NONLINEAR FREE VIBRATIONS AND THERMAL BUCKLING OF POLYGONAL PLATES AT ELEVATED TEMPERATURE BY CONFORMAL TRANSFORMATION

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ABSTRACT

von Karman's coupled nonlinear equations extended to thermal loading for the dynamic case have been transformed into the complex domain and with the help of conformal mapping and Galerkin procedure the vibrational characteristics of different regular polygonal plates have been investigated for clamped immovable edges. Critical buckling temperatures for such plates have been deduced as limiting cases and compared with available results.

INTRODUCTION

PROBLEMS of nonlinear vibrations of heated elastic plates frequently occur in different engineering fields—particularly in aeronautics and high-speed space vehicles. Several authors[1-4] have investigated nonlinear vibration problems of specific plate-shapes. The present paper is devoted to the analysis of nonlinear free vibrations of regular polygonal plates of different shapes using the von Karman's coupled equations in the dynamical case under thermal loading and transformed into complex co-ordinates. Conformal mapping and Galerkin procedures have been adopted throughout the analysis.

BASIC GOVERNING EQUATIONS

With notations as in Chia[5] von Karman's dynamical equations including the thermal effect can be expressed as

$$D\nabla^4 W + \rho h W_{,tt} + \frac{\alpha_t E}{1-\nu}\nabla^2 M_T = $$
$$= W_{,xx}\phi_{,yy} - W_{,xy}\phi_{,xy} + W_{,yy}\phi_{,xx}, \quad (1)$$

$$\nabla^4\phi = Eh\left[W_{,xy}^2 - W_{,xx}W_{,yy}\right] - \alpha_t E\nabla^2 N_T. \quad (2)$$

TRANSFORMATION INTO COMPLEX CO-ORDINATES

The time variable is separated with the substitution $W = w(x, y)F(t)$ in (1) and (2) which are transformed into complex co-ordinates $(z, \bar{z})$ where $z = x + iy$, and then the domain is mapped onto a unit circle by the mapping function $z = f(\xi)$. The above two equations finally reduce to the forms

$$16\,D F(t)\left[\frac{\partial^4 w}{\partial\xi^2\partial\bar{\xi}^2}\frac{dz}{d\xi}\frac{dz}{d\bar{\xi}} - \frac{\partial^3\omega}{\partial\xi^2\partial\bar{\xi}}\cdot\frac{dz}{d\xi}\frac{d^2\bar{z}}{d\bar{\xi}^2} - \frac{\partial^3\omega}{\partial\xi\partial\bar{\xi}^2}\cdot\frac{d\bar{z}}{d\bar{\xi}}\frac{d^2 z}{d\xi^2} + \frac{\partial^2\omega}{\partial\xi\,\partial\bar{\xi}}\frac{d^2 z}{d\xi^2}\frac{d^2\bar{z}}{d\bar{\xi}^2}\right]$$
$$+ \rho h\omega(\xi,\bar{\xi})\left(\frac{dz}{d\xi}\frac{d\bar{z}}{d\bar{\xi}}\right)^3 \ddot{F}(t) = 4F(t)\left(2\frac{\partial^2\omega}{\partial\xi\,\partial\bar{\xi}}\frac{\partial^2\phi}{\partial\xi\,\partial\bar{\xi}} - \frac{\partial^2\omega}{\partial\xi^2}\frac{\partial^2\phi}{\partial\bar{\xi}^2} - \frac{\partial^2\omega}{\partial\bar{\xi}^2}\frac{\partial^2\phi}{\partial\xi^2}\right)\frac{dz}{d\xi}\frac{d\bar{z}}{d\bar{\xi}}$$
$$+\left(\frac{\partial^2\omega}{\partial\bar{\xi}^2}\frac{\partial\phi}{\partial\xi} + \frac{\partial^2\phi}{\partial\bar{\xi}^2}\frac{\partial\omega}{\partial\xi}\right)\frac{d^2 z}{d\xi^2}\frac{d\bar{z}}{d\bar{\xi}} + \left(\frac{\partial^2\omega}{\partial\xi^2}\frac{\partial\phi}{\partial\xi} + \frac{\partial^2\phi}{\partial\xi^2}\frac{\partial\omega}{\partial\bar{\xi}}\right)\frac{d^2\bar{z}}{d\bar{\xi}^2}\frac{dz}{d\xi}$$
$$-\left(\frac{\partial\omega}{\partial\xi}\frac{\partial\phi}{\partial\bar{\xi}} + \frac{\partial\omega}{\partial\bar{\xi}}\frac{\partial\phi}{\partial\xi}\right)\frac{d^2 z}{d\xi^2}\frac{d^2\bar{z}}{d\bar{\xi}^2} - \frac{\alpha_t E}{(1-\nu)}\frac{\partial^2 M_T}{\partial\xi\,\partial\bar{\xi}}\left(\frac{dz}{d\xi}\right)^2\left(\frac{d\bar{z}}{d\bar{\xi}}\right)^2 \quad (3)$$

$$4\left[\frac{\partial^4\phi}{\partial\xi^2\partial\bar{\xi}^2}\frac{dz}{d\xi}\frac{d\bar{z}}{d\bar{\xi}} - \frac{\partial^3\omega}{\partial\xi^2\partial\bar{\xi}}\frac{d^2 z}{d\xi^2}\frac{d\bar{z}}{d\bar{\xi}} - \frac{\partial^3\omega}{\partial\xi\,\partial\bar{\xi}^2}\frac{d^2\bar{z}}{d\bar{\xi}^2}\frac{dz}{d\xi} + \frac{\partial^2\omega}{\partial\xi\,\partial\bar{\xi}}\frac{d^2 z}{d\xi^2}\frac{d^2\bar{z}}{d\bar{\xi}^2}\right]$$
$$= EhF^2(t)\left[\left(\frac{\partial^2\omega}{\partial\xi^2}\frac{\partial^2\omega}{\partial\bar{\xi}^2} - \left(\frac{\partial^2\omega}{\partial\xi\,\partial\bar{\xi}}\right)^2\right)\frac{dz}{d\xi}\frac{d\bar{z}}{d\bar{\xi}} - \frac{\partial^2\omega}{\partial\xi^2}\frac{\partial\omega}{\partial\bar{\xi}}\frac{d^2\bar{z}}{d\bar{\xi}^2}\frac{dz}{d\xi} - \frac{\partial^2\omega}{\partial\bar{\xi}^2}\frac{\partial\omega}{\partial\xi}\frac{d^2 z}{d\xi^2}\frac{d\bar{z}}{d\xi}\right]$$
$$-\alpha_t E\frac{\partial^2 N_T}{\partial\xi\,\partial\bar{\xi}}\left(\frac{dz}{d\xi}\right)^2\left(\frac{d\bar{z}}{d\bar{\xi}}\right)^2 \quad (4)$$

FREE VIBRATIONS UNDER THERMAL LOADING

For free vibrations it is not exactly true that $M_{T=0}$; it is an assumption based on the neglect of temperature variation in depth due to compression – nor does it follow from Majumder *et al*[6] who considered $M_T = 0$. For free thermal vibrations the temperature field should be taken to depend on the radial co-ordinate as considered by Buckens[7] and Biswas[8–9] for vibrations of thermally-stressed plates.

METHOD OF SOLUTION AND BOUNDARY CONDITIONS

For plates with immovable edges and clamped along the boundary the appropriate form of $w(\xi, \bar{\xi})$ should be

$$w(\xi, \bar{\xi}) = w_0(1 - \xi\bar{\xi})^2, \; \xi = r \exp(i\theta) \tag{5}$$

Since N_T is constant and appears in the boundary condition for in-plane displacement we can take $\nabla^2 N_T = 0$, and considering only one term of the mapping function, namely $z = a\delta\xi$ where δ is the mapping function co-efficient and a is the characteristic dimension of the plate, the solution of (4) is expressed in the form

$$\phi = A\xi\bar{\xi} + Eh w_0^2 F^2(t)\left[-\frac{\xi\bar{\xi}}{4} + \frac{(\xi\bar{\xi})^3}{9} - \frac{(\xi\bar{\xi})^4}{48} \right], \tag{6}$$

in which A is a constant determined from the condition for inplane displacement for immovable edge of the plate in the form

$$A = \frac{Eh w_0^2 F^2(t)(5 - 3\nu)}{12(1 - \nu)} - \frac{E\alpha_t a^2 N_T}{2(1 - \nu)}. \tag{7}$$

Again retaining only one term of the mapping function and inserting the expressions for ϕ and A into (3) one gets the error function. Applying Galerkin procedure one gets

$$\frac{d^2F(t)}{dt^2} + C_1 F(t) + C_2 F^3(t) = 0 \tag{8}$$

where

$$C_1 = \frac{D}{a^4 \rho h \delta^4}\left[\frac{320}{3} - \frac{20}{3} \cdot \frac{\alpha_t E a^2 \delta^2 N_T}{(1 - \nu)D}\right] \tag{9}$$

$$C_2 = \frac{D}{a^4 \rho h \delta^4}(53.62)(w_0/h)^2. \tag{10}$$

The solution of (8) is given by Nash and Moderer[11] and one gets

$$T^*/T = \frac{2\Theta}{\pi}(1 + C_2/C_1)^{-1/2} \tag{11}$$

where

$$C_2/C_1 = 0.5026875\,(w_0/h)^2 \Big/ \left(1 - \frac{\delta^2 N_N^*}{16}\right) \tag{12}$$

$$N_T^* = \alpha_t E a^2 N_T / D(1 - \nu) \tag{13}$$

Table 1 *Critical buckling temperature for polygonal plates.*

Plate shape	Value of δ	Critical buckling temperature
Equilateral triangle	1.353	8.74
Square	1.08	13.42
Pentagon	1.0526	14.44
Hexagon	1.0376	14.86
Circle	1.000	16.00

Table 2 *Variations of non-dimensional time-periods for different values of non-dimensional amplitudes and thermal loading parameter.*

	w_0/h	0	0.5	1	1.5	2
T^*/T	Equilateral triangular plate $N_T^* = 0.5$	1	0.93935	0.8076	0.6742	0.5600
T^*/T	Square plate $N_T^* = 0.5$	1	0.9404	0.81065	0.6802	0.5648
T^*/T	Circular plate $N_T^* = 0.5$	1	0.9412	0.8124	0.6801	0.5703
T^*/T	Circular plate $N_T^* = 0$	1	0.9430	0.8161	0.6852	0.5763 [P.S.]
		1	0.9436	0.8165	0.68599	0.5773 [Ref 13]

P.S.—Present study.

CRITICAL BUCKLING TEMPERATURE, RESULTS AND DISCUSSION

For the pre-buckling state non-dimensional time-periods T^*/T can be obtained from (11) and (12) by taking the values of $(\delta^2/16)N_T^*$ sufficiently near to unity. Buckling occurs when $\delta^2 N_T^*/16$ equals to unity and the critical buckling temperature $(N_T^*)_{cr}$ for polygonal plates can be expressed as

$$(N_T^*)_{cr} = 16/\delta^2. \tag{14}$$

It is observed from table 1 that critical buckling temperature of polygonal plates increases as the number of sides increases.

Variations of non-dimensional time-periods T^*/T for variations of non-dimensional amplitudes (w_0/h) and temperature parameter N_T^* have been presented in table 2. It is observed that the values of T^*/T are less for plates with thermal effect than for those without thermal effect, *i.e.*, the effect of N_T^* is to diminish the relative time-periods. Moreover, the nature of the effect of N_T^* on the relative time-periods is similar to that of plates subjected to in-plane compressive stresses discussed by Biswas[12].

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ANNOUNCEMENT

JAWAHARLAL NEHRU VISITING PROFESSORSHIP OF THE CAMBRIDGE UNIVERSITY, UK

Professor S. Chandrasekhar, FRS, Raman Research Institute, Bangalore, has been invited to be the Jawaharlal Nehru Visiting Professor of Physics in the University of Cambridge for the academic year 1986–87. He will be joining the Cavendish Laboratory in October 1986.