SHORT COMMUNICATIONS

AXIALLY SYMMETRIC VACUUM SOLUTIONS OF THE BIMETRIC RELATIVITY THEORY

G. S. KHADEKAR and T. M. KARADE
Department of Mathematics, Nagpur University, Nagpur 440 010, India.

A new theory of gravitation proposed by Rosen[1] displays the attractive features of general relativity without singularity. The bimetric theory of gravitation is based on two metric tensors g_{ij} (a Riemann tensor describing the gravitational field) and γ_{ij} (a tensor of flat space–time describing inertial forces).

The field equations of bimetric relativity are

$$K_{ij} = -8\pi k T_{ij}, \tag{1}$$

where $K_{ij} = N_{ij} - (1/2)N g_{ij}$, and

$$N^i_j = (1/2)\gamma^{\alpha\beta}(g^{hi} g_{hj}/\alpha)/\beta, \tag{2}$$

where the (/) stands for covariant differentiation with respect to γ_{ij}, $k = (g/\gamma)^{1/2}$, $g = \det(g_{ij})$, $\gamma = \det(\gamma_{ij})$ and T_{ij} is the energy momentum tensor. In our earlier work[2,3] we observed that Rosen bimetric theory does not admit a plane-symmetric massive scalar field, an axially symmetric complex field as well as complex scalar field coupled with electromagnetic field. Moreover the Maxwell tensor vanishes in all cases contributing nothing to the gravitational field, resulting in vacuum solution. An attempt was made to derive non-static axially symmetric vacuum solutions exhibiting cylindrical waves. In this note we look for solutions of bimetric relativity with a cosmological constant, that is

$$N^i_j = \lambda \delta^i_j, \tag{3}$$

similar to the Einstein field equations $R_{ij} = \lambda g_{ij}$ of general relativity, where λ corresponds to a cosmological term. We choose the Einstein–Rosen non-static metric

$$ds^2 = e^{2\alpha - 2\beta}(dT^2 - dR^2) - R^2 e^{-2\beta} d\phi^2 - e^{2\beta} dZ^2, \tag{4}$$

where α and β are functions of R and T only and $x^1 = R$, $X^2 = \phi$, $X^3 = Z$, $X^4 = T$. Now the flat space-time corresponding to (4) will be

$$d\sigma^2 = dT^2 - dR^2 - R^2 d\phi^2 - dZ^2. \tag{5}$$

The non-vanishing γ-Christoffel symbols for metric (4) are

$$\Gamma^2_{12} = \Gamma^2_{21} = 1/R \text{ and } \Gamma^1_{22} = -R.$$

Now the field equations in (3) yield

$$\lambda = -(\alpha'' - \beta'') - (1/R)(\alpha' - \beta') + (\alpha^{\cdot\cdot} - \beta^{\cdot\cdot}) + (1/R^2)\sinh(2\alpha), \tag{6}$$

$$\lambda = \beta'' + \beta'/R - \beta^{\cdot\cdot} - (1/R^2)\sinh(2\alpha), \tag{7}$$

$$\lambda = -(\beta'' + \beta'/R - \beta^{\cdot\cdot}), \tag{8}$$

$$\lambda = -(\alpha'' - \beta'') - (1/R)(\alpha' - \beta') + (\alpha^{\cdot\cdot} - \beta^{\cdot\cdot}), \tag{9}$$

where $\alpha' = \partial\alpha/\partial R$, $\alpha^{\cdot} = \partial\alpha/\partial T$, etc.

These equations imply

$$\alpha = 0 \tag{10}$$

$$\beta'' + \beta'/R - \beta^{\cdot\cdot} - \lambda = 0. \tag{11}$$

Case (1): Let $\beta = H(r, t) + g(t)$ then (11) gives the Bessel equation

$$H'' + H'/R - H^{\cdot\cdot} = 0, \tag{12}$$

representing cylindrical waves,

$$\text{provided} \quad g^{\cdot\cdot} = \lambda. \tag{13}$$

The Bessel equation (12) results in

$$H = a j_0(hR)\cos(ht + e_1) + bN_0(hR)\sin(ht + e_2). \tag{14}$$

From (13) we get

$g = (\lambda/2)t^2 + c_1 t + c_2$, where c_1 and c_2 are arbitrary constants of integration. Then

$$\beta = a j_0(hR)\cos(ht + e_1) + bN_0(hR)\sin(ht + e_2) + (\lambda/2)t^2 + c_1 t + c_2, \tag{16}$$

where $j_0(hR)$ and $N_0(hR)$ are the Bessel functions of the first and second kind of zero order respectively, h the frequency and a, b, e_1, e_2 are arbitrary constants.

Case (2): Let $\beta = H(r, t) + f(r)$
In this case we get (12)

$$\text{provided} \quad f'' + f'/R = \lambda. \tag{17}$$

The solution of (17) is

$$f = \lambda R^2/4 + c_3 \log R + c_4,$$

where c_3 and c_4 are arbitrary constants of integration. Now the solution turns out to be

$$\beta = aj_0(hR)\cos(ht+e_1) + bN_0(hR)\sin(ht+e_2) + \lambda R^2/4 + c_3 \log R + c_4. \quad (18)$$

For Marder's metric,

$$ds^2 = e^{2\alpha-2\beta}(dT^2 - dR^2) - R^2 e^{-2\beta} d\phi^2 - (e^{2\beta} - e^{2\gamma}) dZ^2, \quad (19)$$

the vacuum field equations yield

$$\alpha = 0$$
$$\beta'' + \beta'/R - \beta^{\cdot\cdot} = \lambda$$
$$\gamma'' + \gamma'/R - \gamma^{\cdot\cdot} = \lambda.$$

The form of γ is the same as that of β and hence we take $\gamma = n\beta$, n = constant etc. The solutions of (19) will be as in (16) and (18).

13 June 1988

1. Rosen, N., *Ann. Phys. N.Y.*, 1974, **84**, 455.
2. Karade, T. M. and Dhoble, Y. S., *Curr. Sci.*, 1979, **48**, 675.
3. Karade, T. M. and Dhoble, Y. S., *Acta Phys.*, 1979, **47**, 357.

^{13}C NMR AND NEW KINETIC EVIDENCE FOR STERIC ENHANCEMENT OF RESONANCE

V. BALIAH*, B. THEYMOLI and A. MANGALAMUDAIYAR
Department of Chemistry, Annamalai University, Annamalainagar 608 002, India.
**Present address: 79 3rd Cross, Venkatanagar, Pondicherry 605 011, India.*

AFTER the discovery[1] that there is steric enhancement of resonance in *ortho*-substituted 4-nitroanisoles and similar compounds, more evidence was found[2-11] for the phenomenon. In this communication we present further evidence for it.

The second-order rate constants for the reaction between some substituted anilines and 2,4-dinitrochlorobenzene are given in table 1. This is a reaction in which extra electron displacements accompany activation in consequence of electrical demands of the reaction centre. Thus electron-releasing groups, present *para* or *meta* to the amino group, increase the rate. The 4-methoxy group increases the rate by about ten times and the 3-methyl group by 1.3 times. When a methyl group is present *ortho* to methoxyl, the observed rate is 17.1×10^{-4}, l mol^{-1} s^{-1}, while the rate calculated on the basis of additivity of group effects[12] shows an increase of about 13%. The increased rate is due to the enhanced resonance of the methoxy group. When the methoxyl orients away from the *ortho* methyl, its rotation gets restricted and its chances of becoming coplanar with the benzene ring increase, enhancing its resonance.

Table 1 *Second-order rate constants for the reaction between substituted anilines and 2,4-dinitrochlorobenzene at 30°C*

Substituent(s)	10^4 k$_2$ (l mol^{-1} s^{-1}) Obs.	Calc.
H	1.10	–
4-Methoxy	11.4	–
3-Methyl	1.46	–
4-Methoxy-3-methyl	17.1	15.1
4-Methoxy-3,5-dimethyl	5.75	20.1

The observed rate of 4-methoxy-3,5-dimethylaniline is much less than the calculated rate, indicating the expected steric inhibition of resonance.

Proof for the existence of steric enhancement of resonance may also be seen from the ^{13}C NMR spectra of the substituted 4-nitroanisoles listed in table 2. A 2-methyl substitution in 4-nitroanisole increases the electron density at C-4, compared with 4-nitroanisole, as seen from the upfield shift and $\Delta\delta$ value, reflecting steric enhancement of resonance. For 2,6-dimethyl-4-nitroanisole there is a downfield shift, indicating decreased electron density and the expected steric inhibition of resonance.

The compounds available commercially were purified rigorously before use and others were

Table 2 *Chemical shifts of the benzene ring for substituted anisoles*

Anisole	Chemical shift of C-4 (ppm) Obs.	Calc.*	$\Delta\delta$
4-Nitro	141.5	140.8	+0.7
2-Methyl-4-nitro	140.6	140.7	−0.1
2,6-Dimethyl-4-nitro	143.2	140.6	+2.6

*Calculation is based on δ_c for benzene, 128.5, and substituent effects from ref. 15.

*For correspondence.