It is clear that (3) involves $\hat{u}$ and $\hat{b}$ and (4) is function of only $\hat{b}$. Although in theory, it should be possible to find out the value of $\hat{b}$ from (4) but in practice, this is not so due to the involvement of summations and $\hat{b}$ as a power of an exponential function. Equation (4) can only be solved by iteration method. The first approximation for $\hat{b}$ was obtained by the method of moments and given as

$$\hat{b}_1 = \frac{S}{\left[\frac{\Gamma''(m)}{\Gamma(m)} - \left[\frac{\Gamma'(m)}{\Gamma(m)}\right]^2\right]^{1/2}}, \qquad (6)$$

where S is the standard deviation of the sample observations and Γ' and Γ'' are first and second derivatives of gamma function. The estimates for different value of m can be obtained from (3) and (4).

To make a comparative study the variances are calculated for $m = 1, 2, 3, ---, 30$. It has been observed that the efficiency, as indicated by variances, increased significantly: (n var$(\hat{u})/\hat{b}^2$ decreased from 1.108665 to 0.975751 and n var$(\hat{b})/\hat{b}^2$ decreased from 0.607927 to 0.558701), when instead of first maximum (minimum), 2nd maximum (minimum) order statistics are used to estimate the parameters of Gumbel distribution. However, when third maximum (minimum) order statistics are used to estimate the parameter of Gumbel distribution, the efficiency as indicated by variance, reduced significantly in case of location parameter and increased insignificantly in case of scale parameter; (n var$(\hat{u})/\hat{b}^2$ increased from 0.975751 to 1.185539 and n var$(\hat{b})/\hat{b}^2$ decreased from 0.558701 to 0.540112). If 4th maximum (minimum) order statistics are used instead of third, n var$(\hat{u})/\hat{b}^2$ increased from 1.185539 to 1.453204 and n var$(\hat{b})/\hat{b}^2$ decreased from 0.540112 to 0.530422. Similarly for other lower order statistics, n var$(\hat{u})/\hat{b}^2$ increases significantly but on the contrary the extent of reduction in n var$(\hat{b})/\hat{b}^2$ is insignificant. When the 30th minimum (maximum) order statistics are used for estimation, n var$(\hat{u})/\hat{b}^2$ and n var$(\hat{b})/\hat{b}^2$ were observed as 5.922497 and 0.504159 respectively which clearly indicated the rapid increase in the values of n var$(\hat{u})/\hat{b}^2$ and slow reduction in the values of n var$(\hat{b})/\hat{b}^2$. On the basis of variances it can be deduced that to obtain efficient maximum likelihood estimates of u and b the second maximum (minimum) can be fruitfully utilized. The correlation coefficients reach ± 1 as m increases. Here again, the change is maximum when second maximum (minimum) order statistics are used in place of first maximum (minimum) order statistics. Later on it reduces to zero.

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ON THE ESTIMATION OF MEAN USING SUPPLEMENTARY INFORMATION ON TWO AUXILIARY CHARACTERS IN DOUBLE SAMPLING

H. P. SINGH

Department of Mathematics and Statistics,
College of Agricultural Engineering,
Jawaharlal Nehru Krishi Vishwa Vidyalaya,
Jabalpur 482004, India.

CONSIDER finite population of size N (N is assumed large here). Let y_1 and y_2 denote the two auxiliary characters and let y_0 denote character under study. Now when two auxiliary characters are used in a survey there may be three possible sampling schemes[1-4].

1. A preliminary sample of size n' is selected by simple random sampling without replacement (SRSWOR) for observing the auxiliary characters y_1, y_2 and smaller subsample of size n is selected from the n' observations.
2. The preliminary sample is selected as in above but the smaller sample of size n is selected independently.
3. Many times the auxiliary information may be collected by two different agencies and hence two independent preliminary samples of sizes n'_1 and n'_2 are selected for observing y_1 and y_2 and the smaller sample of size n is also selected independently from the population by SRSWOR. To facilitate comparison we take $n'_1 = n'_2 = n'$.

Further, in what follows we shall use the following notations throughout the investigation:

$\bar{y}_i$: sample means for y_i ($i = 0, 1, 2$) based on sample of size n,
$\bar{y}'_1$ and $\bar{y}'_2$: sample means of y_1 and y_2 respectively based on the larger sample of size n',
$\bar{Y}_i$: population mean of y_i ($i = 0, 1, 2$),
σ_i: standard deviation of y_i ($i = 0, 1, 2$),
C_i ($= \sigma_i/\bar{Y}_i$): population coefficient of variation of y_i ($i = 0, 1, 2$),

$R_i = \bar{Y}_0/\bar{Y}_i$, $(i = 1, 2)$; $C_{0i} = \mathrm{cov}(y_0, y_i)/(\bar{Y}_0, \bar{Y}_i)$;
$i = 1, 2$; $C_{12} = \mathrm{cov}(y_1, y_2)/(\bar{Y}_1, \bar{Y}_2)$,
$\delta_0 = (C_1^2 C_2^2 - C_{12}^2)$; $\delta'_0 = (C_1^2 C_2^2 - f^2 C_{12}^2)$,
$\delta_1 = (C_{02} C_{12} - C_{01} C_2^2)$;
$\delta_2 = (C_{02}^2 C_1^2 - C_{01} C_{12})$; $\delta'_1 = (C_{01} C_2^2 - f C_{02} C_{12})$;
$\delta'_2 = (C_{02} C_1^2 - f C_{01} C_{12})$;
$\delta = [A_0 \delta_0 + \lambda (C_{01} \delta_1 - C_{02} \delta_2)]$,
$\delta' = [A_0 \delta_0 + n^{-1} f (C_{01} \delta_1 - C_{02} \delta_2)]$,
$\delta'' = [A_0 \delta'_0 + n^{-1} f (C_{01} \delta'_1 - C_{02} \delta'_2)]$,
$\lambda = (n^{-1} - n'^{-1})$, $\lambda^* = (n + n')/n'$,
$f = n'(n' + n)^{-1}$, $A_0 = (1 + n^{-1} C_0^2)$,

$$A_1 = \left(\sum_{i=1}^{2} W_i^2 R_i^2 C_i^2 + 2 W_1 W_2 R_1 R_2 C_{12} \right) \text{and}$$

$$A_2 = \sum_{i=1}^{2} W_i R_i C_{0i}.$$

THE ESTIMATOR

We propose the following class of estimators defined as

$$\bar{y}_d = W_0 \bar{y}_0 + W_1 (\bar{y}'_1 - \bar{y}_1) + W_2 (\bar{y}'_2 - \bar{y}_2), \quad (1)$$

where W_i $(i = 0, 1, 2)$ is a constant to be determined such that MES's of $\bar{y}_d$ is minimum, $\bar{y}_d$ reduces to Singh's[4] estimator

$$\bar{y}_s = \bar{y}_0 + W_1 (\bar{y}'_1 - \bar{y}_1) + W_2 (\bar{y}'_2 - \bar{y}_2) \quad (2)$$

for $W_0 = 1$.

The expressions for biases and MSE's of $\bar{y}_d$ are given in table 1, for the three different sampling schemes (stated above), which can be easily established.

The minimum MSE's along with optimum weights and biases under optimum conditions of $\bar{y}_d$ are presented in table 2.

Comparisons: For comparison we give the minimum MSE's along with optimum weights of estimator $\bar{y}_s$ proposed by Singh[4] in table 3.

To show the better performance of our estimator $\bar{y}_d$ over $\bar{y}_s$, we obtained the difference of minimum MSE's of $\bar{y}_d$ and $\bar{y}_s$. They are given in table 4 which clearly demonstrates that the proposed class of estimators $\bar{y}_d$ is more efficient than that of estimator envisaged by Singh[4] in all the three sampling schemes without using additional information.

Table 1 *Biases and MSE's of $\bar{y}_s$ under three different sampling schemes*

Sampling Schemes	Bias	Mean squared errors
1.	$B(\bar{y}_d) = (W_0 - 1)\bar{Y}_0$	$M(\bar{y}_d) = \bar{Y}_0^2 [A_0 W_0^2 + \lambda (A_1 - 2 W_0 A_2) - 2 W_0 + 1]$,
2.	$B(\bar{y}_d) = (W_0 - 1)\bar{Y}_0$	$M^*(\bar{y}_d) = \bar{Y}_0^2 [A_0 W_0^2 + n^{-1} (\lambda^* A_1 - 2 W_0 A_2) - 2 W_0 + 1]$,
3.	$B(\bar{y}_d) = \bar{Y}_0 (W_0 - 1)$	$M^{**}(\bar{y}_d) = \bar{Y}_0^2 \left[A_0 W_0^2 + n^{-1} \left(\lambda^* \sum_{i=1} W_i^2 R_i^2 C_i^2 - 2 W_0 A_2 + 2 W_1 W_2 R_1 R_2 C_{12} \right) - 2 W_0 + 1 \right]$

Table 2 *Minimum MSE's and biases of $\bar{y}_d$*

Sampling Schemes	Minimum MSE	Optimum weights	Biases
1.	$M_0(\bar{y}_d) = \bar{Y}_0^2 (\delta - \delta_0)/\delta$	$W_{00} = \delta_0/\delta$ $W_{10} = \delta_1/(R_1 \delta)$ $W_{20} = \delta_2/(R_2 \delta)$	$B_0(\bar{y}_d) = -\bar{Y}_0^{-1} M_0(\bar{y}_d)$
2.	$M_0^*(\bar{y}_d) = \bar{Y}_0^2 (\delta' - \delta_0)/\delta'$	$W_{00} = \delta_0/\delta'$ $W_{10} = f\delta_1/(R_1 \delta')$ $W_{20} = f\delta_2/(R_2 \delta')$	$B_0^*(\bar{y}_d) = -\bar{Y}_0^{-1} M_0^*(\bar{y}_d)$
3.	$M_0^{**}(\bar{y}_d) = \bar{Y}_0^2 (\delta'' - \delta'_0)/\delta''$	$W_{00} = \delta'_0/\delta''$ $W_{10} = f\delta'_1/(R_1 \delta'')$ $W_{20} = f\delta'_2/(R_2 \delta'')$	$B_0^{**}(\bar{y}_d) = -\bar{Y}_0^{-1} M_0^{**}(\bar{y}_d)$

Table 3 *Minimum* MSE*'s and optimum weights of* $\bar{y}_s$

Sampling Schemes	Minimum MSE's	Optimum weights
1.	$M_0(\bar{y}_s) = \bar{Y}_0^2(\delta - \delta_0)/\delta_0$	$W_{10} = R_1\delta_1/\delta_0, W_{20} = R_2\delta_2/\delta_0$
2.	$M_0^*(\bar{y}_s) = \bar{Y}_0^2(\delta' - \delta_0)/\delta_0$	$W_{10} = R_1\delta_1 f/\delta_0, W_{20} = R_2 f\delta_2/\delta_0$
3.	$M_0^{**}(\bar{y}_s) = \bar{Y}_0^2(\delta'' - \delta_0')/\delta_0'$	$W_{10} = R_1 f\delta_1'/\delta_0'$, $W_{20} = R_2 f\delta_2'/\delta_0'$,

Table 4 *Difference of minimum* MSE*'s of* $\bar{y}_d$ *and* $\bar{y}_s$

Sampling Schemes	Difference of Minimum MSE's
1.	$M_0(\bar{y}_s) - M_0(\bar{y}_d) = \bar{Y}_0^2(\delta - \delta_0)^2/(\delta\,\delta_0) > 0$
2.	$M_0^*(\bar{y}_s) - M_0^*(\bar{y}_d) = \bar{Y}_0^2(\delta' - \delta_0)^2/(\delta'\,\delta_0) > 0$
3.	$M_0^{**}(\bar{y}_s) - M_0^{**}(\bar{y}_d) = \bar{Y}_0^2(\delta'' - \delta_0')^2/(\delta''\,\delta_0') > 0$

Remarks

1. Following the procedure opted by Singh[4] one can easily obtain the conditions under which one sampling scheme provides better estimators than another one.
2. The results given for two auxiliary characters may also be generalized to p (> 2) auxiliary characters.

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SPASMOLYTIC AND ANTIOXYTOCIC COUMARINS FROM *HERACLEUM THOMSONI* (Linn)

KHALID A. KHAN, A. SHOEB and G. K. PATNAIK
Central Drug Research Institute,
Lucknow 226001, India.

As a sequel to a programme of research aimed at the pharmacological evaluation of Indian medicinal plants in this Institute[1], a 50% aqueous EtOH extract of the aerial parts of *H. thomsoni* exhibited spasmolytic and antioxytocic activities in test animals. Subsequent examination of the C_6H_6- (12 g) and EtOAc-soluble (8 g) fractions of the crude extract (169 g from 2 kg dried plant material) where the entire activity was lodged, yielded the following constituents whose characterization and pharmacological evaluation have been described in the present communication.

Chemical investigation: The C_6H_6-soluble fraction on column chromatography over silica gel afforded four constituents, A, B, C and D which were characterized by the study of their spectral data and confirmed by comparing with the respective authentic samples.

The compound A (0.2 g), m.p.138–40°, M^+ m/e 186 $C_{11}H_6O_3$, eluted from 50% C_6H_6 in hexane, was identified as angelicin[2]. Continued elution with the same solvent system afforded B (0.05 g), m.p.161–63°, M^+ m/e 186, $C_{11}H_6O_3$, was identified as psoralen[3]. Compounds C and D were eluted successively from 75% C_6H_6 in hexane. The compound C (0.02 g), m.p.108–9°, M^+ m/e 270, $C_{16}H_{14}O_4$, was characterized as heratomin[4], while the compound D (0.1 g), m.p.189–91°, M^+ m/e 216, $C_{12}H_8O_4$, was identified as sphondin[5].

The EtOAc-soluble fraction similarly yielded additional quantities of A (0.1 g), B (0.03 g), C (0.04 g) and D (0.10 g). Further elution of the column with 2% MeOH in $CHCl_3$ gave bergaptol[6] (0.015 g), m.p. 180–82°, M^+ m/e 202, $C_{11}H_6O_4$.

The *n* BuOH-soluble fraction (1.35 g) of the crude alcoholic extract yielded by droplet counter-current chromatography ($CHCl_3$: MeOH : H_2O, 7 : 13 : 8), apterin[7], m.p. 247–48°, M^+ m/e 424, $C_{20}H_{24}O_{10}$.

Pharmacological study: The plant materials were tested for spasmolytic activity on the isolated guinea pig ileum preparation[8] initially using a concentration of 50 μg/ml against contractions induced by a submaximal concentration of spasmogens, such as ac-