SHORT COMMUNICATIONS

NONLINEAR VIBRATIONS OF PARABOLIC PLATES AT ELEVATED TEMPERATURE

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STUDIES on nonlinear vibrations of thermally-stressed elastic plates are very few when compared with those without thermal effect. In aerospace engineering and in vibrations of machine parts, the problems have to be treated with nonlinear theory, when complementary stresses in the middle plane of the plate are taken into account. As a follow-up of an earlier paper[1] on nonlinear vibration analysis of triangular plates at elevated temperature, the present study is analyzed with Berger's approximation[2].

Governing equations:
Free thermal vibrations of heated elastic plates are governed by the following equations[1]

$$D\nabla^4 w + K^2\nabla^2 w + \rho h w_{,tt} = 0, \quad (1)$$

$$\frac{N_T}{1-\nu} - \frac{12De_1}{h^2} = K^2, \quad (2)$$

where

$$e_1 = u_{,x} + v_{,y} + \tfrac{1}{2}w_{,x}^2 + \tfrac{1}{2}w_{,y}^2 \quad (3)$$

$$N_T = \alpha_t E \int_{-h/2}^{+h/2} T(x,y,z)\,dz, \quad (4)$$

and u, v, w are displacement components, α_t the coefficient of thermal expansion, ρ the density per unit mass, ν the Poisson's ratio, E the Young's modulus and $T(x,y,z)$ is the temperature distribution within the plate given by[3]

$$T(x,y,z) = \tau_0(x,y) + z\tau(x,y), \quad (5)$$

in which $\tau_0(x,y)$ and $\tau(x,y)$ satisfy certain temperature distribution differential equations[3] and K^2 is independent of x and y but involves time t.

Method of solution for a parabolic plate

We consider a parabolic plate with boundary given by

$$x^2 = \frac{a}{2}(2a-y), \quad y = 0. \quad (6)$$

For this plate-shape clamped along the boundary the deflection w is expressed in the form

$$w = \frac{Ay^2}{a^6}\left[\frac{a}{2}(2a-y) - x^2\right]^2 F(t). \quad (7)$$

Combining (1) and (4) and applying Galerkin procedure one gets

$$\int_{y=0}^{2a}\int_{x=-[(a/2)(2a-y)]^{1/2}}^{[(a/2)(2a-y)]^{1/2}} \left[D\left\{\frac{24}{a^6}y^2 AF(t) + \frac{2A}{a^6}(24x^2 - 5a^2 + 12ay)F(t)\right\} + K^2(2a^4 - 6a^3y - a^2y^2 + 2ay^3 + 12x^2y^2 - 4a^2x^2 + 6ax^2y + 2x^4)\frac{AF(t)}{a^6} + \frac{A}{a^6}\rho h y^2\left\{\frac{a}{2}(2a-y) - x^2\right\}^2 \ddot{F}(t)\right] \times \left\{y^2\left[\frac{a}{2}(2a-y) - x^2\right]^2\right\} dx\,dy = 0. \quad (8)$$

Performing the necessary integrations we arrive at the equation

$$F(t)[6.694D - 2.0231a^2K^2] + 0.0288a^4\rho hA\ddot{F}(t) = 0, \quad (9)$$

in which K^2 is still unknown which is obtained by integrating (2) over the area of the plate leading to

$$\iint \frac{N_T}{1-\nu}dx\,dy - \frac{6D}{h^2}\iint\left\{(\partial\omega/\partial x)^2 + (\partial\omega/\partial y)^2\right\}dx\,dy = K^2\iint dx\,dy \quad (10)$$

with limits of integrations as in (8). Since u and v vanish on the boundary of the plate clamped along the immovable edges, (10) ultimately leads to

$$\frac{1}{a^2}\iint\frac{N_T}{(1-\nu)D}dx\,dy - \frac{7.13358\,A^2F^2(t)}{h^2} = 2.66K^2/D. \quad (11)$$

Eliminating K^2 with the help of (9) and (11) one gets the well-known cubic equation in the form

$$\ddot{F}(t) + C_1F(t) + C_2F^3(t) = 0, \quad (12)$$

where

$$C_1 = D(232{\cdot}2 - 26{\cdot}29\, N_T^*)/a^4 \rho h, \qquad (13)$$

$$C_2 = 187{\cdot}57\, D(A/h)^2/a^4 \rho h, \qquad (14)$$

$$N_T^* = \frac{1}{a^2}\iint \frac{N_T}{D(1-\nu)}\, dx\, dy. \qquad (15)$$

The solution of (12) with the initial conditions $F(O) = 1$, $dF(0)/dt = 0$ has been given by Nash and Modeer[4] in terms of Jacobian elliptic functions of cosine type and obtained the ratio of the time-periods for nonlinear and linear vibrations of elastic plates. In the present case such ratio is given by

$$T^*/T = \frac{2\Theta}{\pi}(1 + C_2/C_1)^{-1/2} \qquad (16)$$

in which T and T^* denote the time-periods for linear and nonlinear vibrations.

Numerical results and discussion: Variations of non-dimensional time-periods T^*/T for different values of non-dimensional amplitudes A/h and temperature parameter N_T^* have been computed and presented graphically. It is seen that the effect of increasing N_T^* is to diminish the relative time-periods. As expected, the nonlinear behaviour of plates due to elevated temperature, obtained here, is similar in nature to that of the plates subjected to in-plane compressive forces investigated by Biswas[5].

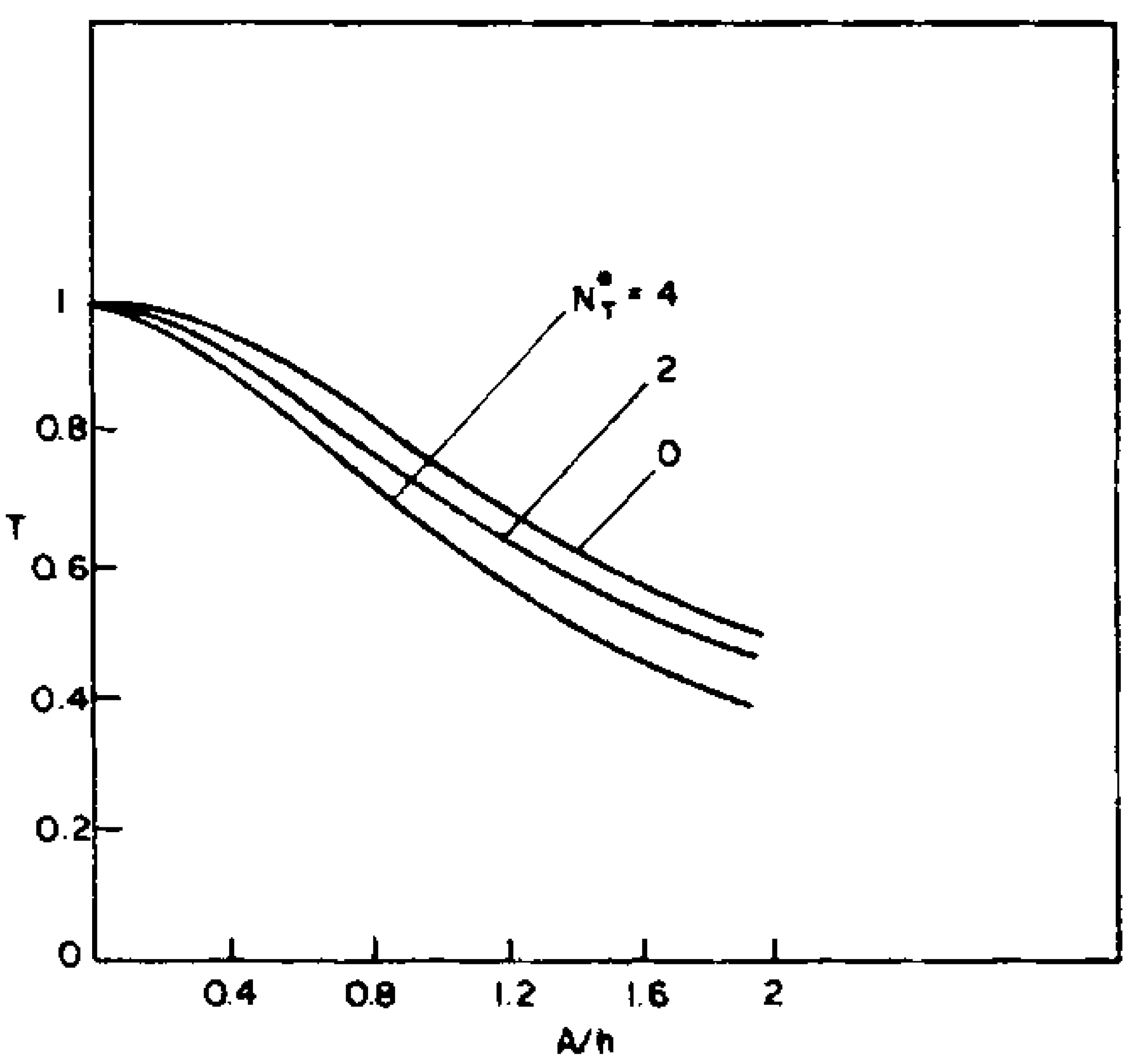

Figure 1. Variations of non-dimensional time-periods T^*/T for different values of non-dimensional amplitudes A/h and thermal loading parameter N_T^*.

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CORRECTIONS-TO-SCALING FROM LIGHT SCATTERING INTENSITY MEASUREMENTS IN A BINARY LIQUID MIXTURE

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ONE of the aims of light scattering studies near a critical point is to verify the universality hypothesis. Near the critical point, the susceptibility corresponding to the order-parameter fluctuations diverges as $t^{-\gamma}$ where $t = (T - T_c)/T_c$, is the reduced temperature, T_c being the critical temperature and γ a critical exponent. This exponent is obtained experimentally from the light scattering intensity measurements.

The true values of critical exponents are obtained by approaching very close to the critical point where $q\xi \gg 1$, q being the momentum transfer vector and ξ the correlation length. Away from T_c, in the hydrodynamic regime, where $q\xi < 1$, corrections-to-scaling terms may be present. Singular thermodynamic functions such as the order parameters, susceptibility etc are expected to be of the form

$$f - f_C = A|t|^{\lambda}(1 + B|t|^{\Delta} + \ldots),$$

where the first term represents the 'pure' scaling term and the second and higher order terms, corrections to scaling. For instance, in binary liquid mixtures, λ corresponds to β, the order parameter exponent obtained from coexistence-curve measurements.