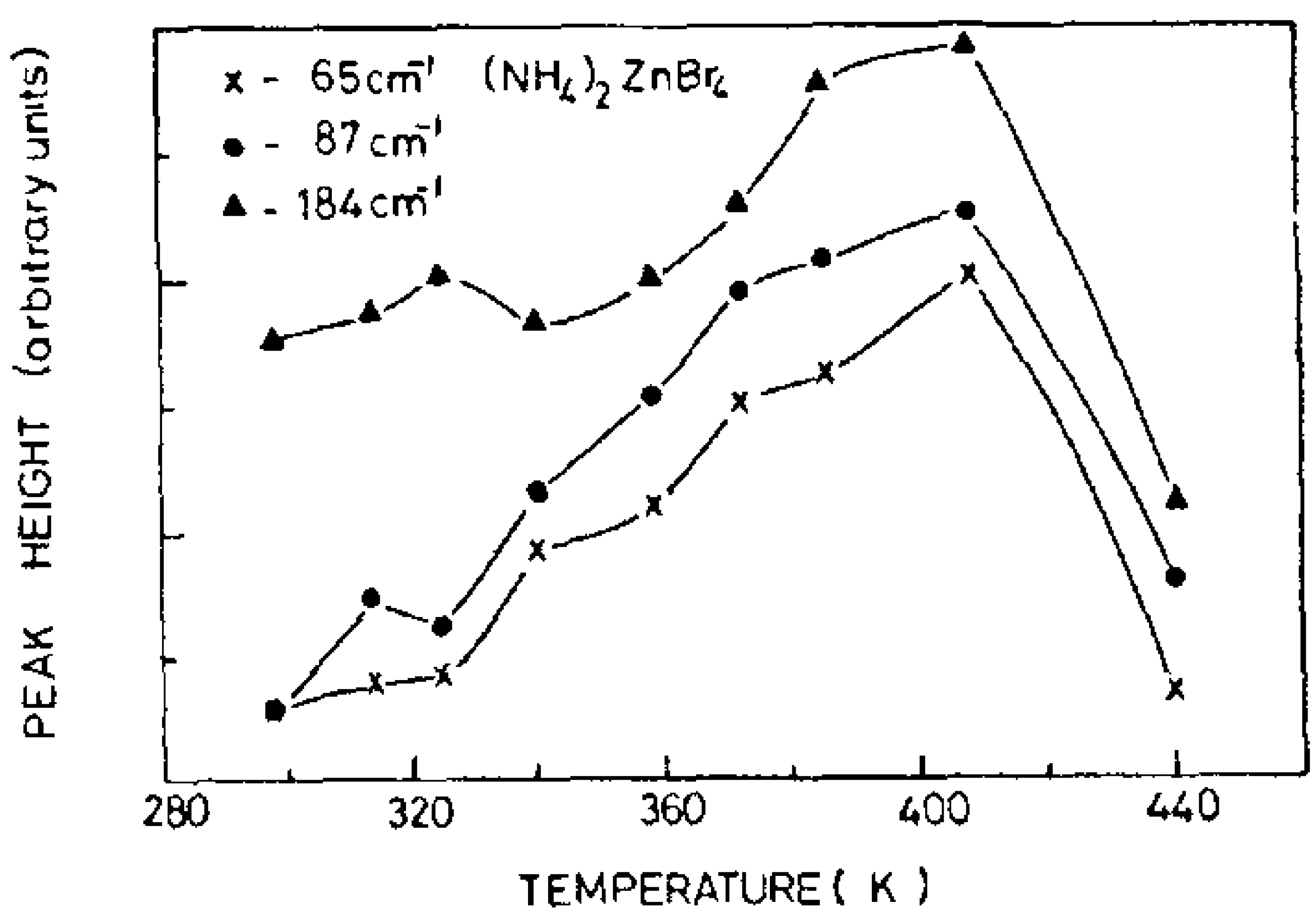

Figure 2. Peak height *vs* temperature

under two Raman bands (figure 3) show a similar behaviour.

The incommensurate phase is characterised by a complex order parameter[2,7] with amplitude fluctuations (amplitudons) and phase fluctuations (phasons). Amplitudons are generally observable by Raman spectroscopy, whereas phasons contribute to the Central Peak[7]. A soft amplitude mode was observed in